Esercitazioni di Analisi 1 - A.A. 21/22 - Callegari

Lezione 14

10 Gennaio 2022

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DATA $\boxed{f(x)=\ln\left(\frac{e^{x+1}+e^{-x}}{e^{x}+e^{-x}}\right)}$

A) STUDIARLA

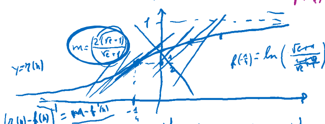
B) DIRE QUANTE SOL HA L'EQUAZIONE $f(x)=\frac{1}{2}+\frac{1}{5}\left(x+\frac{1}{4}\right)$

C) PER QUALI m ESISTONO RETTE CON PENDENZA m CHE INTERSECANO IL GRAF. DI f IN ~~ALMENO~~ ESATTAMENTE 3 PUNTI

DOMINIO $=\mathbb{R}$

$$\lim_{x\to+\infty}\ln\left(\frac{e^{x+1}+e^{-x}}{e^{x}+e^{-x}}\right)=\ln e=1$$

$$\lim_{x\to-\infty}\ln(\dots)=\ln 1=0$$



$y=\frac{1}{2}+\boxed{\frac{1}{5}}\left(x-\left(-\frac{1}{4}\right)\right)$

$f'\left(-\frac{1}{4}\right)=\boxed{2\frac{\sqrt{e}-1}{\sqrt{e}+1}}$

$m=\frac{2(\sqrt{e}-1)}{\sqrt{e}+1}$

$y=r(x)$

$f\left(-\frac{1}{4}\right)=\ln\left(\frac{\sqrt{e}+1}{\frac{1}{\sqrt{e}}+1}\right)=\ln\sqrt{e}=\frac{1}{2}$

$(r(x)-f(x))'=m-f'(x)$

$$f'(x)=\left(\ln\frac{e^{x+1}+e^{-x}}{e^{x}+e^{-x}}\right)'=\left(\ln\frac{e^{2x+1}+1}{e^{2x}+1}\right)'=$$

$$=\left(\ln(e^{2x+1}+1)-\ln(e^{2x}+1)\right)'=$$

$$=\frac{2e^{2x+1}}{e^{2x+1}+1}-\boxed{\frac{2e^{2x}}{e^{2x}+1}}=$$

$$=\boxed{\frac{2e^{2x}}{e^{2x}+\frac{1}{e}}-\frac{2e^{x}}{e^{2x}+1}}>0$$

$$f''(x)=\left(\frac{2e^{2x+1}+2-2}{e^{2x+1}+1}-\frac{2e^{2x}+2-2}{e^{2x}+1}\right)'=$$

$$=\left(\cancel{2}-\frac{2}{e^{2x+1}+1}-\cancel{2}+\frac{2}{e^{2x}+1}\right)'=$$

$$=2\left(-\frac{1}{e^{2x+1}+1}+\frac{1}{e^{2x}+1}\right)'=$$

$$=2\cdot\left(\frac{2e^{2x+1}}{(e^{2x+1}+1)^2}-\frac{2e^{2x}}{(e^{2x}+1)^2}\right)>0$$

$$\frac{e^{2x+1}}{(e^{2x+1}+1)^2}>\frac{e^{2x}}{(e^{2x}+1)^2}$$

$$\frac{e}{(e^{2x+1}+1)^2}>\frac{1}{(e^{2x}+1)^2}$$

$$(e^{2x+1}+1)^2<e\cdot(e^{2x}+1)^2$$

$$e^{2x+1}+1<\sqrt{e}\cdot(e^{2x}+1)$$

$$e\cdot e^{2x}+1<\sqrt{e}\cdot e^{2x}+\sqrt{e}$$

$$\sqrt{e}(\sqrt{e}-1)e^{2x}<\sqrt{e}-1$$

$e\cdot e^{2x}-\sqrt{e}\cdot e^{2x}<\sqrt{e}-1$

$(e-\sqrt{e})e^{2x}<\sqrt{e}-1$

$$\to\sqrt{e}\,e^{2x}<1$$

$$\boxed{e^{2x}<\frac{1}{\sqrt{e}}=e^{-\frac{1}{2}}}$$

$$e^{2x}<e^{-\frac{1}{2}}\qquad 2x<-\frac{1}{2}\qquad\to\boxed{x<-\frac{1}{4}}$$

$f'(x)=\frac{-2}{e^{2x+1}+1}+\frac{2}{e^{2x}+1}$

$f'\left(-\frac{1}{4}\right)=\frac{-2}{\sqrt{e}+1}+\frac{2}{\frac{1}{\sqrt{e}}+1}=$

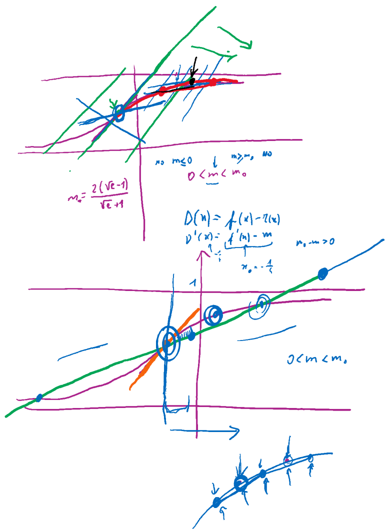
$=\frac{-2}{\sqrt{e}+1}+\frac{2\sqrt{e}}{\sqrt{e}+1}=$

$=\frac{-2+2\sqrt{e}}{\sqrt{e}+1}=$

$=\frac{2(\sqrt{e}-1)^2}{e-1}=$

$=\frac{2(\sqrt{e}-1)^2}{(\sqrt{e}-1)(\sqrt{e}+1)}=$

$=2\frac{\sqrt{e}-1}{\sqrt{e}+1}$ (m)



$m=\frac{2(\sqrt{e}-1)}{\sqrt{e}+1}$

$m\le 0$ $m\ge m_0$ NO

$0<m<m_0$

$D(x)=f(x)-r(x)$

$D'(x)=f'(x)-m$

$m_0>0$

$x_0=-\frac{1}{4}$

$0<m<m_0$

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QUANTE SOL. HA L'EQUAZIONE

$$\boxed{e^{x^4}+\cos x=2}$$

$f(x)=0$

$f(x)=(e^{x^4}-1)+(\cos x-1)=0$

$f(0)=0$

$$f(x)=\left[e^{x^4}-1+\cos x-1\right]$$

$$=x^4+o(x^8)-\frac{x^2}{2}+\frac{x^4}{24}+o(x^6)$$

$$=-\frac{x^2}{2}+\frac{25}{24}x^4+o(x^6)=$$

$$\frac{x^2}{2}\left(-1+\frac{25}{12}x^2+o(x^4)\right)$$

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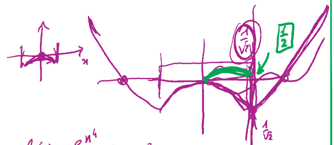
PER QUALI $\alpha>1$ L'EQUAZIONE

$\alpha\arctan x=\sin(\alpha x)$

HA UNA SOLA SOLUZIONE?

$$\frac{x^2}{2}\left(-1+\frac{25}{12}x^2+o(x^4)\right)$$



$$f(x)=e^{x^4}+\cos x-2$$

$$f'(x)=4x^3e^{x^4}-\sin x>0$$

$x>\frac{1}{\sqrt{2}}$

$\frac{4}{(\sqrt{2})^2}=\sqrt{2}>1$

$\boxed{x\ge\frac{1}{\sqrt{2}}}$

$$f(x)=e^{x^4}+\cos x-2=\;=-\frac{x^2}{2}+\left(\frac{25x^4}{24}+o(x^6)\right)$$

$$-\frac{x^2}{2}\left(-1+\dots\right)$$

$0<\xi<x^4$

$0<\xi<t$

$$e^t=1+t+\boxed{\frac{f''(\xi)}{2}t^2}\qquad\boxed{\frac{e^{\xi}}{2}t^2}$$

$$e^{x^4}=1+x^4+\frac{e^{\xi}}{2}x^8<$$

$$<1+x^4+\frac{\sqrt[4]{e}}{2}x^8$$

$0\le x\le\frac{1}{\sqrt{2}}$

$0<\xi<x$

$\cos x=1-\frac{x^2}{2}+\frac{x^4}{24}+\frac{-\cos\xi}{720}x^6\le$

$\le 1-\frac{x^2}{2}+\frac{x^4}{24}$

$$f(x)=e^{x^4}+\cos x-2\le$$

$$\le\cancel{1}+x^4+\frac{\sqrt[4]{e}}{2}\cdot x^8+\cancel{1}-\frac{x^2}{2}+\frac{x^4}{24}-\cancel{2}$$

$x<\frac{1}{\sqrt{2}}$

$$=-\frac{x^2}{2}+\frac{25}{24}x^4+\frac{\sqrt[4]{e}}{2}x^8=$$

$x<\frac{1}{2}$

$$=+\frac{x^2}{2}\left(-1+\frac{25}{12}\cdot x^2+\sqrt[4]{e}\,x^6\right)<$$

$$=\frac{x^2}{2}\left(-1+\frac{25}{24}\cdot\frac{1}{2}+\sqrt[4]{e}\cdot\frac{1}{64}\right)<$$

$$<-1+\frac{1}{2}+\frac{1}{48}+\frac{1}{32}<0$$

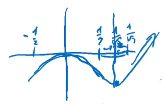
$\frac{1}{\sqrt{2}}$

$0<x<\frac{1}{\sqrt{2}}$

$0<x<\frac{1}{2}$

$\frac{1}{\sqrt{3}}$

$f'($



$\frac{4}{\sqrt{3}}>\frac{1}{\sqrt{3}}$

$f'(x)=4x^3e^{x^4}-\sin x>0$

$\frac{4}{3\sqrt{3}}\quad e^{\frac{1}{9}}$

$\frac{1}{\sqrt{3}}$

$>\boxed{\frac{4}{3\sqrt{3}}e^{\frac{1}{9}}-\frac{\sqrt{3}}{2}}=$

$=\frac{\sqrt{3}}{2}\left(\frac{8}{9\sqrt{3}}e-1\right)$

>1