ESERCITAZIONI DI ANALISI 1

Exe. 4

19 OTTOBRE 2022

ESERCIZI PROPOSTI

1) DATA $f:\mathbb{R}\to\mathbb{R}$, SIA $\mathcal{T}=\{T>0 \mid T \text{ È PERIODO DI } f\}\neq\emptyset$ E SIA $\tau=\inf(\mathcal{T})$. MOSTRARE CHE SE $\tau>0$ ALLORA $\mathcal{T}=\{k\tau \mid k\in\mathbb{N}-\{0\}\}$

2) COSA SUCCEDE IN (1) SE $\tau=0$? ESIBIRE ALCUNE $f:\mathbb{R}\to\mathbb{R}$ TALI CHE $\tau=0$. CHE PROPRIETÀ HA $\mathcal{T}$?

DIRE SE LE FUNZIONI CHE SEGUONO SONO PERIODICHE E IN CASO AFFERMATIVO TROVARNE IL PERIODO.

3) $(\sin x)^2$

4) $\sin(x^2)$

5) $\sin\left(\frac{3x+5}{11}\right)$

6) $\sin\frac{2}{13}x+\cos\frac{3}{11}x$

7) $-\cos\frac{\pi x}{5}+3x-\lfloor 3x\rfloor+\chi_{\mathbb{Q}}(x)$

8) $(x-\lfloor x\rfloor)-\cos x$

PER CASA: 63-82 PAG. 11

NOTO IL GRAFICO DI $g(x)$ DISEGNARE QUELLO DI $f(x)$ NEI SEGUENTI CASI:

9) $g(x) = \sin x \qquad f(x) = \left| 2\sin\left(2|x| + \frac{\pi}{6}\right) + 1 \right|$

10) $g(x) = x^2 \qquad f(x) = \left| 5 - \left| 3 - \left| x^2 - 4|x| + 3 \right| \right| \right|$

$\boxed{\text{PER CASA: } 99\text{-}113 \text{ PAG. } 12\text{-}13}$

DIRE SE SONO VERE O FALSE LE SEGUENTI AFFERMAZIONI

11) DEFINITIVAMENTE IN n, $\forall \lambda \in \mathbb{R}$ $n > \lambda$ F

12) $\forall \lambda \in \mathbb{R}$, DEFINITIVAMENTE IN n, $n > \lambda$ V

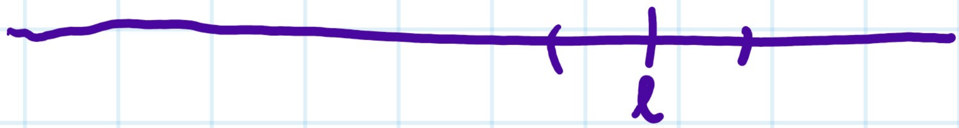
13) FREQUENTEMENTE IN n, $\forall \lambda \in \mathbb{R}$ $n > \lambda$

14) $\forall \varepsilon > 0$, FREQ. IN n, DEFIN. IN m, $\left(\frac{3}{2} + \sin n\right)^m < \varepsilon$

PERMUTARE

$\boxed{\text{PER CASA: } 11\sim18 \text{ PAG. } 64\text{-}65}$

$a_n \to l$ per $n \to +\infty$



$$\forall \varepsilon > 0 \ \underbrace{\exists n_0 \in \mathbb{N} \text{ t.c. } \forall n \geq n_0}_{\text{DEF. IN } n} \text{ si ha } \underbrace{|x_n - l| < \varepsilon}$$

$$\neg(\forall x \ \ x \text{ è buono})$$

$$\exists x \ \neg(x \text{ è buono})$$

$P(n)$ VALE FREQ. IN n

$$\underbrace{\forall n_0 \in \mathbb{N} \ \exists n > n_0 \text{ t.c.}}_{\text{FREQ. IN } n} \ P(n)$$

$$\neg(a_n \to \ell)$$

$$\neg\left(\forall \varepsilon > 0 \quad \text{DEF. IN } n \quad |a_n - \ell| < \varepsilon\right)$$

$$\boxed{\exists \varepsilon_0 > 0 \ \text{ t.c. } \ \text{FREQ. IN } n \quad |a_n - \ell| \geq \varepsilon}$$

$$\neg\left(\exists \ell \in \mathbb{R} \ \text{ t.c. } \ a_n \to \ell\right)$$

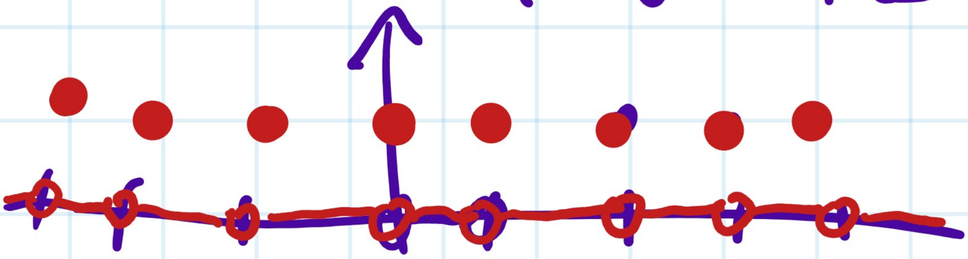
$$\forall \ell \in \mathbb{R} \ \boxed{\neg(a_n \to \ell)}$$

DEF. DATA $f:\mathbb{R}\to\mathbb{R}$ DIREMO CHE $T>0$ È PERIODO PER f SE

$$\forall x\in\mathbb{R} \quad f(x)=f(x+T)$$

INDICHIAMO CON $\mathcal{T}=\{T\in(0,+\infty) \mid T \text{ È PERIODO PER } f\}$

$$\chi_{\mathbb{Z}}(x)=\begin{cases}1 & x\in\mathbb{Z}\\ 0 & x\notin\mathbb{Z}\end{cases}$$



$$\chi_{\mathbb{Q}}(x)=\begin{cases}1 & x\in\mathbb{Q}\\ 0 & x\notin\mathbb{Q}\end{cases}$$

$\boxed{T \in \mathbb{Q}}$ $T > 0$

$\forall x \in \mathbb{R}$ $\quad \chi_{\mathbb{Q}}(x+T) \overset{?}{=} \chi_{\mathbb{Q}}(x)$

$x+T \in \mathbb{Q} \Leftrightarrow x \in \mathbb{Q}$

$\mathcal{T} = \{T > 0 \mid T \in \mathbb{Q}\} = \mathbb{Q}^+$

$\exists f$ t.c. $\forall \varepsilon > 0$ f HA PERIODO $T < \varepsilon$

$\forall \varepsilon > 0$ $\exists f$ t.c f "

1) DATA $f:\mathbb{R}\to\mathbb{R}$, SIA $\mathcal{T}=\{T>0 \mid T \text{ È PERIODO DI } f\}\neq\emptyset$ E SIA $\tau=\inf(\mathcal{T})$. MOSTRARE CHE SE $\tau>0$ ALLORA $\mathcal{T}=\{k\tau \mid k\in\mathbb{N}-\{0\}\}$

$\boxed{\text{DIM}}$

1) $\tau\in\mathcal{T}$

P.A. $\tau\notin\mathcal{T}$

2τ NON È MINORANTE DI $\mathcal{T}$

QUINDI $\exists\, T_1\in\mathcal{T}$ t.c. $\tau<T_1<2\tau$

ANCHE T_1 NON È MINORANTE QUINDI

$\exists\, T_2\in\mathcal{T}$ t.c. $\tau<T_2<T_1$

T_1 È PER. $\iff \forall x \in \mathbb{R} \quad \overline{f(x+T_1)} = f(x)$:

T_2 È PER. $\iff \forall x \in \mathbb{R} \quad \overline{f(x+T_2)} = f(x)$:

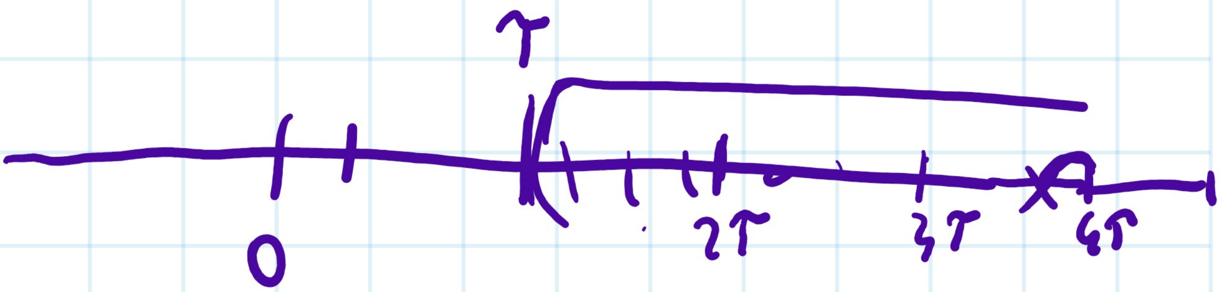
$$\boxed{\forall x \in \mathbb{R} \quad f(x+T_1) = f(x+T_2)}$$

$$f(\underbrace{x+T_2}_{y} + (T_1 - T_2)) = f(\underbrace{x+T_2}_{y})$$

$$\forall y \in \mathbb{R} \quad f(y + \overline{(T_1 - T_2)}) = f(y)$$

$$\Downarrow$$

$T_1 - T_2 \in \mathcal{T}$ MA $T_1 - T_2 < T$



$\sin^2 x$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$\boxed{\sin^2 x} = \frac{1 - \cos 2x}{2}$$

$$\forall x \in \mathbb{R} \quad \cos(\underbrace{2x}_{y}) = \cos(2(x+T))$$

$$= \cos(\underbrace{2x}_{y} + 2T)$$

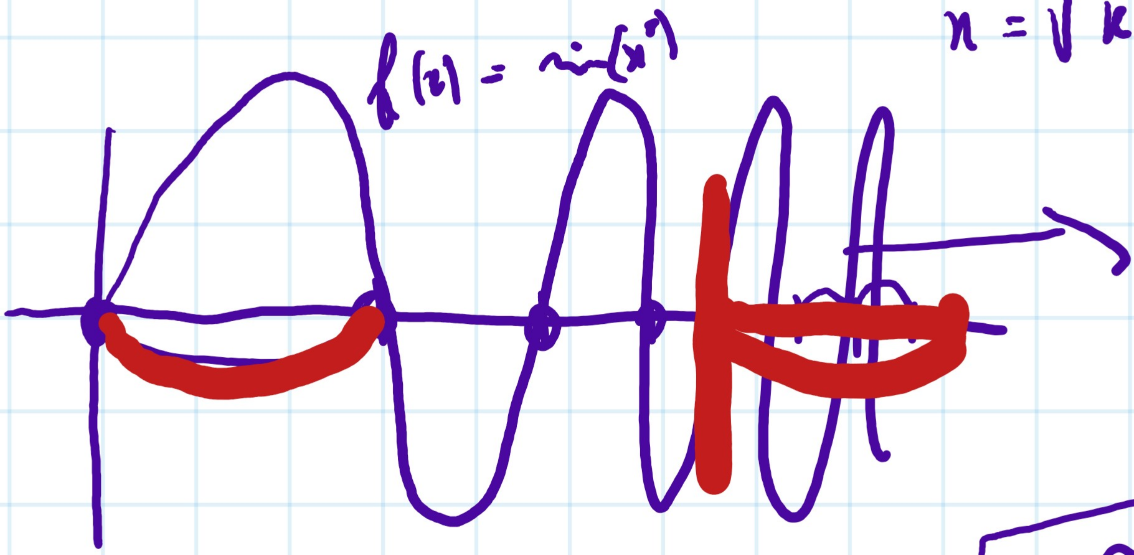
$$\forall y \in \mathbb{R} \quad \cos(y) = \cos(y + 2T)$$

$$2T = 2k\pi \qquad k \in \mathbb{N} - \{0\}$$

$$T = k\pi$$

$$\sin(x^2) = 0 \qquad x^2 = k\pi$$

$$x = \sqrt{k\pi}$$



$$\frac{\sqrt{(k+1)\pi} - \sqrt{k\pi}}{1} = \boxed{\frac{\pi}{\sqrt{(k+1)\pi} + \sqrt{k\pi}}}$$

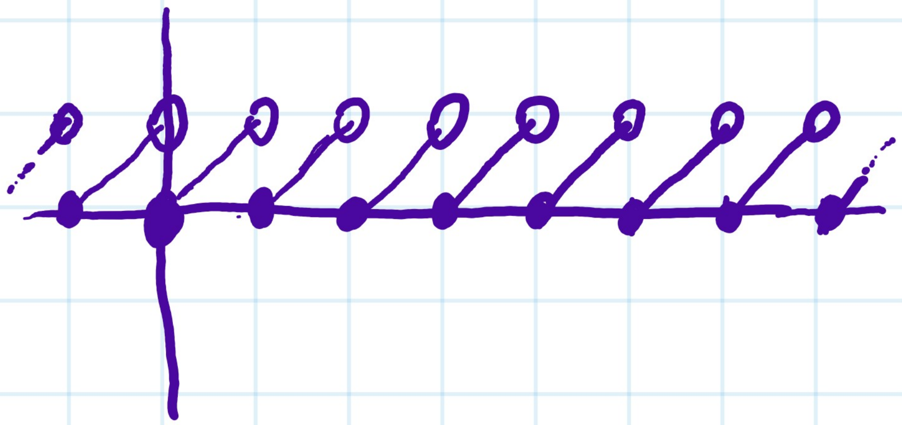
$f(x) > 0$ PER $x \in (0, \sqrt{\pi})$

$$\boxed{f(T) = f(0)}$$

$f(T+x) = f(x) > 0$ PER $x \in (0, \sqrt{\pi})$

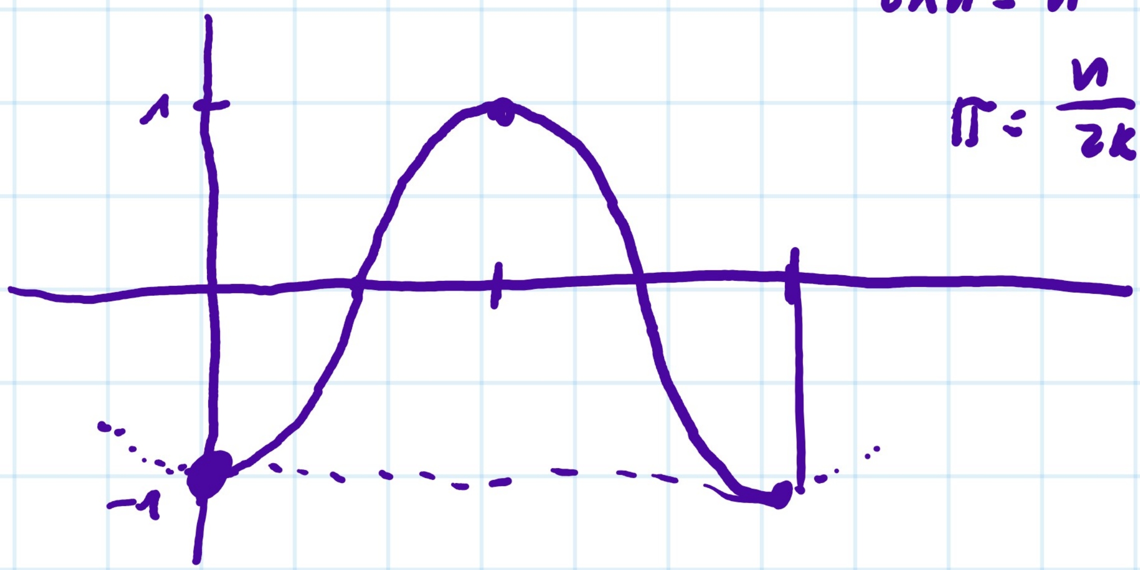
$$f(x) = \underbrace{x - \lfloor x \rfloor}_{} \overset{+}{-} \underline{\cos x}$$

$$\mathrm{MIN}(f(x)) = -1 = f(0) = f(k\pi) \quad \forall k \in \mathbb{Z}$$



$$2k\pi = n$$

$$\pi = \frac{n}{2k} \in \mathbb{Q}$$



$$f(x) = -1 \iff \Big(\big(\underline{x - \lfloor x \rfloor \text{ VALE } 0} \big) \text{ E } \big(-\cos x \text{ VALE } -1 \big) \Big)$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$x \in \mathbb{N} \qquad\qquad x = 2k\pi$$

$$x = n$$

$$f(x) = \left| 5 - \left| 3 - \underbrace{\left| x^2 - 4|x| + 3 \right|}_{h(x)} \right| \right|$$

$$h(x) = \left| x^2 - 4|x| + 3 \right| = \left| \overline{|x|^2 - 4|x| + 4} - 1 \right| =$$

$$= \left| \left(|x| - 2 \right)^2 - 1 \right|$$

$$g(x) = x^2$$

$$(x-2)^2$$

$$(x-2)^2 - 1$$

$$\left| (x-2)^2 - 1 \right|$$

$$\left| \left(|x| - 2 \right)^2 - 1 \right|$$

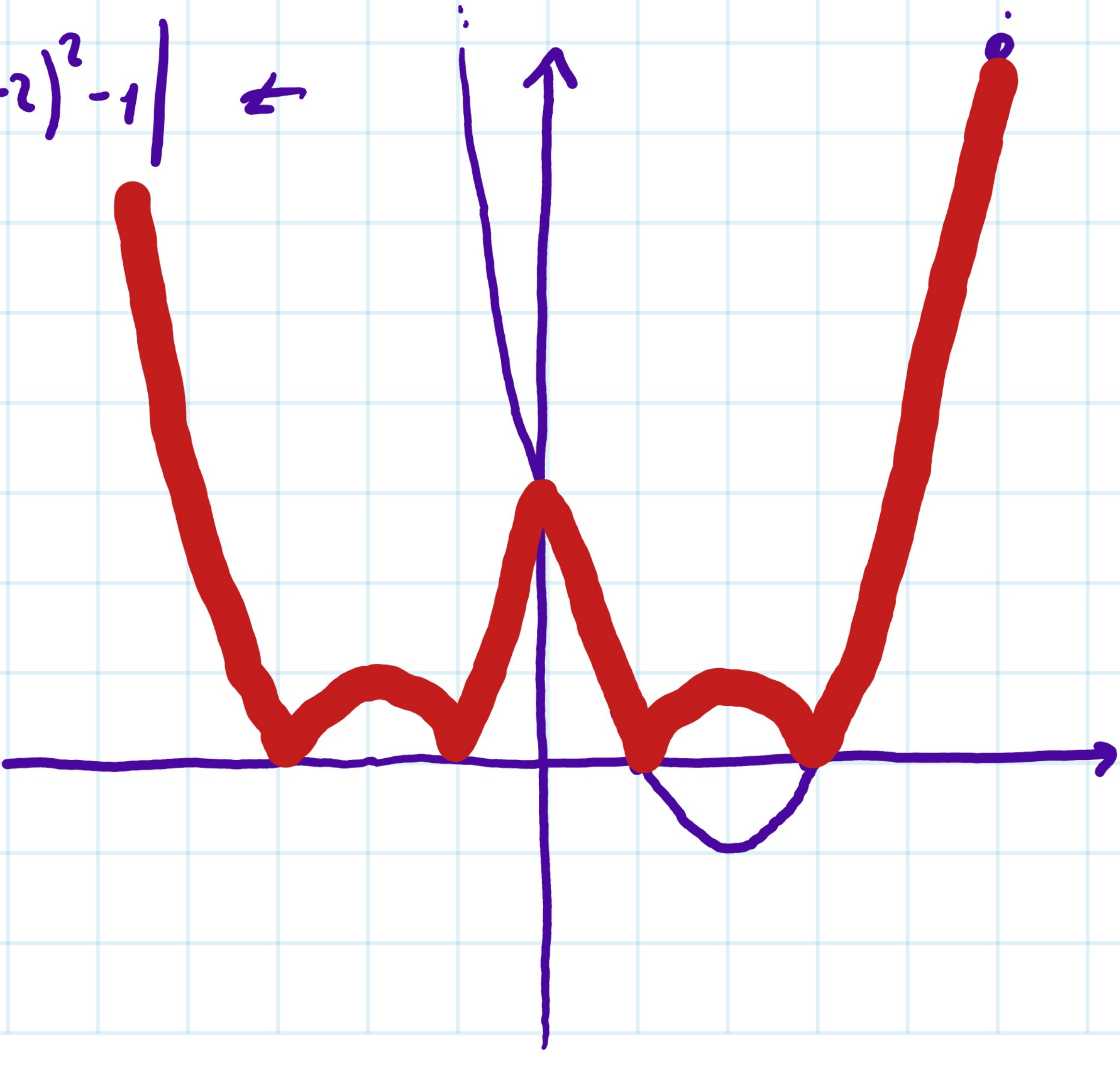
x^2

$(x-2)^2$

$\rightarrow (x-2)^2 - 1$

$\rightarrow \left|(x-2)^2 - 1\right| \leftarrow$

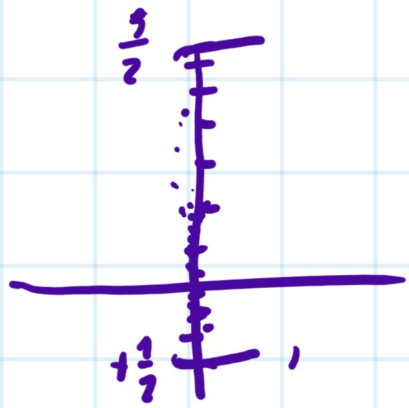
$\left|(|x|-2)^2 - 1\right| \leftarrow$



14) $\forall \varepsilon > 0$, FREQ. IN n, DEFIN. IN m, $\left(\frac{3}{2} + \sin n\right)^m < \varepsilon$

PERMUTARE

$$\forall \varepsilon > 0 \left(\text{FREQ IN } n \left(\text{DEFIN. IN } m \quad \underbrace{\left(\frac{3}{2} + \sin n\right)}_{<1}{}^{m} < \varepsilon \right)\right)$$



$0 < A < 1$

$A^m \to 0$

FREQ IN n, DEF. IN m, $\forall \varepsilon > 0$ $\left(\frac{3}{2} + \sin n\right)^m < \varepsilon$

$$\text{FREQ. IN } n \left(\forall \varepsilon > 0, \left(\text{DEF. IN } m \quad \left(\frac{3}{2} + \sin n\right)^m < \varepsilon \right)\right)$$

DATO $A \subset \mathbb{R}$, QUANTI INSIEMI SI POSSONO OTTENERE, AL MASSIMO, PARTENDO DA A E ESTRAENDO, RIPETUTAMENTE, PARTE INTERNA E CHIUSURA, NELL'ORDINE CHE SI PREFERISCE?