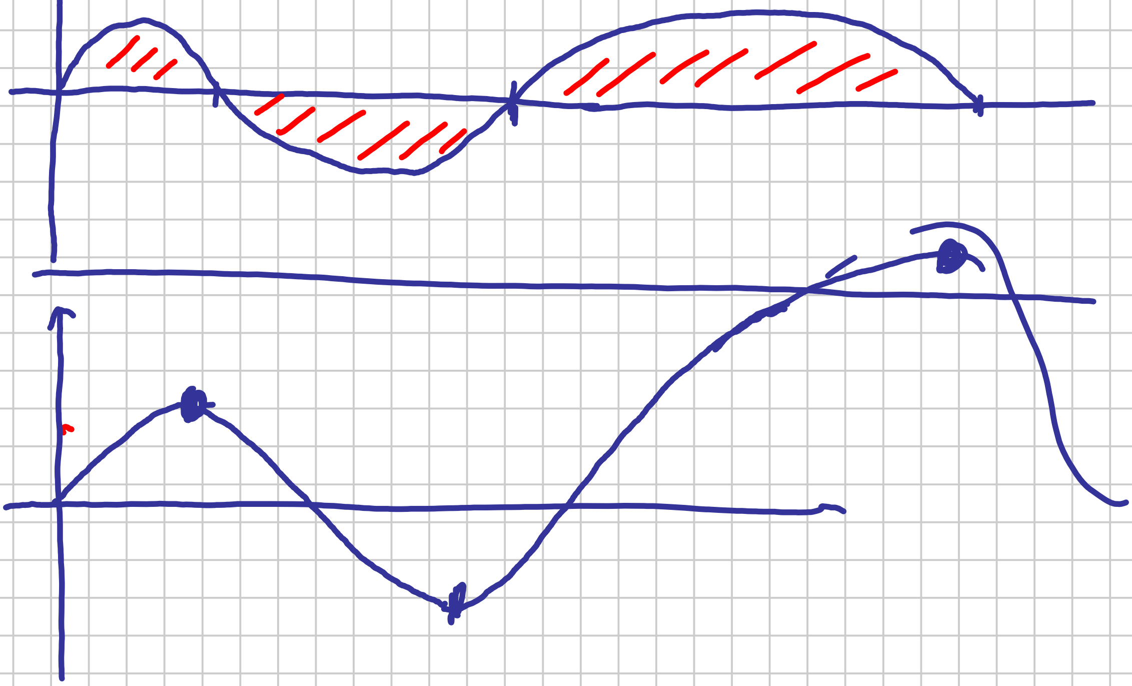
$\boxed{5\ (a)}$ $F(x) = \int_0^x \sin \sqrt[10]{t}\, dt$. $F(x) = 100$

$t = u^{10}$ $= \boxed{\int_0^{\sqrt[10]{x}} \sin u \cdot 10u^9}$



$$F(x) = \int_0^{\sqrt[10]{x}} 10 \cdot t^9 \cdot \sin t\, dt$$

$$\boxed{G(y) = \int_0^y 10\, t^9 \sin t\, dt}$$

$F(x) = 100$

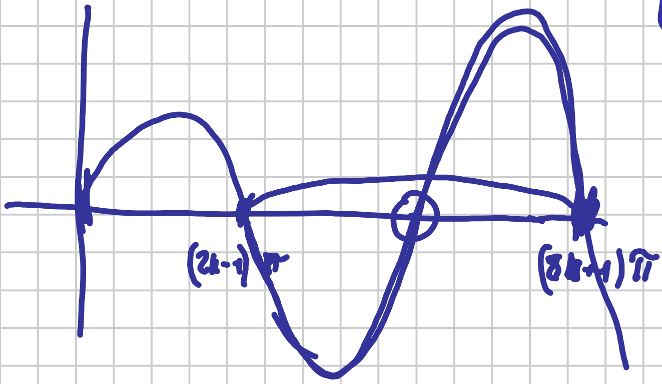
$\uparrow$

$y = \sqrt[10]{x}$

$G(y) = F(\sqrt[10]{x})$

$\boxed{G(y) = 100}$

$\boxed{G(y) = 100}$

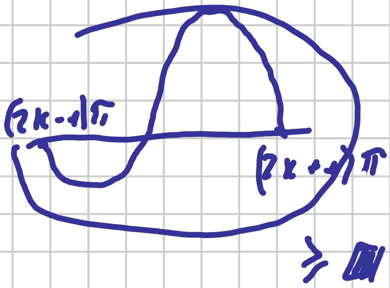


$\boxed{(2k\pi), (k+1)\pi)}$

$\boxed{G((2k\pi)) \to -\infty}$

$\boxed{G((2k+1)\pi) \to +\infty}$

$$G((2n+1)\pi) \to +\infty$$



$$\boxed{G((2n+1)\pi)} = \int_0^{(2n+1)\pi} \underbrace{10x^9 \cdot \sin x}_{f(x)}\, dx =$$

$$= \int_0^{\pi} f(x)\,dx + \sum_{k=1}^{n} \boxed{\int_{(2k-1)\pi}^{(2k+1)\pi} f(x)\,dx} \geq \boxed{n \cdot 20\pi} \to +\infty$$

20π

$$\int_{(2k-1)\pi}^{(2k+1)\pi} 10x^9 \cdot \sin x\, dx = \int_{(2k-1)\pi}^{2k\pi} \boxed{}\, dx + \int_{2k\pi}^{(2k+1)\pi} \boxed{}\, dx =$$

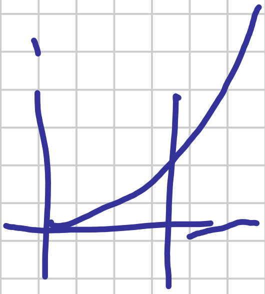
(1) (2)

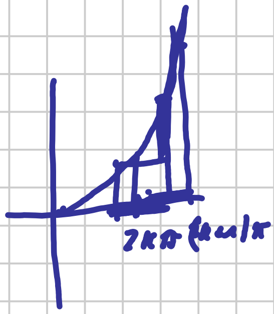
$$\boxed{x = t - \pi}$$

$$\int_{(2k-1)\pi}^{2k\pi} 10x^9 \sin x\, dx = \int_{2k\pi}^{(2k+1)\pi} 10(t-\pi)^9 \overbrace{\sin(t-\pi)}^{-\sin t}\, dt =$$

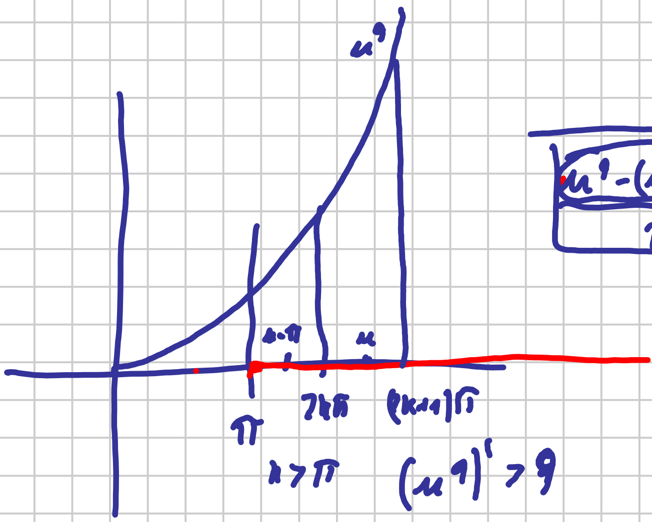
$$\frac{x^9 - (x-\pi)^9}{\pi} \cdot \pi$$

$$\int_{2k\pi}^{(2k+1)\pi} 10\Big(\underbrace{x^9 - (x-\pi)^9}_{\geq \pi}\Big) \sin x\, dx \geq$$





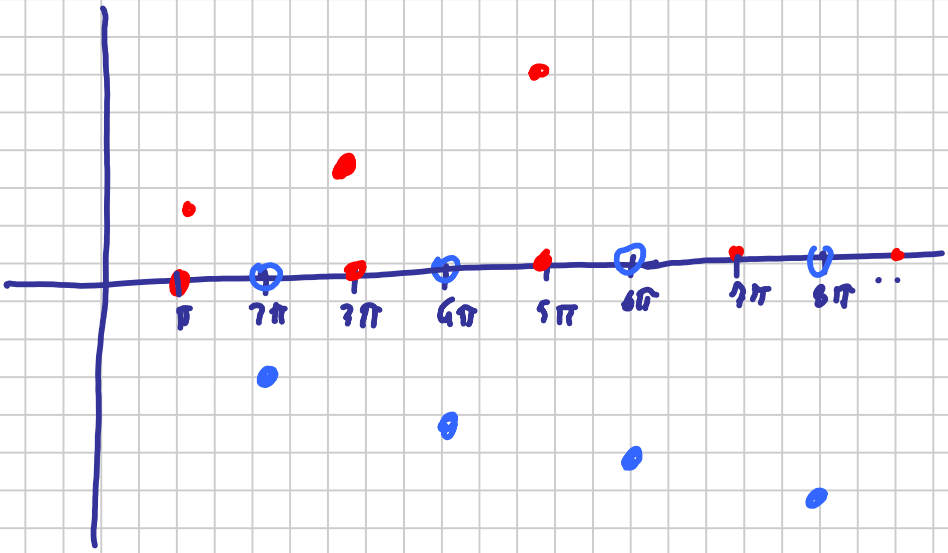
$$\geq \int_{2k\pi}^{(2k+1)\pi} \underbrace{10 \cdot \pi}\cdot \sin x\, dx = \boxed{20\pi}$$



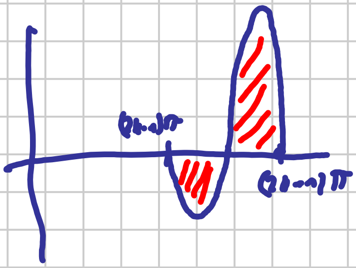
$x > \pi \quad (x^9)' > 9$

$$\geq 9$$
$$\boxed{\frac{x^9 - (x-\pi)^9}{\pi}} \cdot \pi$$

$$G(y)=\int_0^y 10\,t^9 \sin t\,dt \qquad G(y)=100$$



$$\begin{cases} G((2n+1)\pi) \to +\infty & \text{PER } n\to+\infty \\ G(2n\pi) \to -\infty & \text{PER } n\to+\infty \end{cases}$$



$$\boxed{G((2n+1)\pi)} = \int_0^{(2n+1)\pi} f(t)\,dt =$$

$$= \int_0^{\pi} f(t)\,d + \sum_{k=1}^{n} \overbrace{\int_{(2k-1)\pi}^{(2k+1)\pi} f(t)\,dt}^{\geq 20\pi} \geq \int_0^{\pi} f(t)\,dt + 20\pi\cdot n \to +\infty$$

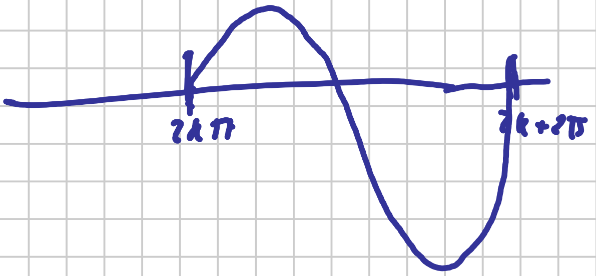
$$\boxed{\int_{(2k-1)\pi}^{(2k+1)\pi} 10\,t^9 \sin t\,dt} = \underbrace{\int_{(2k-1)\pi}^{2k\pi} ◎}_{①} + \int_{2k\pi}^{(2k+1)\pi} ◎ =$$

$$① \int_{(2k-1)\pi}^{2k\pi} 10\,t^9 \sin t\,dt \underset{t=s-\pi}{=} \int_{2k\pi}^{(2k+1)\pi} 10\,(s-\pi)^9 \underbrace{\sin(s-\pi)}_{-\sin s}\,ds = \int_{2k\pi}^{(2k+1)\pi} 10\,(t-\pi)^9\cdot(-\sin t)\,dt$$

$$= \int_{2k\pi}^{(2k+1)\pi} 10\cdot\big(\underbrace{t^9-(t-\pi)^9}_{\geq \pi}\big) \sin t\,dt \geq$$

$$\geq \int_{2k\pi}^{(2k+1)\pi} 10 \cdot \pi \cdot \sin t \, dt \geq 20\pi$$

$\leq c < 0$



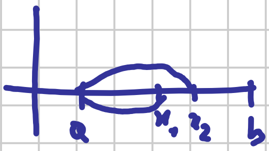
1 (d) $f : [a,b] \to \mathbb{R}$ CONTINUA

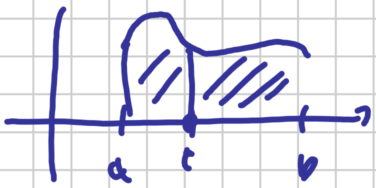
$\forall t \in (a,b) \quad \int_a^t f(x)dx = \int_t^b f(x)dx \quad \not\Rightarrow \quad f \equiv 0$

---

1 (c)

$\forall x_1, x_2 \in (a,b) \int_{x_1}^{x_2} f(x)dx = 0 \quad \Rightarrow \quad f \equiv 0$





$$\int_a^t f(x)dx = \frac{1}{2}\int_a^b f(x)dx$$

$$\overbrace{\int_a^t f(x)dx} = \int_t^b f(x)dx = \overbrace{\int_a^b f(x)dx - \int_a^t f(x)dt}$$

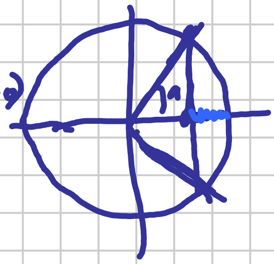
$$2\int_a^t f(x)dx = \int_a^b f(x)dx$$

$$\int_a^t f(x)dx = \frac{1}{2}\int_a^b f(x)dx$$

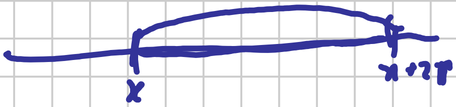
$$F(x) := \int_0^x \cos(\cos t)\,dt \qquad \overline{F'(x)} = \cos(\underbrace{\cos x}) \geq \cos(\cos x) > 0$$

$$F'(x) = \cos(\cos x) - \cos(\cos 0) + \cos(\cos 0)$$

$=$



$$F(x)=\int_0^x \sin(\sin t)\,dt$$



$$F(x+2\pi)=F(x)\qquad \forall x$$

$$F(x+2\pi)-F(x)=\int_0^{x+2\pi}\bullet-\int_0^x\bullet=\int_x^{x+2\pi}\bullet=0$$

$$\int_{-\pi}^{\pi}\sin(\sin t)\,dt=\int_{-\pi}^{0}f(t)\,dt+\int_0^{\pi}f(t)\,dt==0$$

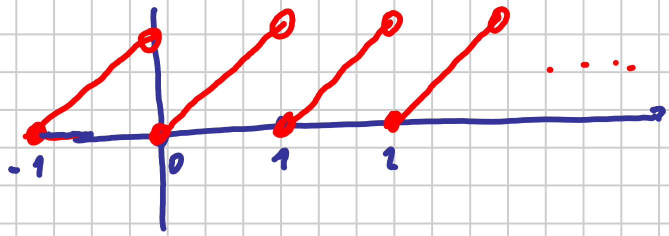
$$u=-t \Leftrightarrow t=-u$$

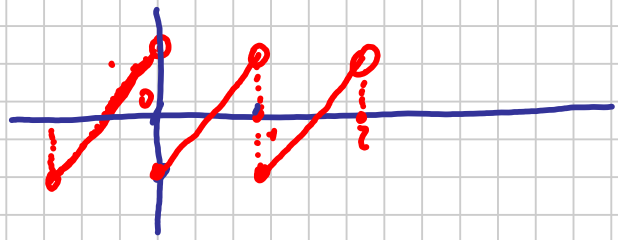
$$\int_{\pi}^{0}f(-u)(-1)\,du=\int_{\pi}^{0}-f(u)(-1)\,du=$$

f dispari

$$=\int_{\pi}^{0}f(u)\,du=-\int_0^{\pi}f(u)\,du$$

$$F(x)=\int_0^x\underbrace{\left(x-\lfloor x\rfloor-\frac12\right)}_{f(x)}dx$$





$$\int_0^1 x-\lfloor x\rfloor-\frac12\,dx=$$

$$=\int_0^1 x-\frac12\,dx=\left[\frac{x^2}{2}\right]_0^1-\frac12=$$

$$=\frac12-\frac12=0$$

$$f(x+1)=f(x)\qquad\forall x$$

$$f(x+1)=(x+1)-\overbrace{\lfloor x+1\rfloor}^{\lfloor x\rfloor+1}-\frac12=x+\cancel{1}-\lfloor x\rfloor-\cancel{1}-\frac12=f(x)$$