Esercitazioni di Analisi 1 - A.A. 21/22 - Callegari

Lezione 17

14 Gennaio 2022

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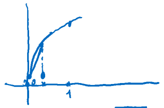
STUDIARE UNIF. CONT. E LIPSC. SU $[0,1]$ E $[1,+\infty]$ DI

$f(x)=\ln(1+\sqrt{x}\cdot e^x)$

$g(x)=\sqrt{x+x^3}$ → U.C. SU $[0,1]$ H.C.

$\sqrt{x}$



$$\lim_{x\to 0^+}\boxed{\frac{f(x)-f(0)}{x-0}}=\lim_{x\to 0^+}\frac{\sqrt{x+x^3}}{x}=\lim_{x\to 0^+}\sqrt{\frac{1+x^2}{x}}=$$

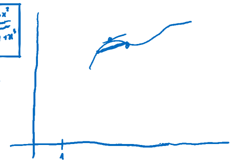
$=+\infty$

$\forall L>0\ \exists x\in(0,1)$ t.c. $\boxed{\frac{f(x)-f(0)}{x-0}>L}$

$g(x)=\sqrt{x+x^3}$

$\boxed{g'(x)=\frac{1+3x^2}{2\sqrt{x+x^3}}}$

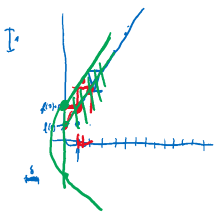
$\lim_{x\to+\infty} g'(x)=+\infty$



$\boxed{\forall L>0}$ SIA x_0 t.c. $|g'(x_0)|>2L$

$\boxed{g'(x_0)}=\lim_{x\to x_0}\boxed{\frac{g(x)-g(x_0)}{x-x_0}}$

def. per $x\to x_0$ $\left|\boxed{\frac{g(x)-g(x_0)}{x-x_0}}>L\right.$



$g(x)=\sqrt{x+x^3}\approx x^{\frac{3}{2}}$

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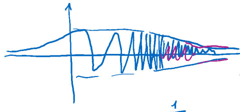
STUDIARE U.C. E LIP. DI

$f(x)=\sin x^2$ $\quad g(x)=\sqrt{|\sin x|}$

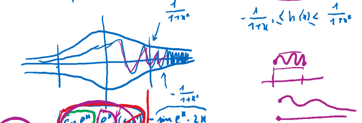
$\boxed{h(x)=\frac{\sin e^x}{1+x^2}}$ $\quad F(x)=|\sin x|^{\frac{1}{x}}$

$G(x)=|\sin x|^x$



$h(x)=\boxed{\frac{\sin e^x}{1+x^2}}$

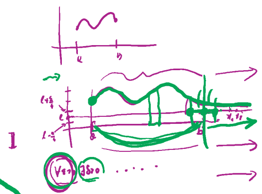
$-\frac{1}{1+x^2}<h(x)<\frac{1}{1+x^2}$



$h'(x)=\frac{\cos e^x\cdot e^x(1+x^2)-\sin e^x\cdot 2x}{(1+x^2)^2}=$

$=\frac{e^x}{1+x^2}\cos e^x-\frac{2x\sin e^x}{(1+x^2)^2}$

$\left|\ \right|\le\frac{2|x|}{(1+x^2)^2}\to 0$



$\forall\varepsilon>0\ \exists\delta>0\ .\ .\ .\ .\ .$

$f(x)=\ln(1+\sqrt{x}\cdot e^x)$

$[0,1]$ $\quad[1,+\infty)$

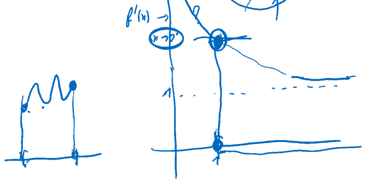
U.C. SU $[0,1]$

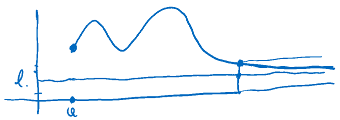
$$f'(x)=\frac{1}{1+\sqrt{x}e^x}\cdot\left(\frac{1}{2\sqrt{x}}e^x+\sqrt{x}e^x\right)=\boxed{\frac{e^x}{2\sqrt{x}}\cdot\frac{1+2x}{1+\sqrt{x}e^x}}\to+\infty$$

$x\to 0^+$

$f'(x)\to$

$x\to 0^+$





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$$\begin{cases}a_{n+1}=\sqrt{1+a_n}\\ a_1=5\end{cases}\overset{?}{\Rightarrow}\lim_{n\to+\infty}a_n=\ell$$

$f(a_n)$

$f(x)=\sqrt{1+x}$

$\ell=\sqrt{1+\ell}$ $\quad\frac{1+\sqrt{5}}{2}$

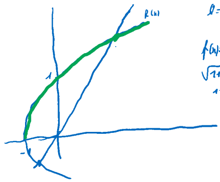
$\ell=\frac{1+\sqrt{5}}{2}$

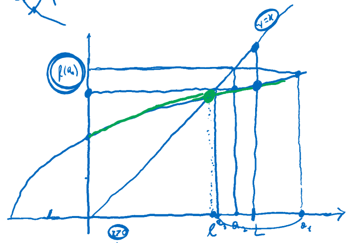
$f(x)=x$

$\sqrt{1+x}=x$

$1+x=x^2$

$x=\frac{1\pm\sqrt{5}}{2}$





$f'(x)=\frac{1}{2\sqrt{1+x}}<\frac{1}{2}$

1) $f(x)<x$ per $x>\ell$
2) $f(x)$ è crescente
3) $f(\ell)=\ell$

$\boxed{\ell<a_2<a_1}$?

$\boxed{f(\ell)<f(a_1)<a_1}$?

$\boxed{\ell<a_1}$ $\quad f(\ell)<f(a_1)$ $\quad\boxed{\ell<a_2}$

$\boxed{\ell<a_n<a_{n-1}}$

⇓ ?

$\boxed{\ell<a_{n+1}<a_n}$

f è crescente per $x\ge\ell$

$f(\ell)<f(a_n)<f(a_{n-1})$

$\ell<a_{n+1}<a_n$

1) a_n DECRESCE

2) INF. LIMITATA DA ℓ

$\lim_{n\to+\infty}a_n$ ESISTE FINITO L

$L=\sqrt{1+L}$

$a_{n+1}=\sqrt{1+a_n}$

$\downarrow\qquad\downarrow$

$L\qquad\sqrt{1+L}$

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$a_n\to L$

$a_{n+1}=\sqrt{1+a_n}$

$\downarrow\qquad\downarrow$

$L\qquad\sqrt{1+L}$