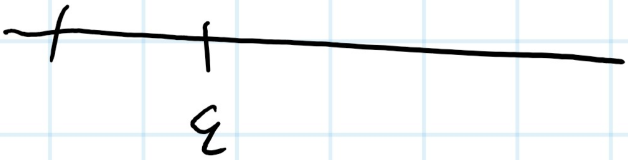
RICEVIMENTO STUDENTI AM1

R. 1

07-10-2022



P.A. $\forall n \in \mathbb{N} \quad \frac{1}{n+1} > \varepsilon$

$$\Updownarrow$$

$$n+1 < \frac{1}{\varepsilon}$$

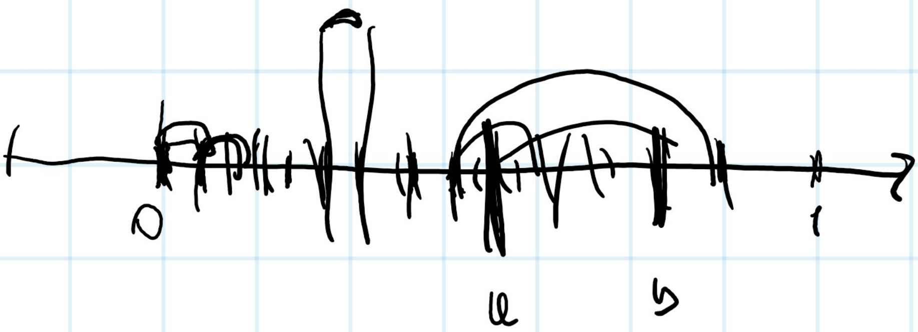
$$\boxed{n < \frac{1}{\varepsilon} - 1}$$

$$\frac{1}{\varepsilon} > 0$$

$\exists\, n \in \mathbb{N}$ t.c. $\quad n > \frac{1}{\varepsilon} - 1$

$$n+1 > \frac{1}{\varepsilon} \qquad \frac{1}{n+1} < \varepsilon$$

$\sqrt{n} - \lfloor \sqrt{n} \rfloor$



$\boxed{n = 9 \qquad n = 15}$

$\sqrt{9} = 3$

$\sqrt{10} = 3,\ldots \quad \sqrt{15} = 3,\ldots \quad \sqrt{16}$

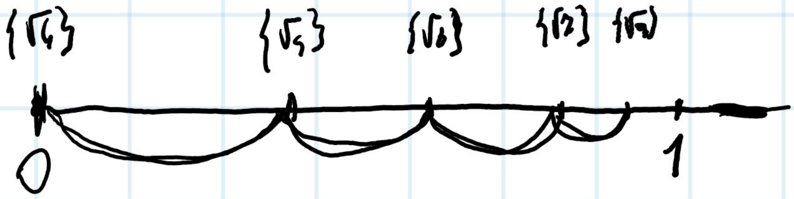
$\sqrt{n}-\lfloor\sqrt{n}\rfloor=\{\sqrt{n}\}$ $\sqrt{(k+1)^2-1}$

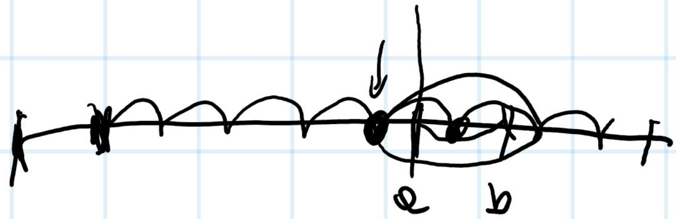
$\sqrt{4}$ . . . . .

$\sqrt{k^2}$

$\{\sqrt{\ }$

$\{\ \}$





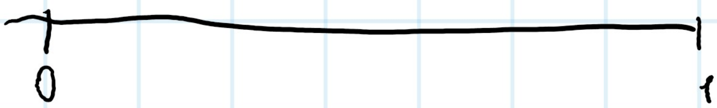
$\boxed{\sqrt{k^2}+\sqrt{k^2+1}}$

POSSO?

1) $\delta = b-a$ PRENDO $k\in\mathbb{N}$ t.c. $\sqrt{k^2+1}-\sqrt{k^2}<\delta$

$$\frac{\sqrt{k^2+1}-\sqrt{k^2}}{1}=\frac{1}{\sqrt{k^2+1}+\sqrt{k^2}}<\frac{1}{k}<\delta$$

$\exists k \in \mathbb{N}$ t.c. $\sqrt{k^2+1} - \sqrt{k^2} < \overbrace{b-a}^{\delta}$



II SE $k^2 \le n \le (k+1)^2 - 1$

$$\lfloor \sqrt{n} \rfloor = k$$

$$k^2 \le n < (k+1)^2$$

$$k \le \sqrt{n} < k+1$$

SE $k^2 \le \underline{n_1 < n_2} < (k+1)^2$

$$k \le \sqrt{n_1} < \sqrt{n_2} < k+1$$

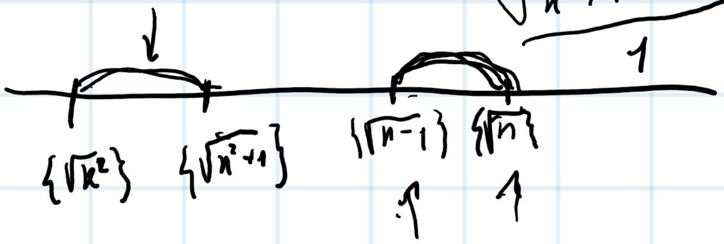
$$0 \le \underbrace{\sqrt{n_1} - \lfloor \sqrt{n_1} \rfloor}_{} < \underbrace{\sqrt{n_2} - \lfloor \sqrt{n_2} \rfloor}_{} < k+1$$

$n = k^2 + h$ $\qquad$ $h < (k+1)^2 - k^2 - 1$

$$k^2 \le n < (k+1)^2 - 1$$

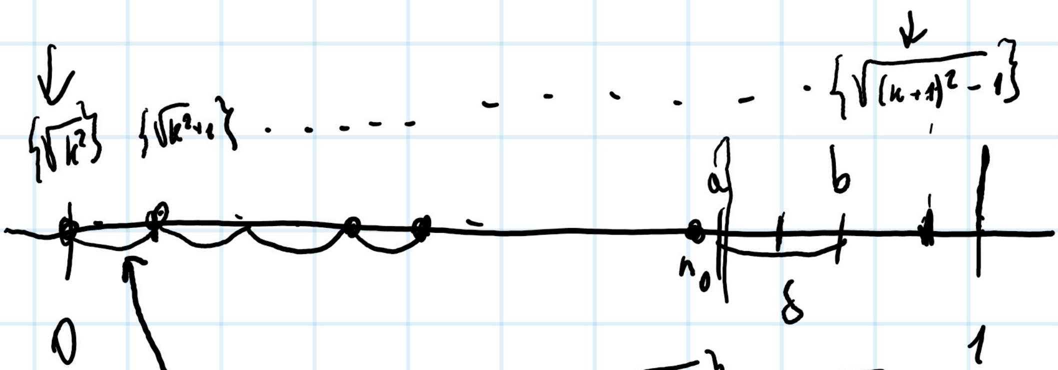
$$\{\sqrt{k^2+h}\} - \{\sqrt{k^2+h-1}\} \le \sqrt{k^2+1} - \lfloor\sqrt{k^2+1}\rfloor \ ?$$

$$= \frac{\sqrt{k^2+1} - k}{1} = \boxed{\frac{1}{\sqrt{k^2+1}+k}}$$



$$\left(\sqrt{k^2+h} - \cancel{\lfloor\sqrt{k^2+h}\rfloor}\right) - \left(\sqrt{k^2+h-1} - \cancel{\lfloor\sqrt{k^2+h-1}\rfloor}\right) =$$

$$= \frac{\sqrt{k^2+h} - \sqrt{k^2+h-1}}{1} = \boxed{\frac{1}{\sqrt{k^2+h}+\sqrt{k^2+h-1}}}$$



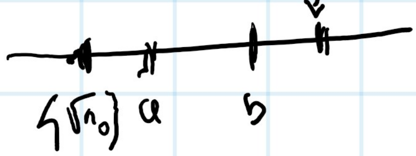
$$\{\sqrt{n}\} - \{\sqrt{n-1}\} \le \boxed{}$$

$$n_0 = \mathrm{MAX}\left(\left\{ n \in \mathbb{N} \mid k^2 \le n \le (k+1)^2 - 1 \quad e \quad \sqrt{n} - \lfloor \sqrt{n} \rfloor \le a \right\}\right)$$

$\{\sqrt{n_0+1}\} > a$

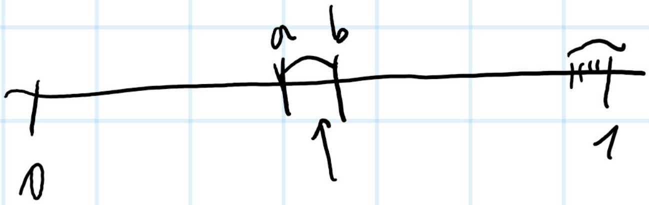
$\{\sqrt{n_0+1}\} < b$

$> b$



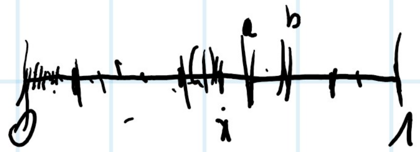
$$\{\sqrt{n_0+1}\} - \{\sqrt{n_0}\} \le \{\sqrt{k^2+1}\} - \{\sqrt{k^2}\} < b - a$$

$> b - a$



$\sqrt{2}\ ,\ 2\sqrt{2}\ ,\ 3\sqrt{2}$

$$\boxed{\left\{ n\sqrt{2} - \lfloor n\sqrt{2} \rfloor \mid n \in \mathbb{N} \right\}} \qquad \sqrt{2}$$



(29) $\bigcap_{n\in\mathbb{N}} \left(\bigcup_{m\in\mathbb{N}} \left[0, 1+\frac{1}{n+1}-\frac{1}{m+1}\right] \right) = \bigcap_{n\in\mathbb{N}} \left[0, 1+\frac{1}{n+1}\right)$

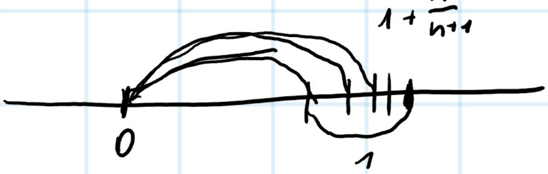
$\forall \varepsilon>0$ $\boxed{1+\frac{1}{n+1}-\varepsilon}$ $\boxed{1+\frac{1}{n+1}}$

t.c. $0>0$

$\bigcup_{m\in\mathbb{N}} \left[0, 1+\frac{1}{n+1}-\frac{1}{m+1}\right] =$ $\boxed{\begin{array}{c}\forall\varepsilon>0\ \exists m\in\mathbb{N} \text{ t.c.} \\ \cancel{1}+\cancel{\frac{1}{n+1}}-\varepsilon < \cancel{1}+\cancel{\frac{1}{n+1}}-\frac{1}{m+1}\end{array}}$

?

$\frac{1}{m+1}<\varepsilon$



$\left[0, 1+\frac{1}{n+1}\right)$

$= \left[0, 1+\frac{1}{n+1}\right)$

$\bigcap_{n\in\mathbb{N}} \left[0, 1+\frac{1}{n+1}\right) = [0, 1]$

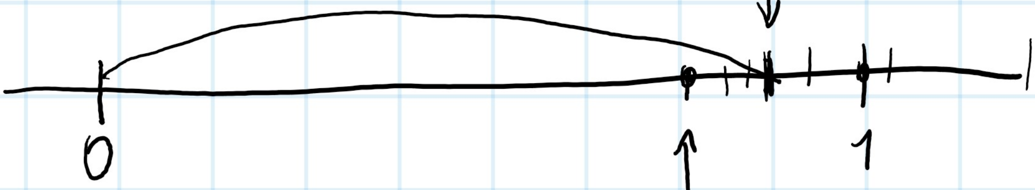
$\frac{1}{n+1}<\varepsilon$

?

$\forall\varepsilon>0$ $\exists n\in\mathbb{N}$ t.c. $1+\varepsilon > 1+\frac{1}{n+1}$

$$\bigcup_{m\in\mathbb{N}}\left(\bigcap_{n\in\mathbb{N}}\left[0, 1+\frac{1}{n+1}-\frac{1}{m+1}\right]\right) = \bigcup_{m\in\mathbb{N}}\left[0, 1-\frac{1}{m+1}\right]$$

$1-\frac{1}{m+1}+\frac{1}{n+1}$



$1-\frac{1}{m+1}$

$\left[0, 1-\frac{1}{m+1}\right]$

$1-\frac{1}{m+1}+\varepsilon$

? $\forall \varepsilon>0\ \exists n\in\mathbb{N}$ t.c. $\underbrace{\cancel{1}-\cancel{\frac{1}{m+1}}+\frac{1}{n+1} < \cancel{1}-\cancel{\frac{1}{m+1}}+\varepsilon}$

$\frac{1}{n+1}<\varepsilon$

$\left[0, 1-\frac{1}{m+1}\right]$

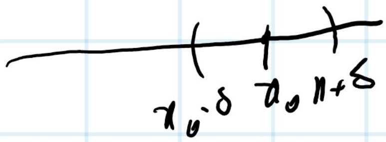
$$\bigcup_{m\in\mathbb{N}} \underbrace{\left[0, 1-\frac{1}{m+1}\right]} = [0,1)$$

$1-\varepsilon$

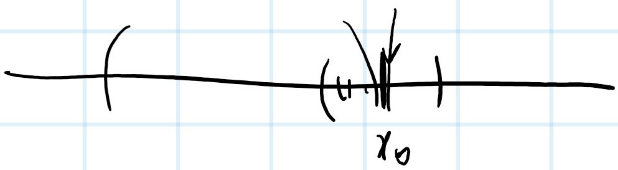
$\forall \varepsilon > 0 \ \exists n \in \mathbb{N}$ t.c. $\underbrace{1-\varepsilon < 1-\frac{1}{n+1}}$ (?)

$$\frac{1}{n+1} < \varepsilon$$

$\forall \delta > 0$

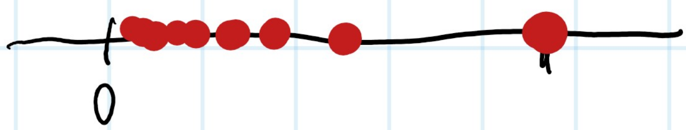


$$\boxed{\begin{array}{l} A \subset \mathbb{R},\ x_0 \in \mathbb{R} \quad x_0 \text{ È DI ACC. PER } A \\ \forall \varepsilon > 0 \ \exists x \in A - \{x_0\} \text{ t.c. } x \in (x_0-\varepsilon, x_0+\varepsilon) \end{array}}$$

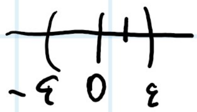


$$A = \left\{ \frac{1}{n} + \frac{1}{m} \;\middle|\; n, m \in \mathbb{N} - \{0\} \right\}$$

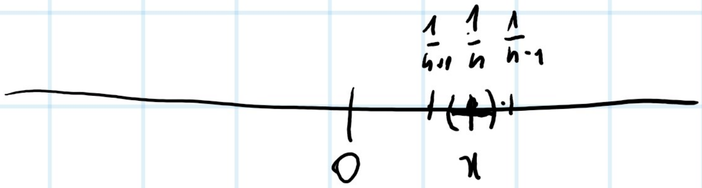
$$B = \left\{ \frac{1}{n} \;\middle|\; n \in \mathbb{N} - \{0\} \right\}$$



$$0 < \frac{1}{n} < \varepsilon$$



$\frac{1}{n}$



$\frac{1}{n}$