# Analisi Matematica (II modulo)- Exe. 2

Titolo nota

18 marzo 2020 (14.00-16.00) - docente: Prof. **Emanuele Callegari** - Università di Roma ***Tor Vergata***

$\boxed{P.X}$

$$\int_a^b f(x)\,dx = \int_\alpha^\beta f(\varphi(t))\,\varphi'(t)\,dt$$

QUESTO È IL PDF "GREZZO" DI CIÒ CHE HO SCRITTO ALLA LAVAGNA MENTRE SPIEGAVO

$f$ continua

$\varphi : [\alpha,\beta] \to [a,b]$ $\quad \underline{C^1 \text{ e strett. crescente}}$

$\boxed{f = \chi_{[c,d]}}$

$\boxed{f = \text{fun. semplice}}$

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$\boxed{21}$

$$\int_{-1}^{0} (3x^2+4x+1)\cdot \ln(x+2)^3 \cdot \arctan x \, dx = \qquad \varphi(t)=t-2$$

$$= \int_1^2 3\left(3(t-2)^2+4(t-2)+1\right)\cdot \ln t \cdot \arctan(t-2)\,dt =$$

$$= 3\int_1^2 (3t^2-8t+5)\cdot \ln t \cdot \arctan(t-2)\,dt =$$

$$= 3\int_1^2 (t^3-4t^2+5t)'\cdot \ln t \cdot \arctan(t-2)\,dt =$$

$$= 3\cdot\Big[(t^3-4t^2+5t)\ln t \cdot \arctan(t-2)\Big]_1^2 -$$

$$-3\cdot\int_1^2 (t^3-4t^2+5t)\left(\frac{1}{t}\cdot\arctan(t-2)+\ln t\cdot\frac{1}{1+(t-2)^2}\right)dt =$$

$$= [\dots] - 3\int_1^2 (t^3-4t^2+5t)\cdot\frac{1}{t}\cdot\arctan(t-2)\,dt -$$

$$\underbrace{-3\int_1^2 \frac{t^3-4t^2+5t}{1+(t-2)^2}\ln t\,dt}_{1}$$

$$\textcircled{1} = \int_1^2 \frac{t\cancel{(t^2-4t+5)}}{\cancel{1+t^2-4t+4}} \ln t \, dt =$$

$$= \int_1^2 t \cdot \ln t \, dt =$$

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$\boxed{\cancel{\varphi(t) = 2\operatorname{arctg} t}}$

$$20 \quad \int_{-\frac{\pi}{2}}^{0} \frac{6\cos x - 6\cos^3 x + \overbrace{4\sin 2x}^{8\sin x\cos x}}{\cos^2 x - 6\sin^3 x} \, dx =$$

$$= \int_{-\frac{\pi}{2}}^{0} \frac{6 - 6\cos^2 x + 8\sin x}{\cos^2 x - 6\sin^3 x} \cdot (\cos x) \, dx =$$

$\varphi(x) = \sin x$

$$= \int_{-\frac{\pi}{2}}^{0} \frac{6\sin^2 x + 8\sin x}{1 - \sin^2 x - 6\sin^3 x} (\sin x)' \, dx =$$

$$\overbrace{1 - 4y^2} + 3y^2 - 6y^3 =$$

$$= (1-2y)(1+2y) + 3y^2(1-2y)$$

$$= \underline{(1-2y)(1+2y+3y^2)}$$

$$= \int_{-1}^{0} \frac{6y^2 + 8y}{1 - y^2 - 6y^3} \, dy =$$

$$\overbrace{3y^2 + 2y + 1 \,|\, +2y - 1}$$

$$= 2\int_{-1}^{0} \frac{3y^2 + 4y}{(1-2y)(1+2y+3y^2)} \, dy = 2\int_{-1}^{0} \frac{(3y^2+2y+1) + (2y-1)}{(1-2y)(1+2y+3y^2)} \, dy =$$

$$= -\underbrace{\int_{-1}^{0} \frac{2}{2y-1} \, dy}_{\textcircled{1}} - 2\underbrace{\boxed{\int_{-1}^{0} \frac{1}{1+2y+3y^2} \, dy}}_{\textcircled{2}} = \boxed{\ln 3 - \sqrt{2} \cdot \frac{\pi}{2}}$$

$$\sqrt{2} \cdot \frac{\pi}{4}$$

$$\textcircled{1} = -\Big[\ln|2y-1|\Big]_{-1}^{0} = +\ln 3$$

$$\textcircled{2} = \frac{1}{3}\int_{-1}^{0} \frac{1}{y^2 + 2\cdot\frac{1}{3}\cdot y + \frac{1}{9} + \left(\frac{1}{3} - \frac{1}{9}\right)} \, dy =$$

$\varphi(t) = t - \frac{1}{3}$

$$= \frac{1}{3}\int_{-1}^{0} \frac{1}{\left(y + \frac{1}{3}\right)^2 + \frac{2}{9}} \, dy =$$

$$= \frac{1}{3}\int_{-\frac{2}{3}}^{\frac{1}{3}} \frac{1}{t^2+\frac{2}{9}}\,dt = \qquad \boxed{\varphi(u)=\frac{\sqrt{2}}{3}\cdot u}$$

$$= \frac{1}{3}\int_{-\sqrt{2}}^{\frac{1}{\sqrt{2}}} \frac{1}{\left(\frac{\sqrt{2}}{3}u\right)^2+\frac{2}{9}}\cdot\frac{\sqrt{2}}{3}\,du =$$

$$= \frac{\sqrt{2}}{\cancel{3}}\cdot\frac{1}{\cancel{3}}\cdot\frac{\cancel{9}}{2}\int_{-\sqrt{2}}^{\frac{1}{\sqrt{2}}} \frac{1}{u^2+1}\,du = \frac{1}{\sqrt{2}}\Big[\arctan u\Big]_{-\sqrt{2}}^{\frac{1}{\sqrt{2}}} =$$

$$= \frac{1}{\sqrt{2}}\left(\arctan\frac{1}{\sqrt{2}}+\arctan\sqrt{2}\right) = \boxed{\sqrt{2}\cdot\frac{\pi}{4}}$$

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$\boxed{T.1}$ DATA $f:[a,b]\to\mathbb{R}$ $\boxed{\text{CONTINUA}}$, E $\varphi:[\alpha,\beta]\to[a,b]$ $C^1$ strett. crescente

$f(x)=\chi_{[c,d]}$

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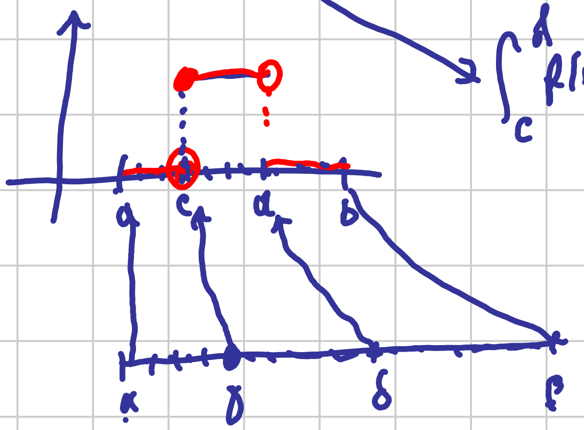
$$\int_a^b f(x)\,dx \underset{\le}{=} \int_\alpha^\beta f(\varphi(t))\cdot\varphi'(t)\,dt$$

$\boxed{DIM}$




$$\int_c^d f(x)\,dx = \int_c^d 1\,dx = (d-c)$$

$$\boxed{\int_\alpha^\beta f(\varphi(t))\,\varphi'(t)\,dt} = \int_\alpha^\gamma + \int_\gamma^\delta + \int_\delta^\beta (\dots)\,dt = \int_\gamma^\delta f(\varphi(t))\,\varphi'(t)\,dt =$$

$$= \int_\gamma^\delta \underbrace{1\cdot\varphi'(t)}\,dt = \Big[\varphi(t)\Big]_\gamma^\delta = \varphi(\delta)-\varphi(\gamma) = \boxed{d-c}$$

$\boxed{\text{PROP.}}$ $f, g$ "BUONE" $\Rightarrow$ $\underbrace{Af + Bg \text{ è "BUONA"}}$ $\forall A, B \in \mathbb{R}$

$$\int_a^b Af(x) + Bg(x)\,dx \stackrel{?}{=} \int_\alpha^\beta \left(Af(\varphi(t)) + Bg(\varphi(t))\right)\varphi'(t)\,dt$$

$$= A\int_a^b f(x)\,dx + B\int_a^b g(x)\,dx =$$

$f, g$ BUONE

$$\stackrel{}{=} A\int_\alpha^\beta f(\varphi(t))\varphi'(t)\,dt + B\int_\alpha^\beta g(\varphi(t))\cdot\varphi'(t)\,dt =$$

$$= \int_\alpha^\beta \left(Af(\varphi(t)) + Bg(\varphi(t))\right)\varphi'(t)\,dt$$