Analisi Matematica 1- Lista n. 12

Esercizi di riepilogo sui limiti di funzioni

Titolo nota

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Calcolare, SENZA usare gli sviluppi di Taylor, i seguenti limiti:

1) $\displaystyle\lim_{x\to+\infty} x \sin\left(\ln\left(\frac{x+1}{x+2}\right)\right) = -1$

2) $\displaystyle\lim_{x\to+\infty} \frac{\frac{\pi}{2}-\arctan x}{\ln\left(\frac{x+1}{x}\right)} = 1$

3) $\displaystyle\lim_{x\to0^-} \frac{\pi+2\arctan\frac{1}{x}}{e^x-\sqrt{1+x}} = -4$

4) $\displaystyle\lim_{x\to+\infty} x\cdot\left(\sqrt[3]{1+\sin\frac{1}{x}}-\cos\frac{1}{x}\right) = \frac{1}{3}$

5) $\displaystyle\lim_{x\to0} \left(1+\arctan 2x\right)^{\frac{3+\sin x}{x}} = e^6$

6) $\displaystyle\lim_{x\to0} \left(1+|x|\right)^{\frac{1}{x}}$ $\boxed{\text{NON ESISTE}}$

7) $\displaystyle\lim_{x\to0} \left(\frac{1}{\cos x}\right)^{\frac{1}{x^2}} = \sqrt{e}$

8) $\displaystyle\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x} = 1$

9) $\displaystyle\lim_{x\to\frac{\pi}{2}^-} (\tan x)^{\cos x} = 1$

10) $\displaystyle\lim_{x\to e} (\ln x)^{\ln|x-e|} = 1$

11) $\displaystyle\lim_{x\to e} (\ln x)^{\frac{1}{\ln^3(\ln x)}} = +\infty$

12) $\displaystyle\lim_{x\to e} (\ln x)^{\frac{1}{\ln^4(\ln x)}}$ $\boxed{\text{NON ESISTE}}$

13) $\displaystyle\lim_{x\to0^+} x^{-\frac{1}{x}} = +\infty$

14) $\displaystyle\lim_{x\to+\infty} x^{\frac{1}{x}} = 1$

15) $\displaystyle\lim_{x\to+\infty} |\sin x|^x$ $\boxed{\text{NON ESISTE}}$

16) $\displaystyle\lim_{x\to+\infty} |\sin(\sin x)|^x = 0$

17) $\displaystyle\lim_{x\to0^+} \left(1+\sin\frac{1}{x}\right)^{\sin x}$ $\boxed{\text{NON ESISTE}}$

18) $\displaystyle\lim_{x\to+\infty} \left(1+\sin\frac{1}{x}\right)^{\sin x} = 1$

19) $\displaystyle\lim_{x\to0^+} \frac{x^\alpha}{e^x-\sin\frac{1}{x}}$ (al variare del parametro $\alpha>0$)
- $\alpha>1 \longrightarrow = 0$
- $0<\alpha\leq1 \longrightarrow$ $\boxed{\text{NON ESISTE}}$

20) $\displaystyle\lim_{x\to0^+} \frac{\alpha+\sin\frac{1}{x}+\cos\frac{1}{x}}{x}$ (al variare del parametro $\alpha>0$)
- $\alpha>\sqrt{2} \longrightarrow = +\infty$
- $0<\alpha\leq\sqrt{2} \longrightarrow$ $\boxed{\text{NON ESISTE}}$

21) $\displaystyle\lim_{x\to+\infty} \frac{\lfloor x\rfloor}{x} = 1$

22) $\displaystyle\lim_{x\to0^+} \left\lfloor\frac{1}{x}\right\rfloor\cdot\sin x = 1$

23) $\displaystyle\lim_{x\to+\infty} \frac{\sqrt{\lfloor x\rfloor}-\lfloor\sqrt{x}\rfloor}{\ln(\ln x)} = 0$

24) $\displaystyle\lim_{x\to+\infty} \frac{\lfloor x^2\rfloor-(\lfloor x\rfloor)^2}{x^\alpha}$ (al variare del parametro $\alpha>0$)
- $\alpha>1 \longrightarrow = 0$
- $0<\alpha\leq1 \longrightarrow$ $\boxed{\text{NON ESISTE}}$

Calcolare, <u>SENZA</u> usare gli sviluppi di Taylor e <u>SENZA</u> usare la formula di Stirling, i seguenti limiti di successioni:

25) $\displaystyle\lim_{n\to+\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} = 1$

26) $\displaystyle\lim_{n\to+\infty} \frac{\sin n}{n} = 0$

27) $\displaystyle\lim_{n\to+\infty} \ln\left(1+n^n\right)\cdot\ln\left(1+e^{-n}\right) = 0$

28) $\displaystyle\lim_{n\to+\infty} \frac{\sqrt[n]{\cos\frac{1}{n}}-1}{\tan\frac{1}{n}-\sin\frac{1}{n}} = -1$

29) $\displaystyle\lim_{n\to+\infty} n^2\cdot\left(\sqrt[n^2]{e}-\sqrt[3]{\frac{n^2-1}{n^2}}+\ln\left(\cos\frac{1}{n}\right)\right) = \frac{5}{6}$

30) $\displaystyle\lim_{n\to+\infty} n\left(\sqrt[n]{3}-\sqrt[n]{2}\right) = \ln\frac{3}{2}$

31) $\displaystyle\lim_{n\to+\infty} \frac{\ln\left(1+\frac{1}{(2n)!}\right)\cdot\ln\left(1+e^{2^n}\right)}{\sin\left(\frac{1}{n^{2n}}\right)\cdot\cos\left(\pi\cdot n!\right)} = +\infty$

32) $\displaystyle\lim_{n\to+\infty} \frac{\sqrt[n]{(n+1)!}-\sqrt[n]{n!}}{\sqrt{n}} = 0$

33) $\displaystyle\lim_{n\to+\infty} n^6\cdot\left(\tan\frac{1}{n^2}-\sin\frac{1}{n^2}\right) = \frac{1}{2}$

34) $\displaystyle\lim_{n\to+\infty} n^3\cdot\left(\tan\frac{1}{n^2}-\left(\sin\frac{1}{n}\right)^2\right) = 0$

35) $\displaystyle\lim_{n\to+\infty} \left(\cos\frac{1}{n!}-\frac{2}{\pi}\arctan n^{\alpha n}\right)\cdot n^{\alpha n}$ (al variare del parametro $\alpha>0$) $\xrightarrow{\alpha\geq 2} = -\infty$; $\xrightarrow[0<\alpha<2]{} = \frac{2}{\pi}$

Nei casi seguenti dire se ciascuna funzione è o-piccolo, O-grande, dello stesso ordine e/o asintoticamente equivalente alle altre, per x che tende al valore a fianco indicato:

36) $f(x)=x^x-e^x$ $\qquad g(x)=x^{x^2}-e^x$ $\qquad h(x)=x^x-e^{x^2}$ $\qquad \boxed{\text{per } x\to0^+}$ $\qquad f(x)\approx h(x) \quad g(x)=o(f(x))$

37) $f(x)=e^{x^5}+(x^2)^{x^2}$ $\qquad g(x)=e^{(\ln x^2)^{\ln x^2}}$ $\qquad h(x)=e^{x^4}$ $\qquad \boxed{\text{per } x\to-\infty}$ $\qquad f(x)=o(h(x)) \quad h(x)=o(g(x))$

38) $f(x)=\left(1+\frac{1}{x}\right)^{x^2}$ $\qquad g(x)=(1+x)^x\left(\frac{\pi}{2}-\arctan x^2\right)$ $\qquad h(x)=(100x)^{\frac{x}{\ln x}}$ $\qquad \boxed{\text{per } x\to+\infty}$ $\qquad f(x)=o(h(x)) \quad h(x)=o(g(x))$

39) $f(x)=(\cos x)^{\sin x}-1$ $\qquad g(x)=(\tan x)^{\sin x}-(\sin x)^{\tan x}$ $\qquad h(x)=\tan(\tan x)-\sin(\sin x)$ $\qquad \boxed{\text{per } x\to0^+}$ $\qquad f(x)\approx-\frac{1}{2}h(x) \quad h(x)=o(g(x))$

40) $f(x)=\sqrt[3]{\frac{8x^2+\ln x}{x^2+1}}-2$ $\qquad g(x)=\ln(1+x^x)+\frac{(x-x^3)\ln x}{x^2+1}$ $\qquad h(x)=\frac{\left(x+\frac{1}{x}\right)^{\ln x}-x^{\ln x}}{x^{100}+x^{\ln x}}$ $\qquad \boxed{\text{per } x\to+\infty}$ $\qquad f(x)\approx\frac{1}{12}h(x) \quad h(x)=o(g(x))$