

ESERCITAZIONI DI ANALISI 1

Exe. **1**
4 OTTOBRE 2022

INFO SUL CORSO

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ESERCIZI E QUESITI DI ANALISI MATEMATICA 1
ESCVLAPIO EDITORE

ORARIO: VEDI PAGINA CORSO, DI NORMA **LUNEDÌ E VENERDÌ**
ORE 11 **AULA 11** SOGENE.

RICEVIMENTO: MARTEDÌ ORE 14 AULA 11 SOGENE (SE LIBERA)
ALTRIMENTI ON LINE MARTEDÌ ORE 18.

ESERCIZI PROPOSTI OGGI:

TROVARE (SE ESISTONO) MAX MIN ^{DEFINIRE} INF E SUP DEI
SEGUENTI INSIEMI:

- 1) $\left\{ \frac{n-1}{n} \mid n \in \mathbb{N} - \{0\} \right\}$
- 2) $\left\{ \frac{n-1}{n} \mid n \in \mathbb{Z} - \{0\} \right\}$
- 3) $\left\{ n + \frac{150}{n} \mid n \in \mathbb{N} - \{0\} \right\}$
- 4) $\left\{ n^k \mid n \in \mathbb{N} - \{0\}, k \in \mathbb{Z} \right\}$
- 5) $\left\{ x \in \mathbb{R} \mid x \text{ È SOLUZIONE DI } \begin{cases} \sqrt{x+2} > \sqrt{x+1} \\ 2x^2 - x^3 + 17 > 0 \end{cases} \right\}$
- 6) $\left\{ \left(1 + \frac{1}{3k+1} \right)^n \mid n \in \mathbb{N}, k \in \mathbb{Z} \right\}$
- 7) $\left\{ \sqrt{m} - \sqrt{n} \mid m, n \in \mathbb{N} - \{0\} \text{ con } m > n \right\}$

SIMILI A QUESTI SONO GLI ESERCIZI DA 5 A 18 DI PAG. 9
DELL'ESERCIZIARIO. SI CONSIGLIA DI FARLI PER CASA.

DETERMINARE I SEGUENTI INSIEMI:

8) $A \cap B$

9) $A \cup B$

10) $A - B$

11) $B - A$

12) $A^c \cup B^c$

13) $(A \cup B)^c$

SUPPONENDO CHE L'INSIEME
AMBIENTE SIA $\mathbb{Z}$

DOVE $A = \{ n \in \mathbb{N} \mid n \geq 2 \text{ e } \log_n 100 \geq 2 \}$

$B = \{ n \in \mathbb{Z} \mid 2^n \leq 100 \}$

→ 14) $(-\infty, \frac{1}{100}] \cap \left(\left\{ |\cos n| + \frac{1}{|\cos n|} \mid n \in \mathbb{Z} \right\} \cup \left\{ x \in \mathbb{R} \mid \sqrt{x-1} > 10 \sin x \right\} \right)$

15) $\bigcap_{n \in \mathbb{N}} \left(-\frac{1}{n+1}, \frac{1}{n+1} \right)$

16) $\bigcap_{n \in \mathbb{N}} \left(\frac{1}{n+1}, \frac{2}{n+1} \right)$

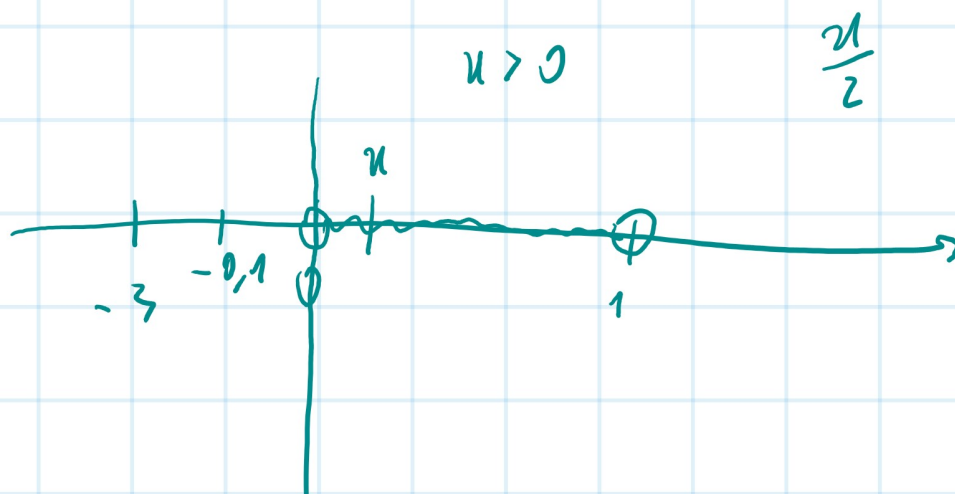
17) $\bigcup_{n \in \mathbb{N}} (n, n^2)$

18) $\bigcup_{n \in \mathbb{N}} \left[\frac{1}{n+1}, n+2 \right]$

19) $\bigcup_{m \in \mathbb{N}} \bigcap_{n \in \mathbb{N}} \left(0, \frac{m+1}{n+1} \right)$

20) $\bigcap_{n \in \mathbb{N}} \bigcup_{m \in \mathbb{N}} \left(0, \frac{m+1}{n+1} \right)$

SIMILI A QUESTI SONO GLI ESERCIZI DA 19 A 30 DI PAG. 10
DELL'ESERCIZIARIO. SI CONSIGLIA DI FARLI PER CASA.



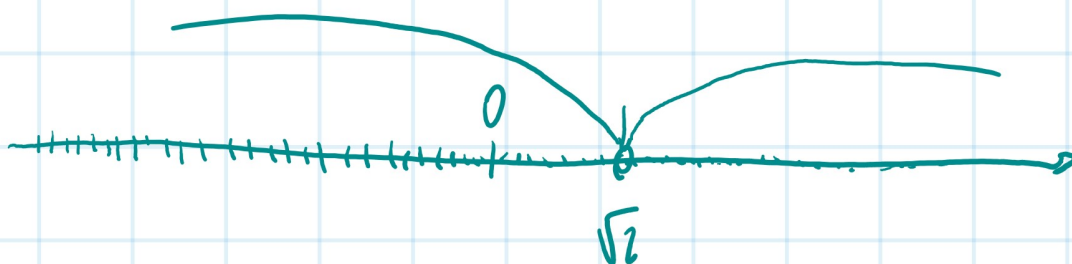
DEF. 1 DATO $A \subset \mathbb{R}$ E $m \in \mathbb{R}$ DIREMO CHE

m È MINORANTE DI A SE $\forall u \in A$ $m \leq u$

SE POI $m \in A$ DIREMO CHE $m = \min(A)$.

DEF. 2 SIA $A \subset \mathbb{R}$ E $V = \{m \in \mathbb{R} \mid m \text{ È MINORANTE DI } A\}$

DEFINIAMO $\inf(A) = \max(V)$



SOLUZIONI

$$1) \left\{ \frac{n-1}{n} \mid n \in \mathbb{N} - \{0\} \right\} = A$$

$$\min(A) = 0 \quad (\text{ovvero})$$

$$\frac{n-1}{n}$$

$$1 - \frac{1}{n}$$

$$A = \left\{ 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots \right\}$$

↑

DEVO DIMOST. CHE $\forall \varepsilon > 0$

$1 - \varepsilon$ NON È MAGG.

$\exists n \in \mathbb{N} - \{0\}$ T.C.

$$\rightarrow \frac{n-1}{n} > 1 - \varepsilon$$

$$1 - \frac{1}{n} > 1 - \varepsilon$$

$$\rightarrow \frac{1}{n} < \varepsilon$$

① IN non HA MAGGIORANTI

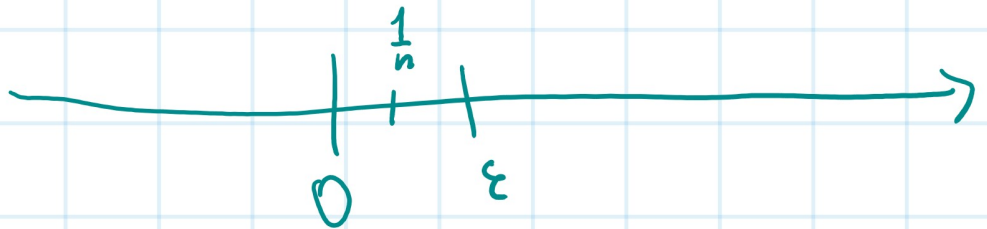




$$\underbrace{\forall x \in \mathbb{R} \exists n \in \mathbb{N} \text{ t.e. } n > x}$$

$$\exists n \in \mathbb{N} \text{ t.e. } n > \frac{1}{\varepsilon}$$

$$\frac{1}{n} < \varepsilon$$



QUINDI $\text{Insieme dei maggiori} = \underbrace{\{x \in \mathbb{R} \mid x \geq 1\}}_U$

$$\text{MIN}(U) = 1$$

$$\text{SUP}(A) = 1$$

$$3) \left\{ \underbrace{n + \frac{150}{n}}_{a_n} \mid n \in \mathbb{N} - \{0\} \right\} = A$$

$$\boxed{a_{n+1} > a_n} \leftarrow$$

$$n+1 + \frac{150}{n+1} > n + \frac{150}{n} \quad ??$$

$$150 \left(\frac{1}{n} - \frac{1}{n+1} \right) < 1 \quad ??$$

$$\frac{1}{n(n+1)} < \frac{1}{150} \quad \leftarrow$$

$$\frac{n^2}{n(n+1)} > 150 \quad \leftarrow$$

$$\boxed{n \geq 12}$$

$$a_1, \dots, a_{10} > a_{11} > \boxed{a_{12}} \leq a_{13} \leq a_{14} \leq \dots$$

$$Q_{12} = 12 + \frac{150}{12} = \boxed{24 + \frac{1}{2}}$$

$$\text{MIN}(A) = 24 + \frac{1}{2}$$

$$\text{INF}(A) = 24 + \frac{1}{2}$$

$$\boxed{\text{SUP}(A) = +\infty}$$

$$\forall \lambda \in \mathbb{R} \quad \exists n \in \mathbb{N} \text{ t.c. } n > \lambda$$

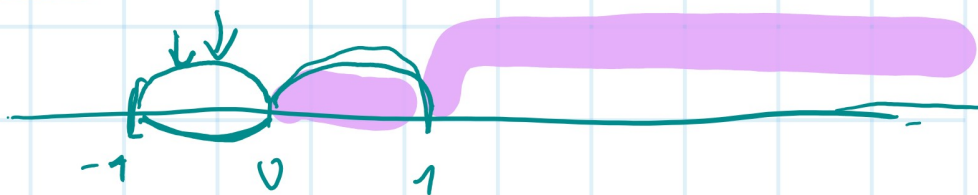
$$n + \frac{150}{n} > n > \lambda$$

5) $\left\{ x \in \mathbb{R} \mid x \text{ È SOLUZIONE DI } \begin{cases} \sqrt{x+2} > \sqrt{x+1} \\ 2x^2 - x^3 + 17 > 0 \end{cases} \right\}$

$$\sqrt{x+2} > \sqrt{x+1}$$

$$x \geq -1$$

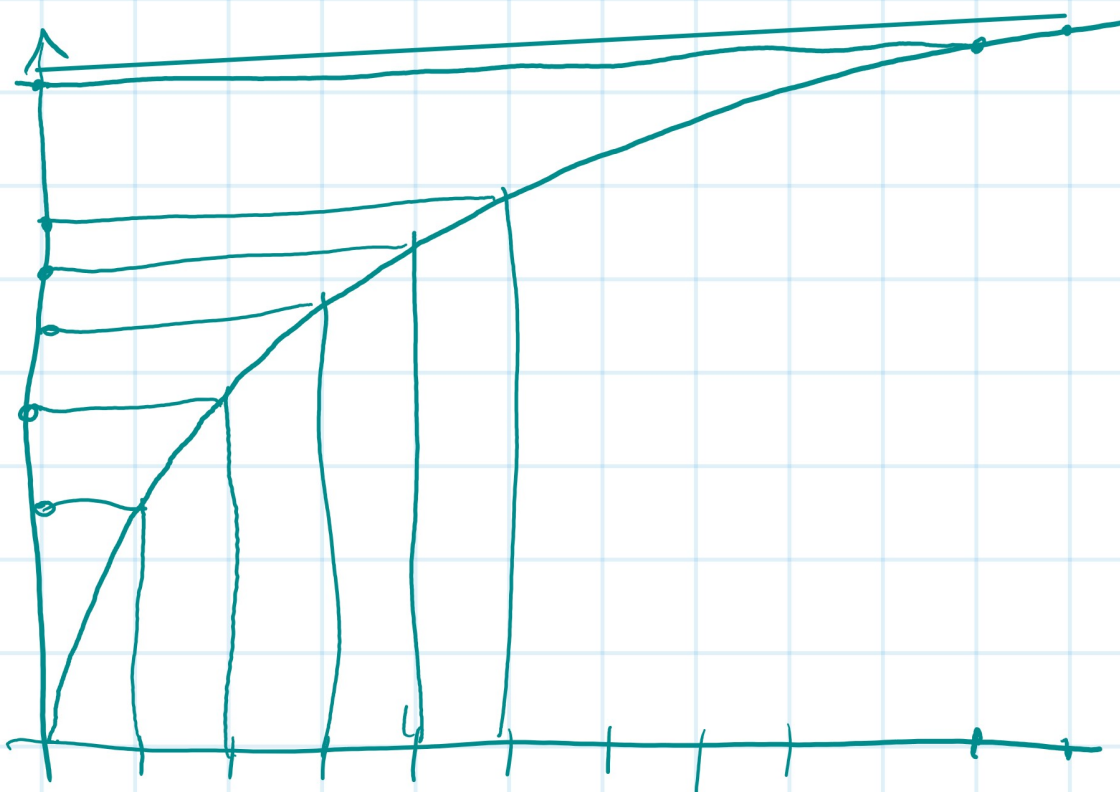
$$[-1, +\infty)$$



$$x \geq 1$$

$$2x^2 - x^3 + 17 = \overbrace{(x^2 - x^3)}^{> 0} + \sqrt{x^2} + \sqrt{17} > 0$$

7) $A = \{\sqrt{m} - \sqrt{n} \mid m, n \in \mathbb{N} - \{0\} \text{ con } m > n\}$



$\inf(A) = 0$ MA $\min(A)$ NON ESISTE

OSC. PER:

1) 0 È MINORANTE (OVVIO)

2) $\forall \varepsilon > 0 \quad \exists \text{ EL. DI } A \text{ MINORE DI } \varepsilon$

$$m = n + 1$$

I POSITIVI
NON SONO
MINORANTI

$$\forall n \in \mathbb{N} - \{0\}$$

$$\frac{\sqrt{n+1} - \sqrt{n}}{1} \in A$$

$$\rightarrow = \frac{(\sqrt{n+1}) - \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}} < \frac{1}{\sqrt{n}}$$

$$\boxed{\forall \epsilon > 0 \exists n \in \mathbb{N} \text{ t.e. } \frac{1}{\sqrt{n}} < \epsilon} \quad (?)$$

$$\boxed{\epsilon^2}$$

$$\exists n \in \mathbb{N} \text{ t.e. } \frac{1}{n} < \epsilon^2$$

$$\frac{1}{\sqrt{n}} < \epsilon$$

$$14) \underbrace{\left(-\infty, \frac{1}{100}\right]}^A \cap \underbrace{\left\{ \left| \cos n + \frac{1}{|\cos n|} \right| \mid n \in \mathbb{Z} \right\}}_B \cup \underbrace{\left\{ x \in \mathbb{R} \mid \sqrt{x-1} > 10 \sin x \right\}}_C$$

$$\underbrace{A \cap (B \cup C)}_{\text{left}} \neq \underbrace{(A \cap B) \cup (A \cap C)}_{\text{right}}$$

$$x \in A \cap (B \cup C) \Leftrightarrow (x \in A \text{ e } x \in B \cup C) \Leftrightarrow$$

$$\Leftrightarrow \left(\overset{p}{x \in A} \text{ e } \left(\overset{q}{x \in B} \text{ o } \overset{r}{x \in C} \right) \right)$$

$$x \in (A \cap B) \cup (A \cap C) \Leftrightarrow (x \in A \cap B \text{ o } x \in A \cap C) \Leftrightarrow$$

$$\Leftrightarrow \left((x \in A \text{ e } x \in B) \text{ o } (x \in A \text{ e } x \in C) \right)$$

$$14) \underbrace{\left(-\infty, \frac{1}{100}\right]}_A \cap \underbrace{\left\{ \left| \cos n + \frac{1}{|\cos n|} \right| \mid n \in \mathbb{Z} \right\} \cup \left\{ x \in \mathbb{R} \mid \sqrt{x-1} > 10 \sin x \right\}}_B \cup \underbrace{\left\{ x \in \mathbb{R} \mid \sqrt{x-1} > 10 \sin x \right\}}_C$$

$$\left(-\infty, \frac{3}{2}\right]$$

$$A \cap B = \emptyset$$

$$\sqrt{x-1} > 10 \sin x$$

$$x + \frac{1}{x} \geq 2 \quad ?$$

$$x + \frac{1}{x} - 2 \geq 0 \quad ?$$

$$\left(\sqrt{x}\right)^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2 \cdot \sqrt{x} \cdot \frac{1}{\sqrt{x}} \geq 0$$

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 \geq 0 \quad \text{SI}$$

$$15) \bigcap_{n \in \mathbb{N}} \left(-\frac{100}{n+1}, \frac{100}{n+1} \right) = \{0\}$$

$$16) \bigcap_{n \in \mathbb{N}} \left(\frac{1}{n+1}, \frac{2}{n+1} \right)$$

$$(-1, 1) \cap \left(-\frac{1}{2}, \frac{1}{2}\right) \cap \left(-\frac{1}{3}, \frac{1}{3}\right) \cap \dots \cap \left(-\frac{1}{n+1}, \frac{1}{n+1}\right) \cap \dots$$

$$\left(1, 2\right) \cap \left(\frac{1}{2}, \frac{2}{2}\right) \cap \left(\frac{1}{3}, \frac{2}{3}\right) \cap \left(\frac{1}{4}, \frac{2}{4}\right) \cap \dots \cap \left(\frac{1}{n+1}, \frac{2}{n+1}\right) \cap \dots$$

$$19) \bigcup_{m \in \mathbb{N}} \left(\bigcap_{n \in \mathbb{N}} \left(0, \frac{m+1}{n+1} \right) \right)$$

$$20) \bigcap_{n \in \mathbb{N}} \left(\bigcup_{m \in \mathbb{N}} \left(0, \frac{m+1}{n+1} \right) \right)$$

$$\left(\left(0, \frac{1}{1} \right) \cap \left(0, \frac{1}{2} \right) \cap \left(0, \frac{1}{3} \right) \cap \dots \cap \left(0, \frac{1}{n+1} \right) \cap \dots \right) \cup \left(\left(0, \frac{2}{2} \right) \cap \left(0, \frac{2}{2} \right) \cap \dots \right) \cup$$

$$\cup \left(\left(0, \frac{3}{1} \right) \cap \left(0, \frac{3}{2} \right) \cap \left(0, \frac{3}{3} \right) \cap \dots \cap \left(0, \frac{3}{n+2} \right) \cap \dots \right) \cup \dots$$

$$\rightarrow 19) \bigcup_{m \in \mathbb{N}} \left(\bigcap_{n \in \mathbb{N}} \left(0, \frac{m+1}{n+1} \right) \right) = \bigcup_{m \in \mathbb{N}} (\emptyset) = \emptyset$$

$$\rightarrow 20) \bigcap_{n \in \mathbb{N}} \left(\bigcup_{m \in \mathbb{N}} \left(0, \frac{m+1}{n+1} \right) \right) = \bigcap_{n \in \mathbb{N}} (0, +\infty) = (0, +\infty)$$

(27) $\bigcup_{q \in \mathbb{Q}} \left(\bigcap_{n \in \mathbb{N}} \left(q - \frac{1}{n+1}, q + \frac{1}{n+1} \right) \right)$



