Esercitazioni di Analisi 1 - A.A. 21/22 - Collegari

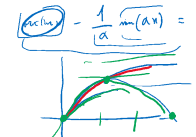
Lezione 15

12 Gennaio 2022

1 Sia $f(x)=\arctan\left(-\frac{1}{\alpha}\sin(\alpha x)\right)$ con α parametro positivo.

(A) Per $\alpha=\sqrt{3}$, quante sol. ha l'equazione $f(x)=0$

(B) Per quali $\alpha>0$ l'eq. $f(x)=0$ ha una sola soluzione.



$$\arctan\left(-\frac{1}{\alpha}\sin(\alpha x)\right)=$$

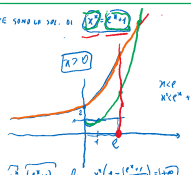
$$=\cancel{x}-\frac{x^3}{3}+\frac{x^5}{5}-\dots\dots-\frac{1}{\alpha}\left(\cancel{\alpha x}-\frac{(\alpha x)^3}{6}+\frac{(\alpha x)^5}{120}-\dots\right)$$

$$=\left(-\frac{1}{3}+\frac{\alpha^2}{6}\right)x^3+\dots\dots$$

$$\text{Asint.} = -Ax^3+\dots$$

$$=\cancel{\text{asint.}}=$$

2 Quante sono le sol. di $\boxed{x^2=e^{x+1}}$



$\boxed{x>0}$

$\forall x\in(0,e^2)\ \ x^2<e^{x+1}$

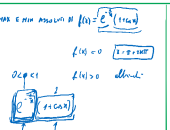
$$\lim_{x\to+\infty}\frac{x^2}{e^{x+1}}=\lim_{x\to+\infty}x^2\left(1-\frac{\left(\frac{e^{x+1}}{x^2}\right)}{x^2}\right)=\boxed{+\infty}$$

$$\left(x^2-e^{x+1}\right)'=\left(e^{2\ln x}-e^{x+1}\right)'=$$

$$=e^{2\ln x}\cdot\left(\frac{2}{x}\right)-e^{x+1}=$$

$$=x^2\left(\frac{2}{x^2}\right)-e^{x+1}\geq 2-e^{x+1}<0$$

3 Trovare, se esistono, max e min assoluti di $f(x)=e^{\frac{1}{x}}(1+\cos x)$ su $(0,+\infty)$



$f(x)=0$ $\boxed{x=\frac{\pi}{2}+2k\pi}$

Occhio! $f(x)>0$ altrimenti...

$$f(2k\pi)=e^{\frac{1}{2k\pi}}\cdot 2\to 2$$

$\forall x>0\ \ f(x)\geq 2$

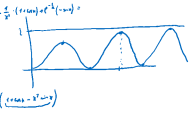
$\exists x_k\ \ f(2k\pi)\to 2$ per $k\to+\infty$

$$\Rightarrow \min_{x\in(0,+\infty)} f(x)=2$$

max non esiste

$$f'(x)=\left(e^{\frac{1}{x}}\cdot(1+\cos x)\right)'=$$

$$=e^{\frac{1}{x}}\cdot\frac{-1}{x^2}\cdot(1+\cos x)+e^{\frac{1}{x}}(-\sin x)=$$



$$=\frac{e^{\frac{1}{x}}}{x^2}\left(-\cos x-x^2\sin x\right)$$

4 Dire se è iniettiva $f:\mathbb{R}\to\mathbb{R}$ definita da $f(x)=\arctan x+\sin\frac{1}{x^2+1}$

$$f'(x)=\left(\arctan x+\sin\frac{1}{x^2+1}\right)'=$$

$$=\frac{1}{1+x^2}+\left(\cos\frac{1}{x^2+1}\right)\cdot\left(-\frac{2x}{(x^2+1)^2}\right)=$$

$$=\frac{1}{1+x^2}\left(1-\frac{2x}{x^2+1}\cdot\cos\frac{1}{x^2+1}\right)>0$$

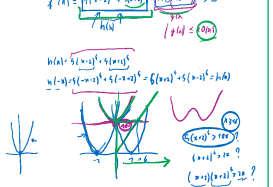
$$\left|\frac{2x}{x^2+1}\cdot\cos\frac{1}{x^2+1}\right|\leq 1\ ?$$

$$2\cdot|x|\cdot 1\leq 1+x^2\ ?$$

$$1+|x|^2-2|x|+1\geq 0\ ?$$

$$(1-|x|)^2\geq 0$$

5 Dire se è iniettiva $f:\mathbb{R}\to\mathbb{R}$ definita da $f(x)=(x+1)^5+(x+2)^5-5\sin x$



$$f'(x)=\underbrace{5(x+1)^4+5(x+2)^4}_{h(x)}\underbrace{+10\cos x}_{\geq -10}>0$$

$f(x)\to\pm\infty$

$h(x)=5(x+1)^4+5(x+2)^4$

$h(-2)=5(-2+1)^4+5(-2+2)^4=5(-1)^4+5(-2+1)^4=2h(x)$

$5(x+1)^4>10$?

$(x+1)^4>2$?

$(x+2)^4\geq 10$?

$(x+1)^4>1$?

$x+1>1$?

6 Data $f(x)=\boxed{\ln\left(\cos x+2\sin^2 x+x^4\right)}$, calcolare $f^{(4)}(0)$ e $\boxed{f^{(4)}(0)}$

$$\text{Pol}=\boxed{a_0+a_1x+a_2x^2+\dots+a_8x^8}$$

$f^{(4)}(0)=P^{(4)}(0)=(a_4x^4)^{(4)}=4!\cdot a_4$

$f^{(6)}(0)\ \ \dots\ \ =6!\cdot a_6$

$f^{(8)}(0)\ \ \dots\ \ =8!\cdot a_8$

$$f(x)=\ln\left(\cos x+2\sin^2 x+x^4\right)=$$

$$=2\ln\left(\cos^2 x+2\sin^2 x+x^4\right)=$$

$$=2\ln\left(1-\sin^2 x+2\sin^2 x+x^4\right)=$$

$$=2\left(-\cancel{x^2}-\frac{x^4}{2}-\frac{\sin^6 x}{3}-\frac{\sin^8 x}{4}+\dots\right)+\cancel{2x^2}+2x^4=$$

$$=x^4-\sin^4 x-\frac{2}{3}\sin^6 x+\dots=$$

$$-\frac{4}{2}\left(x+\dots\right)^4$$

$$-\frac{6}{3}\left(x^3+\dots\right)^3$$

$$\left(x-\frac{x^3}{6}+\dots\right)^6=$$

$$=\left(x-\frac{x^3}{6}\right)^6+\dots$$

$$=\left(x-\frac{x^3}{6}\right)^6+\dots$$

$$=x^6-6x^4\frac{x^3}{6}+\dots=$$

$$=\boxed{x^6-x^4+\dots}$$

$$\ln(x+\sin x)=x-\frac{x^2}{2}+\frac{x^3}{3}+\dots$$

$$\frac{2}{3}x^6-\frac{4}{3}x^8+\dots-\frac{4}{3}x^8+\dots-\frac{4}{2}x^8+\dots=$$

$$=\left(-\frac{4}{3}+\frac{2}{3}-\frac{4}{2}\right)x^8+\dots=$$

$$=\left(-\frac{6}{6}\right)x^8$$

$$\left(x^4-\sin^4 x\right)=$$

$$=\left(x^2-\sin^2 x\right)\left(x^2+\sin^2 x\right)$$

$$=\left(x-\sin x\right)(x+\sin x)\left(x^2+\sin^2 x\right)$$

$$=\left(\frac{x^3}{6}+\dots\right)\left(2x-\frac{x^3}{6}+\dots\right)\left(x^2+\frac{x^3}{6}+\dots\right)^2$$

$$\left(\frac{x^3}{6}+\dots\right)\left(2x+\dots\right)\left(2x^2+\dots\right)$$

$$\left(\frac{x^3}{6}-\frac{x^5}{120}+\dots\right)\left(2x-\frac{x^3}{6}+\dots\right)\left(x^2+\frac{x^3}{6}+\dots\right)^2$$

$$\left(\frac{2}{3}x^6-\frac{x^8}{60}+\dots\right)\left(2x^2-\frac{x^4}{3}+\dots\right)\left(x^4+x^3-\frac{x^2}{3}+\dots\right)=$$

$$=\frac{2}{3}x^6-\frac{1}{30}x^8-\frac{2}{9}x^8-\frac{1}{3}x^8+\dots=$$

$$=\boxed{\frac{2}{3}x^6-\frac{4}{3}x^8+\dots}$$

7

$$f^{(4)}(0)=0$$

$$f^{(6)}(0)=0$$

$$f^{(8)}(0)=\dots\cdot 8!$$