Esercitazioni di Analisi 1 - A.A. 21/22 - Callegari

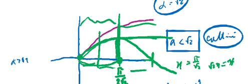
Lezione 16

13 Gennaio 2022

①

$f(x) = \arctan x - \frac{1}{\alpha}\sin \alpha x$ $\alpha > 0$

QUANTE SOL. HA $f(x) = 0$



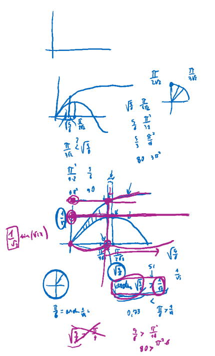
$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots - \frac{1}{\alpha}\left(\alpha x - \frac{(\alpha x)^3}{6} + \frac{(\alpha x)^5}{120} - \dots\right)$

$= \left(-\frac{1}{3} + \frac{\alpha^2}{6}\right)x^3 + \left(\frac{1}{5} - \frac{\alpha^4}{120}\right)x^5$

$\alpha = \sqrt{2}$

$f(x) = \arctan x - \frac{1}{\sqrt{2}}\sin(\sqrt{2}x)$

$\forall x > 0 \quad \exists \xi \in (0, \sqrt{2}x)$ t.c.



②

CONFRONTARE L'ORDINE DI INFINITO

$a_n = \left(\cos\frac{1}{n}\right)^{-\frac{n^5}{4}}$ $b_n = e^{\frac{n^3}{2}}$ $c_n = \left(1 + \frac{1}{n}\right)^{\frac{n^4}{2}}$

$d_n = \left(1 - \frac{1}{n}\right)^{-\frac{n^4}{2}}$ $e_n = \left(1 + \frac{1}{n^5}\right)^{\frac{n^8}{2}}$

$c_n = \left(1 + \frac{1}{n}\right)^{\frac{n^4}{2}} \approx e^{\frac{n^3}{2}}$ NO!

$= e^{\frac{n^4}{2}\ln\left(1 + \frac{1}{n}\right)} = e^{\frac{n^4}{2}\left(\frac{1}{n} - \frac{1}{2}\left(\frac{1}{n}\right)^2 + \frac{1}{3}\left(\frac{1}{n}\right)^3 + O\left(\frac{1}{n^4}\right)\right)} = e^{\frac{n^3}{2} - \frac{1}{4}n^2 + O(n)}$

$\frac{c_n}{e^{\frac{n^3}{2}}} = e^{-\frac{1}{4}n^2 + O(n)} \to 0$

$\left(\cos\frac{1}{n}\right)^{-\frac{n^5}{4}} = e^{-\frac{n^5}{4}\ln\left(1 - \left(1 - \cos\frac{1}{n}\right)\right)}$

$= e^{-\frac{n^5}{4}\left(-\left(1 - \cos\frac{1}{n}\right) - \frac{1}{2}\left(1 - \cos\frac{1}{n}\right)^2 + O\left(\frac{1}{n^6}\right)\right)}$

$= e^{\frac{n^3}{8} + \frac{1}{48}n + O\left(\frac{1}{n}\right)}$

$\frac{\left(\cos\frac{1}{n}\right)^{-\frac{n^5}{4}}}{e^{\frac{n^3}{8}}} = e^{\frac{1}{48}n + O\left(\frac{1}{n}\right)} \to +\infty$

③

STUDIARE UNIF. CONT. E LIPSC. SU $[0,1]$ E $[1,+\infty]$

DI $f(x) = \ln\left(1 + \sqrt{x}\cdot e^x\right)$

$g(x) = \sqrt{x + x^3}$

④

STUDIARE U.C. E LIP. DI

$f(x) = \sin x^2$ $g(x) = \sqrt{|\sin x|}$

$h(x) = \frac{\sin e^x}{1 + x^2}$ $F(x) = |\sin x|^{\frac{1}{x}}$

$G(x) = |\sin x|^x$

$\begin{cases} a_1 = x_0 \\ a_{n+1} = f(a_n) \end{cases}$

$f(x) = 2x$

$\begin{cases} a_{n+1} = 2a_n \\ a_1 = 5 \end{cases}$ $a_n = 5\cdot 2^{n-1} \to +\infty$

$S = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$

$S = \sqrt{1 + S}$

$S^2 = S + 1$

$S^2 - S - 1 = 0$

$S = \frac{1 \pm \sqrt{5}}{2}$ $S = \frac{1 + \sqrt{5}}{2}$

$S = 1 + 2 + 4 + 8 + \dots = +\infty$

$S = 1 + 2(1 + 2 + 4 + \dots)$

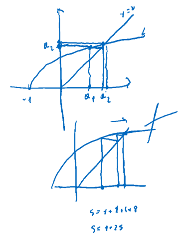
$S = 1 + 2S$ $S = -1$

$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

$S = 1 + \frac{1}{2}(1 + \frac{1}{2} + \dots)$

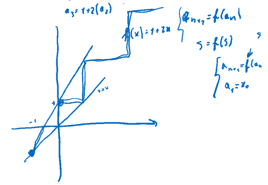
$S = 1 + \frac{1}{2}S$ $S = 2$

$\begin{cases} a_{n+1} = \sqrt{1 + a_n} \\ a_1 = 1 \end{cases}$ $f(x) = \sqrt{1 + x}$



$\begin{cases} a_{n+1} = 1 + 2a_n \\ a_1 = 1 \end{cases}$

$1 + 2 + 4 + 8 + \dots$



$\begin{cases} a_{n+1} = f(a_n) \\ a_1 = x_0 \end{cases}$