Analisi Matematica 1- Lista n. 19

Calcolo della primitiva per sostituzione

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Calcolare le seguenti primitive ricorrendo ad un'opportuna sostituzione $x = g(t)$.

1) $$\int \cos(\ln x)\, dx = \frac{x}{2}\left(\sin(\ln x) + \cos(\ln x)\right)$$

2) $$\int e^{\sqrt[3]{x}}\, dx = \left(3\sqrt[3]{x^2} - 6\sqrt[3]{x} + 6\right) e^{\sqrt[3]{x}}$$

3) $$\int (e^x + 7)^8 e^x\, dx = \frac{1}{9}(e^x + 7)^9$$

4) $$\int (x+2)^{50} x^2\, dx = \frac{(x+2)^{51}}{53}\left(x^2 - \frac{x}{13} + \frac{2}{51\cdot 13}\right)$$

5) $$\int x\sqrt{x+5}\, dx = \left(\frac{2}{5}x^2 + \frac{2}{3}x - \frac{20}{3}\right)\sqrt{x+5}$$

6) $$\int \cos\sqrt{2x+1}\, dx = \cos\sqrt{2x+1} + \sqrt{2x+1}\cdot \sin\sqrt{2x+1}$$

7) $$\int \sin\sqrt[3]{x+2} = \left(6 - 3\sqrt[3]{(x+2)^2}\right)\cdot\cos\sqrt[3]{x+2} + 6\sqrt[3]{x+2}\cdot\sin\sqrt[3]{x+2}$$

8) $$\int \sqrt{3 + \sqrt{4 + \sqrt{x+1}}}\, dx =$$

$$= \frac{8}{9}\left(\sqrt{3+\sqrt{4+\sqrt{x+1}}}\right)^9 - \frac{72}{7}\left(\sqrt{3+\sqrt{4+\sqrt{x+1}}}\right)^7 + \frac{184}{5}\left(\sqrt{3+\sqrt{4+\sqrt{x+1}}}\right)^5 - 40\left(\sqrt{3+\sqrt{4+\sqrt{x+1}}}\right)^3$$

9) $$\int \sqrt{\frac{1+x}{1-x}}\, dx = -\sqrt{1-x^2} + \arcsin x$$

10) $$\int \frac{1}{\sqrt{x^2+9}}\, dx = \ln\left(x + \sqrt{x^2+9}\right)$$

11) $$\int \frac{1}{\sqrt{x^2+2x+2}}\, dx = \ln\left(x + 1 + \sqrt{x^2+2x+2}\right)$$

12) $$\int \frac{1}{\sqrt{x^2+4x+8}}\, dx = \ln\left(x + 2 + \sqrt{x^2+4x+8}\right)$$

Calcolare le seguenti primitive della forma $\int F\left(x, \sqrt{ax^2+bx+c}\right) dx$.

13) $$\int \sqrt{4-x^2}\, dx = 2 \arcsin \frac{x}{2} + \frac{1}{2} x \sqrt{4-x^2}$$

14) $$\int \frac{1}{\sqrt{x^2-1}}\, dx = \ln \left| x + \sqrt{x^2-1} \right|$$

15) $$\int \frac{1}{\sqrt{-x^2-2x}}\, dx = \arcsin (x+1)$$

16) $$\int \frac{1}{\sqrt{x^2+4x-12}}\, dx = \ln \left| x+2+\sqrt{x^2+4x-12} \right|$$

17) $$\int x^2 \sqrt{x^2+1}\, dx = \frac{1}{8} x \left(2x^2+1\right) \sqrt{x^2+1} - \frac{1}{8} \ln \left(x+\sqrt{x^2+1}\right)$$

18) $$\int \frac{\sqrt{1-x^2}}{x^2}\, dx = - \frac{\sqrt{1-x^2}}{x} + \arccos x$$

19) $$\int \left(x^2-1\right) \sqrt{x^2-1}\, dx = \frac{1}{8} x \left(2x^2-5\right) \sqrt{x^2-1} + \frac{3}{8} \ln \left| x+\sqrt{x^2-1} \right|$$

20) $$\int \left(x^3-x\right) \sqrt{x^2-1}\, dx = \frac{1}{5} \left(x^2-1\right)^2 \sqrt{x^2-1}$$

21) $$\int x^2 \sqrt{x^2+2x+2}\, dx = \left(6x^3+2x^2+x-11\right) \cdot \frac{\sqrt{x^2+2x+2}}{24} + \frac{3}{8} \ln \left(x+1+\sqrt{x^2+2x+2}\right)$$

22) $$\int \frac{x^4}{\sqrt{x^2+x}}\, dx = \left(\frac{1}{4}x^3 - \frac{7}{24}x^2 + \frac{35}{96}x - \frac{35}{64}\right) \sqrt{x^2+x} + \frac{35}{128} \ln \left(x+\frac{1}{2}+\sqrt{x^2+x}\right)$$

23) $$\int \frac{x+1}{\sqrt{x^2+8x+17}}\, dx = \sqrt{x^2+8x+17} - 3 \ln \left(x+4+\sqrt{x^2+8x+17}\right)$$

24) $$\int (2x+3) \sqrt{9x^2+6x+5}\, dx = \left(\frac{2}{3}x^2 + \frac{29}{18}x + \frac{41}{54}\right) \sqrt{9x^2+6x+5} + \frac{14}{9} \ln \left(3x+1+\sqrt{9x^2+6x+5}\right)$$

25) $$\int \frac{x^5+8x^3+16x+32}{\sqrt{x^2+4}}\, dx = \frac{1}{5} \left(x^2+4\right)^2 \sqrt{x^2+4} + 32 \ln \left(x+\sqrt{x^2+4}\right)$$