

Esercitazioni di Analisi 1

Exe 9

11 Novembre 2022

PROBLEMI PROPOSTI

1) LIMITI NOTEVOLI (fine)

PROX. LEZ.

2) USO TEO. PONTE (16-26 PAG 111)

3) ALTRI LIMITI (27-60 PAG 111-112)

... DALLA LEZIONE SCORSA
(limiti e limiti)

$$|135| a_n = \left(1 + \frac{\sin \frac{n\pi}{3}}{n}\right)^n$$

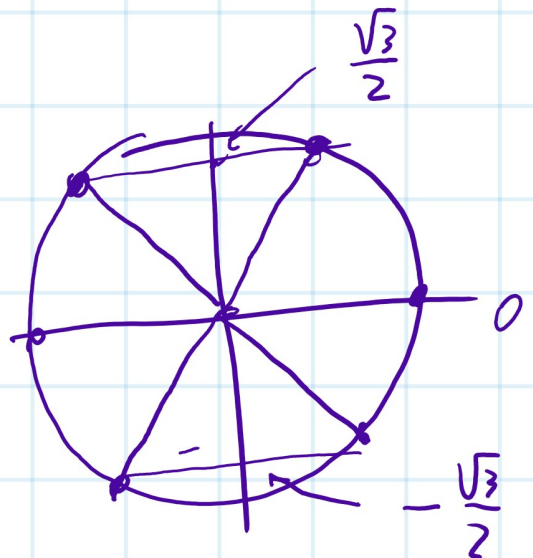
$$|136| a_n = \left(1 + \frac{\sin n}{n}\right)^n$$

$$|139| a_n = \sqrt{n} (\sin \sqrt{n})^2$$

SOLUZIONI

135

$$\left(1 + \frac{0}{n}\right)^n \xrightarrow{n=3k} 1$$



$$n = 6k+1, 6k+2$$

$$\left(1 + \frac{\frac{\sqrt{3}}{2}}{n}\right)^n \rightarrow e^{\frac{\sqrt{3}}{2}} \leftarrow \text{limit.}$$

$$n = 6k+4, 6k+5$$

$$\left(1 - \frac{\frac{\sqrt{3}}{2}}{n}\right)^n \rightarrow e^{-\frac{\sqrt{3}}{2}} \leftarrow \text{limit.}$$

136

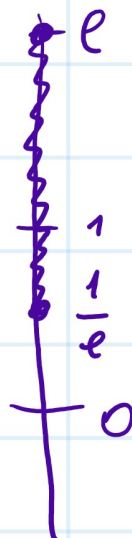
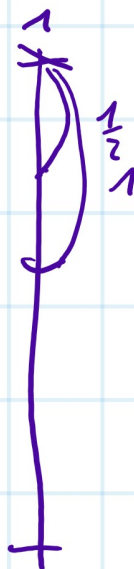
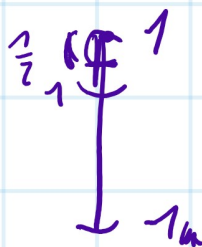
$$136 \quad a_n = \left(1 + \frac{\sin n}{n}\right)^n$$

$\sin n$

$\sin n_k$

$\sin n_1$
 $\sin n_2$
 $n_2 > n_1$
 $\vdots$

$\sin n_k \in \left(1 - \frac{1}{k}, 1\right)$
 $n_k > n_{k-1}$

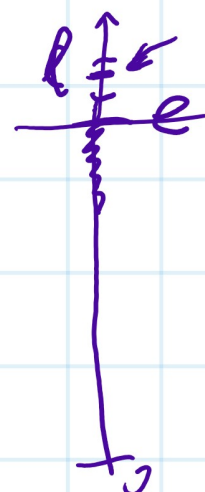


$(n_k) \text{ s.t. } \sin n_k \rightarrow 1$

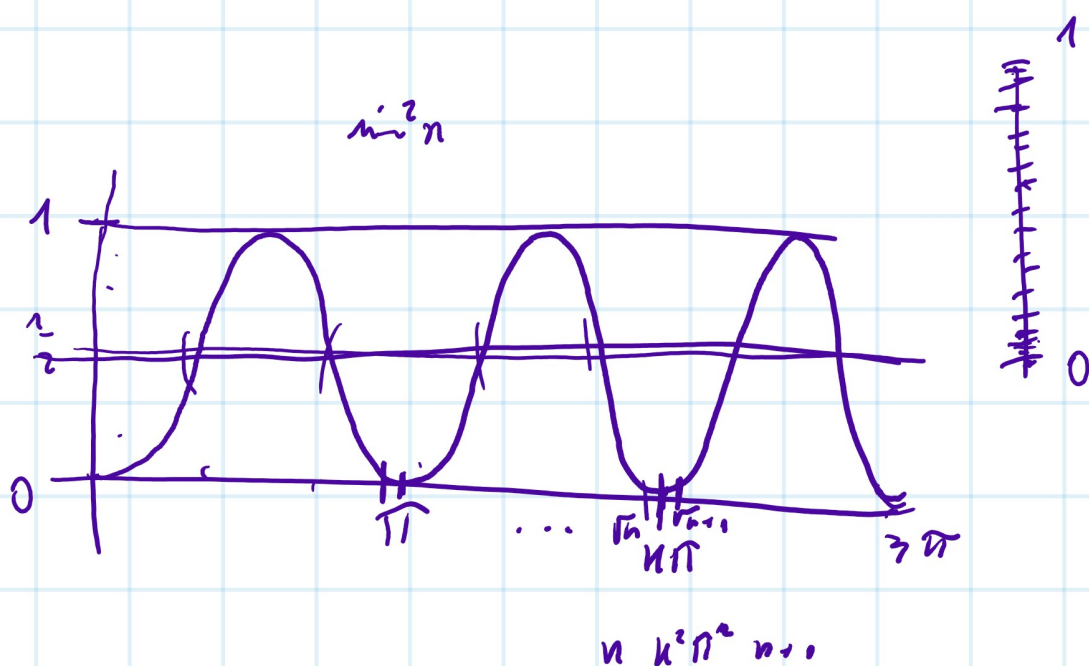
$$\left(1 + \frac{\sin h_k}{h_k}\right)^{h_k} = \left(1 + \frac{1}{\frac{h_k}{\sin h_k}}\right)^{\frac{h_k}{\frac{h_k}{\sin h_k}}}$$

Diagram illustrating the limit definition of e . The expression is shown as a limit of $\left(1 + \frac{1}{n}\right)^n$ as $n \rightarrow \infty$. The diagram shows a sequence of terms $\frac{h_k}{\sin h_k}$ and $\sin h_k$ approaching 1, leading to the conclusion $e^1 = e$.

$$\left(1 + \frac{\sin h}{h}\right)^h \leq \left(1 + \frac{1}{h}\right)^h < e$$



139 $a_n = \sqrt{n} (\sin \sqrt{n})^2$



$$N_k = \lfloor k^2 \pi^2 \rfloor$$

$$\sqrt{n} < k\pi < \sqrt{n+1}$$

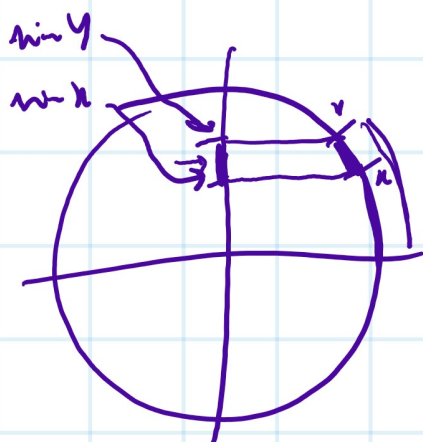
$$n < k^2 \pi^2 < n+1$$

$$Q_n = \sqrt{n} \left(\sin \sqrt{n} \right)^2$$

$$Q_{n_k} = \sqrt{\lfloor k^2 \pi^2 \rfloor} \cdot \left(\sin \sqrt{\lfloor k^2 \pi^2 \rfloor} \right)^2 < k\pi \cdot \frac{1}{\lfloor k^2 \pi^2 \rfloor} < \frac{k\pi}{k^2 \pi^2 - 1}$$

$$\left(\sin \sqrt{\lfloor k^2 \pi^2 \rfloor} - \sin k\pi \right)^2 \leq$$

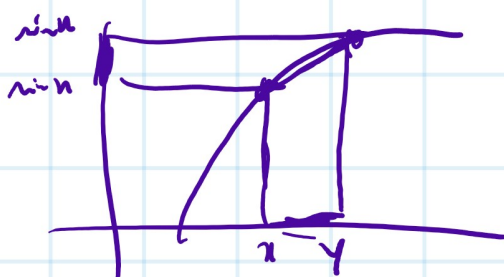
$$\leq \left(\sqrt{\lfloor k^2 \pi^2 \rfloor} - \sqrt{k^2 \pi^2} \right)^2 \leq$$



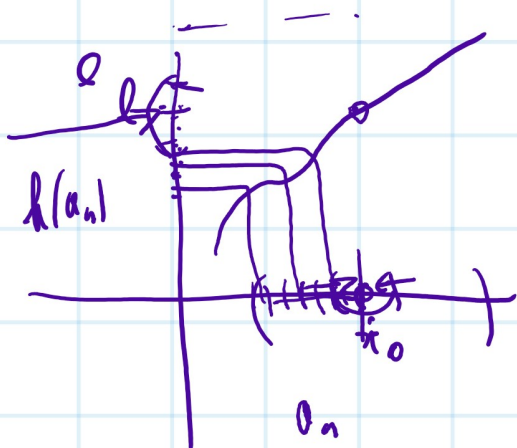
$$|\sin x - \sin y| \leq |x - y|$$

$$\sin(x + \Delta x) = \sin x \cos \Delta x + \cos x \sin \Delta x$$

$$\begin{aligned}
 & \hookrightarrow \leq \left(\sqrt{\frac{[n^2 n^2]}{n_k}} - \sqrt{[n^2 n^2] + 1} \right)^2 = \\
 & \quad \text{to} \\
 & = \left(\sqrt{n_{k+1}} - \sqrt{n_k} \right)^2 = \\
 & = \left(\frac{1}{\sqrt{n_{k+1}} + \sqrt{n_k}} \right)^2 < \left(\frac{1}{\sqrt{n_k}} \right)^2 = \frac{1}{n_k}
 \end{aligned}$$



$$\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = e$$



$$a_n \xrightarrow{(-\infty)} +\infty$$

$$\left(1 + \frac{1}{a_n}\right)^{a_n} \rightarrow e$$

$$f(a_n)$$

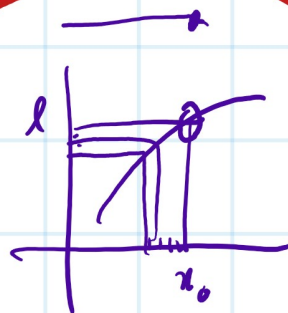
$$f(x) = \left(1 + \frac{1}{x}\right)^x$$

$\neq \infty$

$$\lim_{n \rightarrow 0} \left(1 + n\right)^{\frac{1}{n}} = e$$

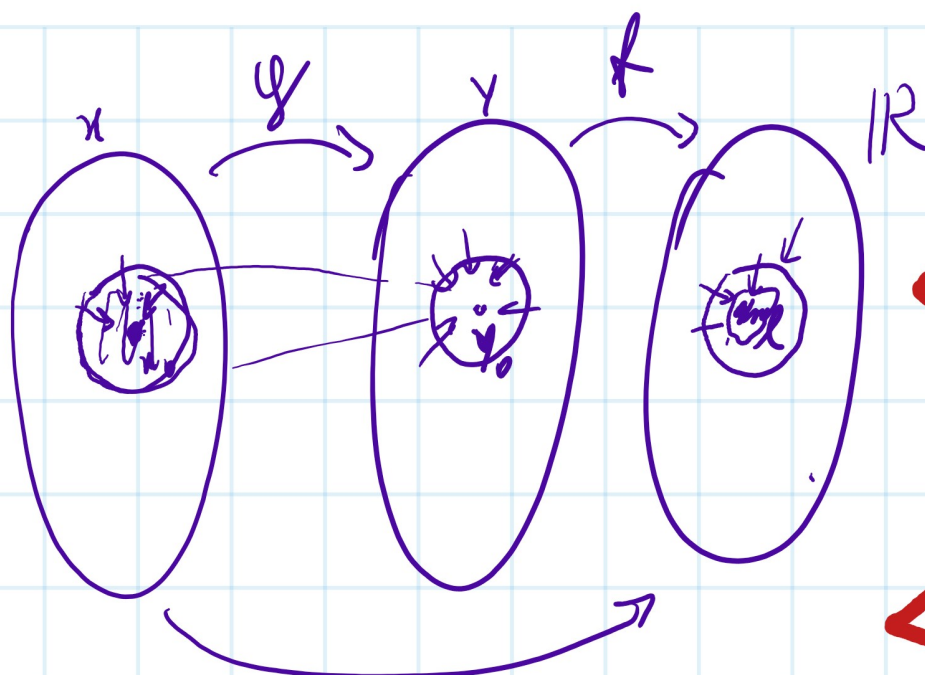
$$\lim_{n \rightarrow 0^+} (1+n)^{\frac{1}{n}} \xrightarrow{\substack{v = \frac{1}{n} \\ \uparrow}} \lim_{y \rightarrow +\infty} \left(1 + \frac{1}{y}\right)^y = e$$

$$\lim_{n \rightarrow x_0} f(g(n)) = l$$



$$\begin{cases} \lim_{n \rightarrow x_0} g(n) = y_0 \\ \lim_{y \rightarrow y_0} f(y) = l \end{cases}$$

$$\text{LIMITE } f(g(n))$$



$$\lim_{x \rightarrow x_0} g(x) = y_0$$

$$\lim_{y \rightarrow y_0} f(y) = l$$

+ CONDIZIONE
AGGIUNTIVA

$$f(g(x))$$

f continua in y_0

oppure
 $g(x) \rightarrow y_0$ "SENZA TOCCARLO"

$$\lim_{x \rightarrow 0^+} \left(1 + x\right)^{\frac{1}{x}} =$$

$$= \lim_{x \rightarrow 0^+} \left(1 + \frac{1}{\left(\frac{1}{x}\right)}\right)^{\left(\frac{1}{x}\right)} \quad \left(\frac{1}{x}\right) \quad \left(y = \frac{1}{x}\right)$$

$$= \lim_{y \rightarrow +\infty} \left(1 + \frac{1}{y}\right)^y = e$$

$$f(y) = \left(1 + \frac{1}{y}\right)^y$$

$$g(x) = \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}$$

$$y = (1+x)^{\frac{1}{x}}$$

$$f(y) = \ln(y)$$

$$g(x) = (1+x)^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \ln\left((1+x)^{\frac{1}{x}}\right)$$

$$= \lim_{y \rightarrow e} \ln y = \ln e = 1$$

$$y = e^x - 1$$

$$f(y) = \frac{y}{\ln(1+y)}$$

$$g(x) = e^x - 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{e^x - 1}{\ln(1 + (e^x - 1))}$$

$$= \lim_{y \rightarrow 0} \frac{y}{\ln(1+y)} = \lim_{y \rightarrow 0} \frac{1}{\frac{\ln(1+y)}{y}} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{e^{\frac{1}{2} \ln(1+x)} - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{e^{\frac{1}{2} \ln(1+x)} - 1}{\frac{1}{2} \ln(1+x)} \cdot \frac{\frac{1}{2} \ln(1+x)}{x} = \frac{1}{2}$$

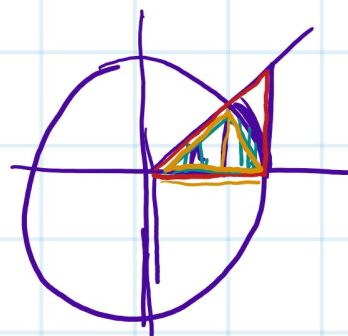
$$\lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} =$$

$$= \lim_{x \rightarrow 0} \frac{e^{\alpha \ln(1+x)} - 1}{\alpha \ln(1+x)} \cdot \frac{\alpha \ln(1+x)}{x} = \alpha$$

$\downarrow$ 1 $\downarrow$ α

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$$

$$\sin x < x < \tan x$$



$$\sin x < x < \tan x$$

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

$\downarrow$ 1 $\downarrow$ 1 $\downarrow$ 1

$$\frac{1 \cdot \sin x}{2} < \frac{1 \cdot x}{2} < \frac{1 \cdot \tan x}{2}$$

$\downarrow$ $\downarrow$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\arcsin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{\sin x \left(\frac{1 - \cos x}{\cos x} \right)}{x^3} =$$

$$= \lim_{x \rightarrow 0} \underbrace{\frac{\sin x}{x}}_1 \cdot \underbrace{\frac{1 - \cos x}{x^2}}_{\frac{1}{2}} \cdot \underbrace{\frac{1}{\cos x}}_1 = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{1 + \cos x} \cdot \frac{1}{x^2} =$$

$$= \lim_{x \rightarrow 0} \underbrace{\left(\frac{\sin x}{x} \right)^2}_{1^2} \cdot \underbrace{\frac{1}{1 + \cos x}}_{\frac{1}{2}} = 1^2 \cdot \frac{1}{2} = \frac{1}{2}$$