

18-10-2022

$$\bigcup A = \bigcup (A^c)$$

↑

 $\forall I \text{ int. } A: x$

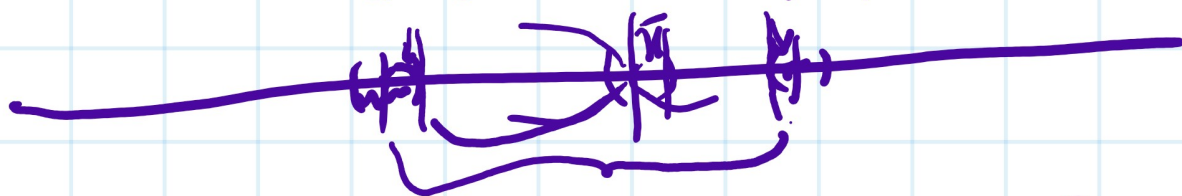
$$\begin{cases} I \cap A \neq \emptyset \\ I \cap A^c \neq \emptyset \end{cases}$$

 $\forall I \text{ int. } A: x$

$$\begin{cases} I \cap A^c \neq \emptyset \\ I \cap (A^c)^c \neq \emptyset \end{cases}$$

 $\parallel$
 A

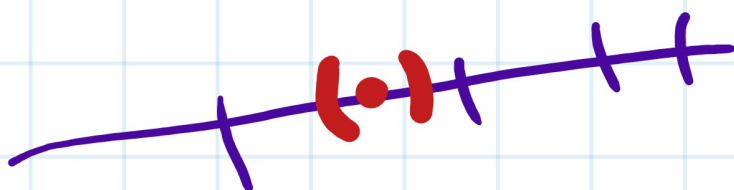
$$\bigcup A = \emptyset \Rightarrow \emptyset, \mathbb{R}$$

 $x \in A$
 $x < y$
 $y \in A^c$


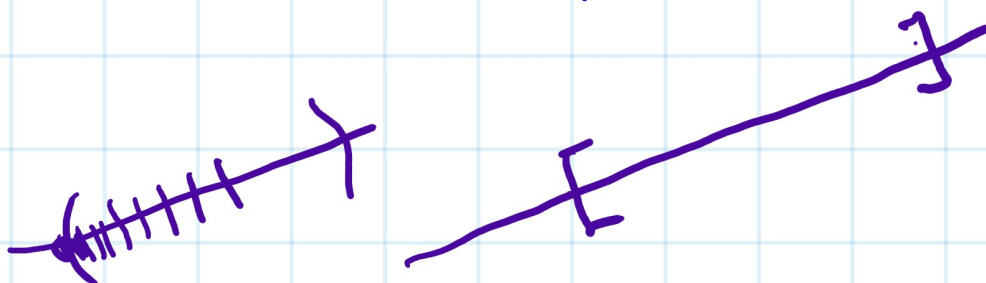
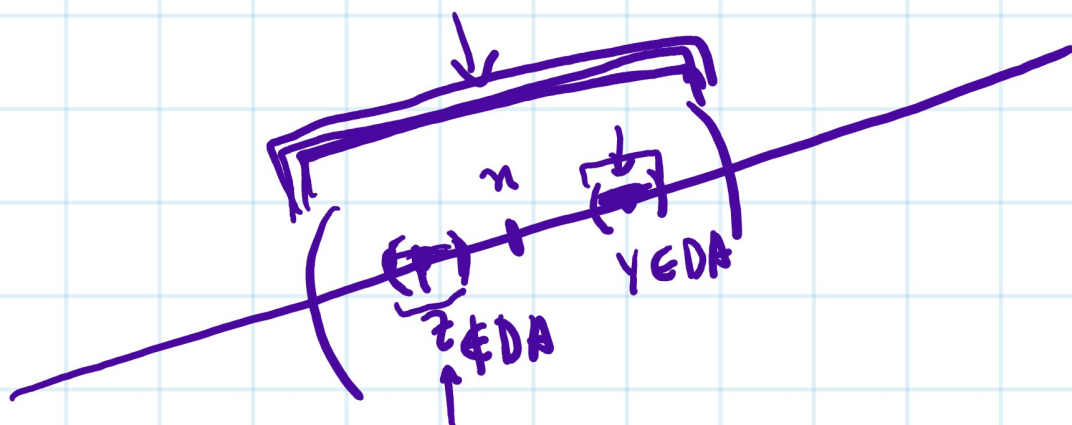
$$\bar{x} = \inf(B)$$

$$B = \{ t \in (x, y) \mid t \in A^c \}$$

$$\mathcal{O}(\mathbb{Q}) = \mathbb{R}$$



$$\mathcal{O}(\mathbb{D}_A) \subset \mathcal{O}_A$$



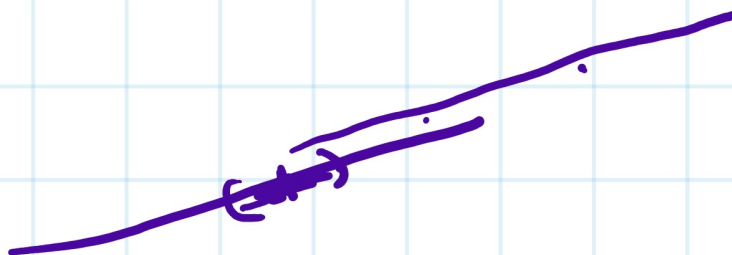
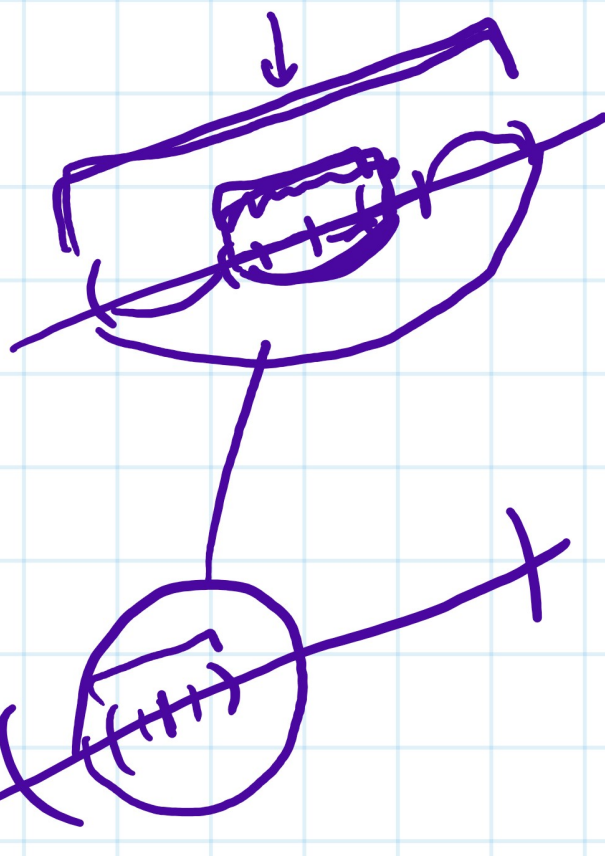
$\mathcal{I}A$ è chiuso

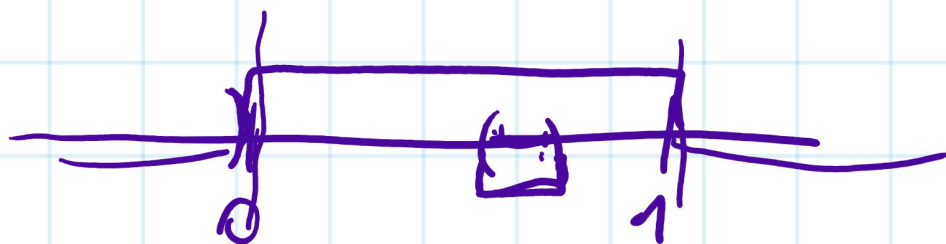
$(\mathcal{I}A)^c$ è aperto



$x \in A$

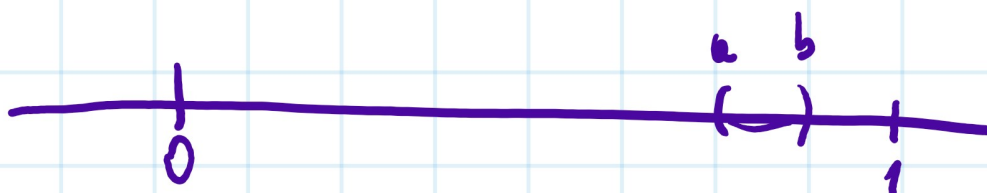
I





$$0 < a < b < 1$$

$$\delta = b - a$$



1) PRENDO $k \in \mathbb{N}$ t.a.

$$\boxed{\sqrt{k^2+1} - \lfloor \sqrt{k^2+1} \rfloor} < \delta$$

?

$$\sqrt{k^2} < \sqrt{k^2+1} < \sqrt{(k+1)^2}$$

$$k < \sqrt{k^2+1} < k+1$$

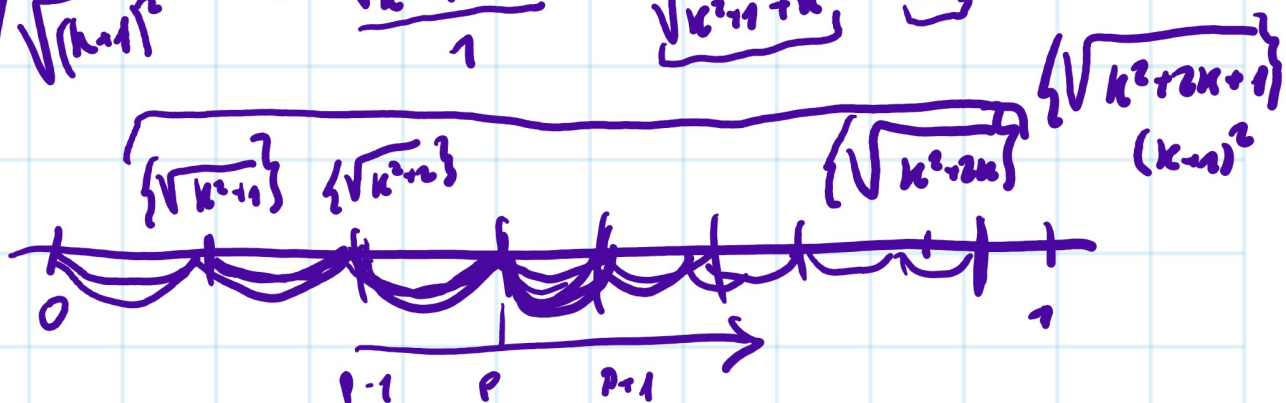
↑

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$$\frac{\sqrt{k^2+1} - k}{1} = \frac{1}{\sqrt{k^2+1} + k} < \frac{1}{k} < \delta$$

$$\sqrt{k^2} \quad \sqrt{(k+1)^2}$$

k



$$\left\{ \sqrt{k^2+p+1} \right\} - \left\{ \sqrt{k^2+p} \right\} < \left\{ \sqrt{k^2+p} \right\} - \left\{ \sqrt{k^2+p-1} \right\}$$

$$\left(\sqrt{k^2+p+1} - \cancel{x} \right) - \left(\sqrt{k^2+p} - \cancel{x} \right)$$

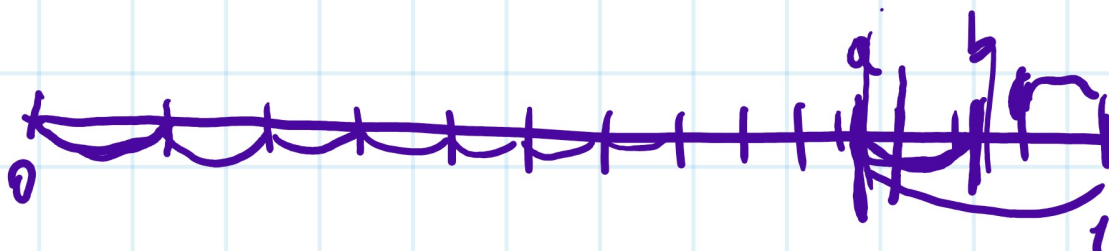
$$\sqrt{k^2+p+1} - \sqrt{k^2+p}$$

$$\frac{\sqrt{k^2+p+1} - \sqrt{k^2+p}}{1} < \frac{\sqrt{k^2+p} - \sqrt{k^2+p-1}}{1}$$

$$\frac{1}{\sqrt{k^2+p+1} + \sqrt{k^2+p}} < \frac{1}{\sqrt{k^2+p} + \sqrt{k^2+p-1}}$$

$$(k+1) - \sqrt{k^2+2k}$$

δ



$$\min(B) \quad B = \{ \sqrt{k^2+p} \mid p \in \{0, \dots, 2k\}, \sqrt{k^2+p} > a \} \neq \emptyset$$

$$\{\sqrt{k^2+p-1}\} \leq a \leq \underbrace{\{\sqrt{k^2+p}\}}_{?} < b$$

