Analisi Matematica 1- Lista n. 15

Limiti da fare con gli sviluppi di Taylor

Titolo nota

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Calcolare i seguenti limiti:

1) $$\lim_{x\to 0} \frac{\ln(1+x^2) - x\arctan x}{\sin^2(x^2)} = -\frac{1}{6}$$

2) $$\lim_{x\to 0} \frac{\ln\left(1+\frac{x}{2}\right) - \sqrt{1+x} + 1}{2\cdot(\tan x - \sin x)} = -\frac{1}{48}$$

3) $$\lim_{x\to 0^+} \frac{\sqrt{1+2x} - \sqrt{x}\arctan\sqrt{x} - 1}{\tan(x^2)} = -\frac{1}{6}$$

4) $$\lim_{x\to 0} \frac{3\sin x - 3x\cos x}{\ln(1+3x) - 3\ln(1+x) + 3x^2} = \frac{1}{8}$$

5) $$\lim_{x\to 0} \frac{(x-\arctan x)\left(\ln(1+x^2) - x\cdot\arctan x\right) + \frac{x^7}{18}}{x^3 - \sin x^3} = \frac{7}{15}$$

6) $$\lim_{x\to 0} \frac{\left(e^{2x} - 1 - \sin 2x\right)\left(3xe^x - \arctan(3x)\right) - 6x^4 - 29x^5}{x^6} = 31$$

7) $$\lim_{x\to 0} \frac{\left(e^x - 1 - \ln(1+x)\right)\left(3xe^x - \sin(3x)\right) - 3x^4 - \frac{11}{2}x^5}{x^6} = \frac{3}{8}$$

8) $$\lim_{x\to 0} \frac{3\sin x - 3x\cos x}{\frac{1}{1-x} - e^x - \frac{x^2}{2}} = \frac{6}{5}$$

9) $$\lim_{x\to 0} \frac{3\sin x - \sqrt{3}\sin(x\sqrt{3})}{\arctan x - \arctan 2x + x} = \frac{3}{7}$$

10) $$\lim_{x\to 0} \frac{2x^5 + x^7\cos\frac{1}{x}}{\sin x\cdot\cos x + \frac{2}{3}x^3 - x} = 15$$

11) $$\lim_{x\to 0} \frac{\cos x - 1 + e^{-\frac{1}{x^2}}}{\cos\sqrt{x} - \sqrt{1-x+x^2}} = \frac{3}{2}$$

12) $$\lim_{x\to 0} \frac{x^8 - (x+x^2)^5}{x\sqrt{1+2x^2} + \cos(x^3+x^4) - x - \sqrt{1+2x^3}} = 2$$

13) $$\lim_{x\to 0} \frac{\sin x - x\cos\left(-\frac{x}{\sqrt{3}}\right)}{-\tan x^5} = -\frac{1}{270}$$

14) $$\lim_{x\to 0} \frac{\left(\cos x - \frac{1}{1-x^2}\right)\arctan x + \frac{3}{2}x^3}{2\sqrt{1-x^5}-2} = \frac{11}{24}$$

15) $$\lim_{x\to 0} \frac{(x\cos x - \sin x)(x^2 - \sin x^2) + \frac{1}{18}x^9}{x^{11}} = \frac{1}{180}$$

16) $$\lim_{x\to 0} \frac{(\arctan x - x\cos x)\left(\sqrt{1+x^4}-1\right) - \frac{1}{12}x^7}{x^9} = \frac{19}{240}$$

17) $$\lim_{x\to +\infty} \frac{x^4 \sin\frac{1}{x^3} - 2x^3\left(1-\cos\frac{1}{x}\right)}{x e^{\frac{1}{x^2}} - x - \ln(x+1) + \ln x} = +\infty$$

18) $$\lim_{x\to +\infty} \left(\frac{x^3}{x+2} e^{\frac{x}{x^2+2}} - x^2 + x\right) = \frac{5}{2}$$

19) $$\lim_{x\to \frac{\pi}{6}} \left(e^{\sin x + \frac{1}{2}} - \left(\frac{6}{\pi}x\right)^{\frac{\pi}{4\sqrt{3}}}\cdot e\right)\tan^2(3x) = \frac{\sqrt{3}}{6\pi} - \frac{e}{36}$$

20) $$\lim_{x\to 0} \frac{\ln(\cos 2x) + 2x^2 + \frac{4}{3}x^4}{x^6} = -\frac{64}{45}$$

21) $$\lim_{x\to +\infty} \left(\arctan x - \frac{\pi}{2} + \frac{1}{x} - \frac{1}{3x^3}\right)\cdot x^\alpha = \begin{cases} 0 & \text{SE } \alpha<5 \\ -\frac{1}{5} & \text{SE } \alpha=5 \\ -\infty & \text{SE } \alpha>5 \end{cases}$$

22) $$\lim_{x\to 0^+} \frac{e^{-\frac{1}{x^2}} + 2 - \cos(x-\sin x) - \sqrt[12]{1+x^6}}{x^\alpha} = \begin{cases} 0 & \text{SE } \alpha<8 \\ -\frac{1}{720} & \text{SE } \alpha=8 \\ -\infty & \text{SE } \alpha>8 \end{cases}$$

23) $$\lim_{x\to 0^+} \frac{\sin(\arctan x) - \arctan(\sin x)}{x^\alpha} = \begin{cases} 0 & \text{SE } \alpha<7 \\ \frac{1}{30} & \text{SE } \alpha=7 \\ +\infty & \text{SE } \alpha>7 \end{cases}$$

24) $$\lim_{x\to 0^+} \frac{\sin(\sin x) - 2\sin x + x}{x^\alpha} = \begin{cases} 0 & \text{SE } \alpha<5 \\ \frac{1}{12} & \text{SE } \alpha=5 \\ +\infty & \text{SE } \alpha>5 \end{cases}$$

25) $$\lim_{x\to 0^+} \frac{\sin(\sin(\sin(\sin x))) - 4\sin(\sin(\sin x)) + 6\sin(\sin x) - 4\sin x + x}{x^\alpha} = \begin{cases} 0 & \text{SE } \alpha<9 \\ \frac{35}{432} & \text{SE } \alpha=9 \\ +\infty & \text{SE } \alpha>9 \end{cases}$$

26) $$\lim_{x\to 0^+} \frac{x^x - \cos x - x\ln x}{x^\alpha\cdot|\ln x|^\beta} = \begin{cases} 0 & \alpha<2 \\ \begin{cases} -\infty & \beta<2 \\ \frac{1}{2} & \beta=2 \\ 0 & \beta>2 \end{cases} & \alpha=2 \\ +\infty & \alpha>2 \end{cases}$$