

ESERCITAZIONI DI ANALISI 1

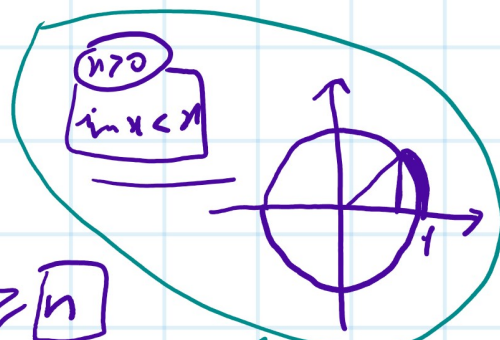
EXE. 6

28 OTTOBRE 2022

(LA NUMERAZIONE DEGLI ESERCIZI È QUELLA DEL CAPITOLO 2 DELL'ESERCIZIARIO)

22) $\lim_{n \rightarrow +\infty} (2n - \sin n) = +\infty$

$$\boxed{2n - \sin n} = n + \underbrace{(n - \sin n)}_{\geq 0} \geq \boxed{n} \rightarrow +\infty$$



NON SERVE
BASTA OSSERVARE CHE
 $\sin n \leq 1$
PER AVERE
INTERO POSITIVO

26) $\lim_{n \rightarrow +\infty} \frac{n^2 - 1}{n - \sqrt{n}}$

$$\boxed{\frac{n^2 - 1}{n - \sqrt{n}}} = \frac{(n+1)(n-1)}{n - \sqrt{n}} \geq \frac{(n+1)(\cancel{n-1})}{\cancel{n-1}} = n+1 > \boxed{n} \rightarrow +\infty$$

28) $\lim_{n \rightarrow +\infty} \log_2 (4^n - 2^n)$

$$\log_2 (2^n \cdot (2^n - 1)) \geq \log_2 2^n = \boxed{n} \rightarrow +\infty$$

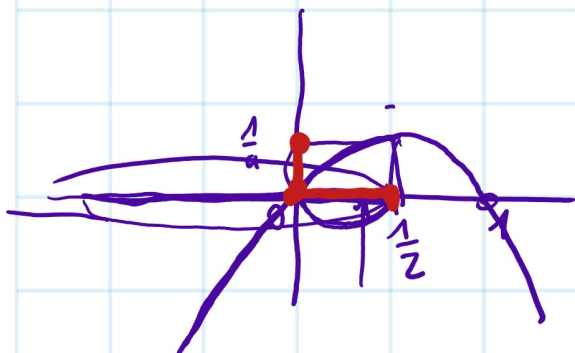
$$33) \quad \begin{cases} a_{n+1} = f(a_n) \\ a_{n+1} = a_n - a_n^2 \quad n \geq 0 \\ a_0 = \frac{1}{2} \end{cases} \Rightarrow \lim_{n \rightarrow +\infty} a_n = 0$$

$$a_n \leq \frac{1}{n+2} \leftarrow$$

$$n=0 \quad \text{SI}$$

$$n=k \Rightarrow n \rightarrow k+1 \quad ??$$

$$f(x) = x - x^2$$



$$a_n \leq \frac{1}{n+2}$$

$$a_k \leq \frac{1}{k+2}$$

$$a_{k+1} = a_k - a_k^2 = f(a_k) \leq f\left(\frac{1}{k+2}\right) =$$

$$= \frac{1}{k+2} - \frac{1}{(k+2)^2} =$$

$$= \frac{k+1}{(k+2)^2} \stackrel{?}{\leq} \frac{1}{k+3}$$

$$0 \leq a_n \leq \frac{1}{n+2}$$

$$f\left[\left[0, \frac{1}{2}\right]\right] = \left[0, \frac{1}{4}\right] \subset \left[0, \frac{1}{2}\right]$$

$$\frac{k+1}{(k+2)^2} \leq \frac{1}{k+3}$$

$$(k+1)(k+3) \leq (k+2)^2$$

$$\cancel{k^2} + 4k + 3 \leq \cancel{k^2} + 4k + 4$$

$$0 \leq a_n \leq \frac{1}{n+2} \leq \frac{1}{n+3}$$

PER
CASA

$$\left\{ \begin{array}{l} a_0 = 1 \\ a_{n+1} = \sin(a_n) \end{array} \right. \quad \lim_{n \rightarrow +\infty} a_n = ??$$

ORD. DI INFINITO / INFINITESIMO

DEF DATE $(a_n), (b_n)$ T.C. $a_n \rightarrow +\infty \quad b_n \rightarrow +\infty$

DIREMO CHE

(1) $a_n = o(b_n)$ SE $\frac{a_n}{b_n} \rightarrow 0$

(2) a_n HA STESSO ORDINE DI b_n
SE $\exists c_1, c_2$ CON $0 < c_1 < c_2$ t.c.

DEF IN n

$$c_1 < \frac{a_n}{b_n} < c_2$$

(3) $a_n \approx b_n$ SE $\frac{a_n}{b_n} \rightarrow 1$

(4) $a_n = \Theta(b_n)$ SE $\exists c > 0$ t.c.

DEF. IN n

$$\frac{a_n}{b_n} \leq c$$

$$a_n = n \quad \parallel \\ b_n = n^2 \quad \parallel$$

$$\frac{a_n}{b_n} = \frac{n}{n^2} = \frac{1}{n} \rightarrow 0$$

$$a_n = n^2(2 + \sin n)$$

$$b_n = n^2 \\ \frac{a_n}{b_n} = (2 + \sin n)$$

$$a_n = n \\ b_n = n+1$$

$$\frac{a_n}{b_n} = \frac{n}{n+1} =$$

$$= 1 - \frac{1}{n+1}$$

(4)

T.1

LE SUCCESSIONI:

$$\rightarrow \log_a(n)$$

$$\rightarrow n^\alpha$$

$$\rightarrow A^n$$

$$n!$$

$$n^n$$

$$a > 1$$

$$\alpha > 0$$

$$A > 1$$

$$\left. \begin{array}{l} a > 1 \\ \alpha > 0 \\ A > 1 \end{array} \right\} \forall$$
TENDONO TUTTE A $+\infty$ E SI HA:

$$\log_a(n) = o(n^\alpha)$$

$$n^\alpha = o(A^n)$$

$$A^n = o(n!)$$

$$n! = o(n^n)$$

INOLTRE

$$\log_a(n) \text{ HANNO TUTTE STESSO ORDINE}$$

$$n^\alpha = o(n^\beta) \text{ SE } \alpha < \beta$$

$$A^n = o(B^n) \text{ SE } A < B$$

$$\rightarrow \log_a(n) \ll n^\alpha \ll A^n \ll n! \ll n^n$$

$\uparrow$ $\uparrow$
 $(n^\alpha)^\alpha \ll A^{n^\alpha}$

$$57) \lim_{n \rightarrow +\infty} \frac{n^{2n}}{2^{n^2}} = 0$$

$$\left(\frac{2^{n^2}}{n^{2n}} \right) = \frac{(2^n)^n}{(n^2)^n} = \left(\frac{2^n}{n^2} \right)^n \geq \left(\frac{2^n}{n^2} \right) = \frac{2^n}{n^2} \rightarrow +\infty$$

DIM ① $n! = o(n^n)$ (?)

$$\frac{n^n}{n!} \rightarrow +\infty$$

$$= \frac{n}{1} \cdot \frac{n}{2} \cdot \frac{n}{3} \cdots \frac{n}{n} \geq \frac{n}{1} \cdot 1 = n \rightarrow +\infty$$

② $\forall A > 1 \quad A^n = o(n!)$

$$o < \frac{A^n}{n!} = \frac{A \cdot A \cdot A \cdots A}{1 \cdot 2 \cdot 3 \cdots n} =$$

$$= \underbrace{\frac{A}{1}}_{\leq A} \cdot \underbrace{\frac{A}{2}}_{< 1} \cdot \frac{A}{n} < \underbrace{\frac{A}{1}}_{\leq A} \cdot \underbrace{\frac{A}{n}}_{\rightarrow 0} \rightarrow 0$$

$$(3) \quad \forall \alpha > 0 \quad \forall A > 1 \quad n^\alpha = o(A^n)$$

PRIMA PER $\alpha = \frac{1}{2}$

$$\sqrt{n} = o(A^n)$$

$\sqrt{n} \rightarrow \infty$

$\sqrt{n} \rightarrow l$

$\sqrt{n} \cdot \sqrt{n} \rightarrow l \cdot l = l^2$

$n \rightarrow l^2 \in \mathbb{R}$

(Assumption)

$$\left(\frac{A^n}{\sqrt{n}} \right) = \frac{(1+\delta)^n}{\sqrt{n}} \geq \frac{1+n\delta}{\sqrt{n}} > \frac{n\delta}{\sqrt{n}} = \delta \cdot \sqrt{n}$$

$$\left(\frac{A^n}{n^\alpha} \right) = \frac{A^n}{(\sqrt{n})^{2\alpha}} = \left(\frac{\left(A^{\frac{1}{2\alpha}} \right)^n}{\sqrt{n}} \right)^{2\alpha}$$

$B > 1$

$\left(\frac{B^n}{\sqrt{n}} \right)^{2\alpha} \rightarrow +\infty$

$$\forall M > 0$$

$$M^{\frac{1}{2\alpha}} > 0$$

$$\left(\frac{B^n}{\sqrt{n}} \right)^{2\alpha} > \left(M^{\frac{1}{2\alpha}} \right)^{2\alpha} = M$$

$$\left(\frac{n^\alpha}{\log_a n} \right) \rightarrow +\infty ?$$

$$= \frac{a^{\log_a n^\alpha}}{\log_a n}$$

$$\boxed{\frac{a^{\log_a(n^a)}}{\log_a n}} = \frac{(a^a)^{\log_a n}}{\log_a n} = \frac{B^{\log_a n}}{\log_a n} \rightarrow +\infty$$

$B > 1$

$$\frac{B^n}{n} \rightarrow +\infty$$

[...]

$$\boxed{\frac{A^n}{n^a} \rightarrow +\infty} \quad \boxed{\frac{A^{a_n}}{(a_n)^a} \rightarrow +\infty} \quad \text{SE } a_n \rightarrow +\infty$$

$a_n \rightarrow +\infty$

$$\boxed{\frac{A^{a_n}}{(a_n)^a} \geq \frac{A^{\lfloor a_n \rfloor}}{(\lfloor a_n \rfloor + 1)^a} = \boxed{\frac{A^{\lfloor a_n \rfloor + 1}}{(\lfloor a_n \rfloor + 1)^a}} \cdot \boxed{\frac{1}{A}} \rightarrow +\infty$$

$k \rightarrow +\infty$

$\frac{1}{A} > 0$

$$\frac{A^k}{k^a} \rightarrow +\infty$$

$$n \rightarrow +\infty$$

$$\left(1 + \frac{1}{n}\right)^n \rightarrow e$$

$$a_n \rightarrow +\infty$$

$$\boxed{\left(1 + \frac{1}{a_n}\right)^{a_n} \rightarrow e}$$

55)

$$\lim_{n \rightarrow +\infty} \frac{100^{\sqrt{n}}}{10^n} = 0$$

$$0 \leq \frac{100^{\sqrt{n}}}{(10^{\sqrt{n}})^{\sqrt{n}}} = \left(\frac{100}{10^{\sqrt{n}}} \right)^{\sqrt{n}} < \frac{100}{10^{\sqrt{n}}} \quad \sqrt{n} > 1$$

$$\downarrow$$

$$0$$

$$10^{\sqrt{n}} \rightarrow +\infty$$

$$\frac{10^{\sqrt{n}}}{\sqrt{n}} \rightarrow +\infty$$

170)

$$\lim_{n \rightarrow +\infty} \frac{n^2}{(\ln(\ln n))^{\ln n}} = 0$$

$$\rightarrow = \frac{n^2}{e^{\ln((\ln(\ln n))^{\ln n})}} = \frac{n^2}{(e^{\ln n})^{\ln(\ln(\ln n))}}$$

$$= \frac{n^2}{n^{\ln(\ln(\ln n))}} \leq \frac{n^2}{n^3} = \frac{1}{n} \rightarrow 0$$