Analisi Matematica 1 - Lezione 5

8 Ottobre 2014 - docente: Prof. **Emanuele Callegari** - Università di Roma ***Tor Vergata***

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FUNZIONI ELEMENTARI

ESERCIZI

P.6.3

$f: \mathbb{Q} \to \mathbb{Q}$ iniettiva SI
$x \to 2x$

$\boxed{\text{suriettiva}}$ $\boxed{\text{SI}}$ perché se $2x \in \mathbb{Q}$
anche $x \in \mathbb{Q}$.

inoltre quindi: Qualsiasi $\frac{m}{n} \in \mathbb{Q}$ è immagine
di $\frac{m}{2n}$.

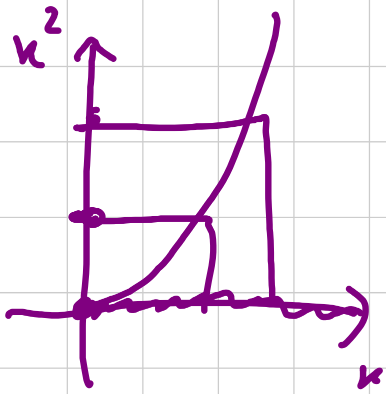
P.5.3

$f: \mathbb{N} \to \mathbb{N}$ iniettiva SI
$x \mapsto 2x$ suriettiva NO

$f(\mathbb{N})$ = numeri pari $\subsetneq \mathbb{N}$

P.B. 10.3

$f: \mathbb{R}^+ \to \mathbb{R}$
$x \mapsto x^2$



$\forall\, x, y$ positivi e diversi
$x^2 \neq y^2$

INIETTIVA SI

SURIETTIVA NO Perché $f(\mathbb{R}) = \mathbb{R}^+$

P. 12.3 bis

$$f: \mathbb{Q} \to \mathbb{Q}^+$$
$$x \mapsto x^2$$

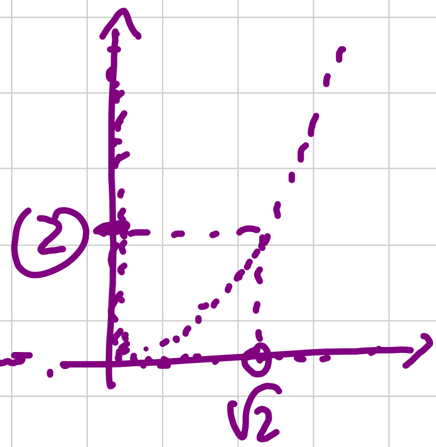
INIETTIVA $\boxed{\text{NO}}$

visto che x e $-x$ vanno nello stesso

SURIETTIVA

$\boxed{\text{NO}}$

Perché ad esempio in 2 non va nessuno.



P. 15.3

$f(x) = x^2 \qquad g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = (\sqrt{x})^2 = x$ (solo per $x \geq 0$)

$(g \circ f)(x) = g(f(x)) = \sqrt{x^2} = |x|$

$(= x)$ NO

P. 16.3

$f(x) = x^4 \qquad g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = (\sqrt{x})^4 = x^2$ (solo con $x \geq 0$)

$(g \circ f)(x) = g(f(x)) = \sqrt{x^4} = x^2$

P. 20.3

$f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x) = -(x^3 + x) = -f(x)$

strett crescente perché somma di funzioni strett. crescenti

Periodica NO perché strett. crescente

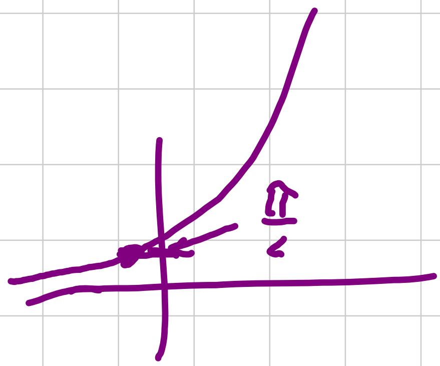
$\boxed{\text{P. 26.3 e 27.3}}$ $\quad f(x) = \sin(e^x) \qquad g(x) = e^{\sin x}$

$x = \ln \frac{\pi}{4}$

$f(x) = \sin\left(e^{\ln \frac{\pi}{4}}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$f(-x) = \sin\left(e^{-\ln \frac{\pi}{4}}\right) = \sin \frac{1}{\frac{\pi}{4}} = \sin \frac{4}{\pi} \neq \begin{cases} -f(x) \\ f(x) \end{cases}$

$\sin(y)$ cresce in $\left(0, \frac{\pi}{2}\right)$
decresce in $\left(\frac{\pi}{2}, \pi\right)$



$\boxed{0 < e^x < \frac{\pi}{2}} \iff \boxed{x < \ln \frac{\pi}{2}}$

quindi in $\left(-\infty, \ln \frac{\pi}{2}\right)$ $\sin(e^x)$

è crescente perché composizione di funzioni crescenti

[$f(g(x))$ è strett. crescente
se f e g lo sono]

Se $\boxed{\ln\frac{\pi}{2} < x < \ln\pi}$ allora $\boxed{\frac{\pi}{2} < e^x < \pi}$

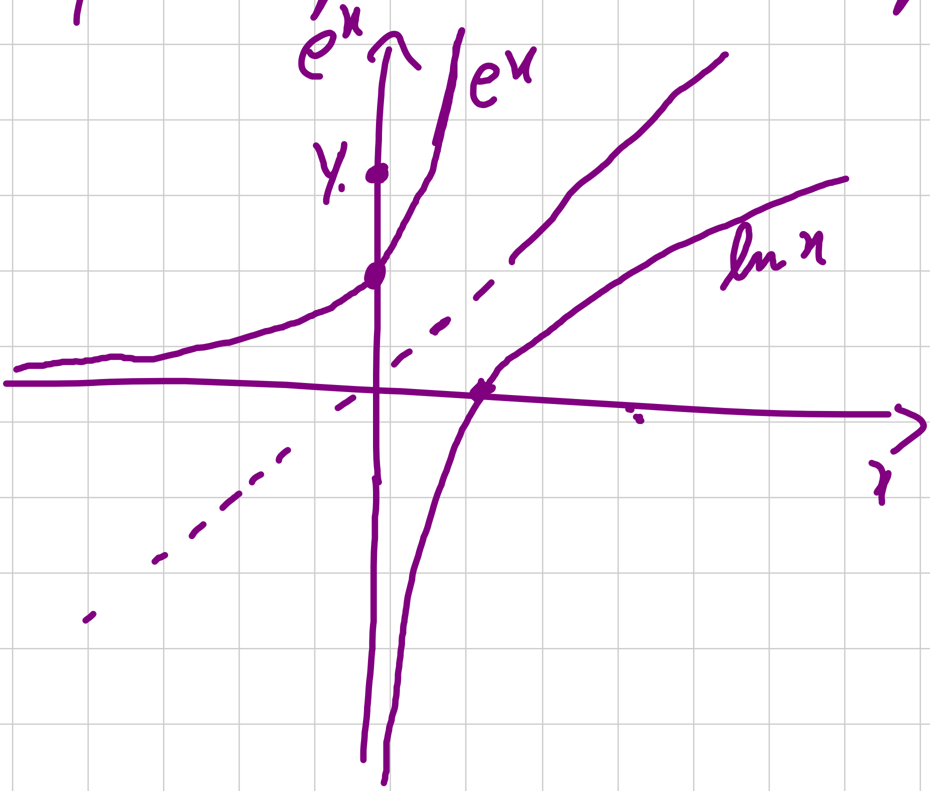
$\sin(e^x)$ con $\boxed{\ln\frac{\pi}{2} < x < \ln\pi}$

$0 < e^x \le 2\pi$ $\qquad x \le \ln(2\pi)$

$2\pi \le e^x \le 4\pi$ $\qquad \ln(2\pi) \le x \le \ln(4\pi)$

PARENTESI : FUNZIONI INVERSE

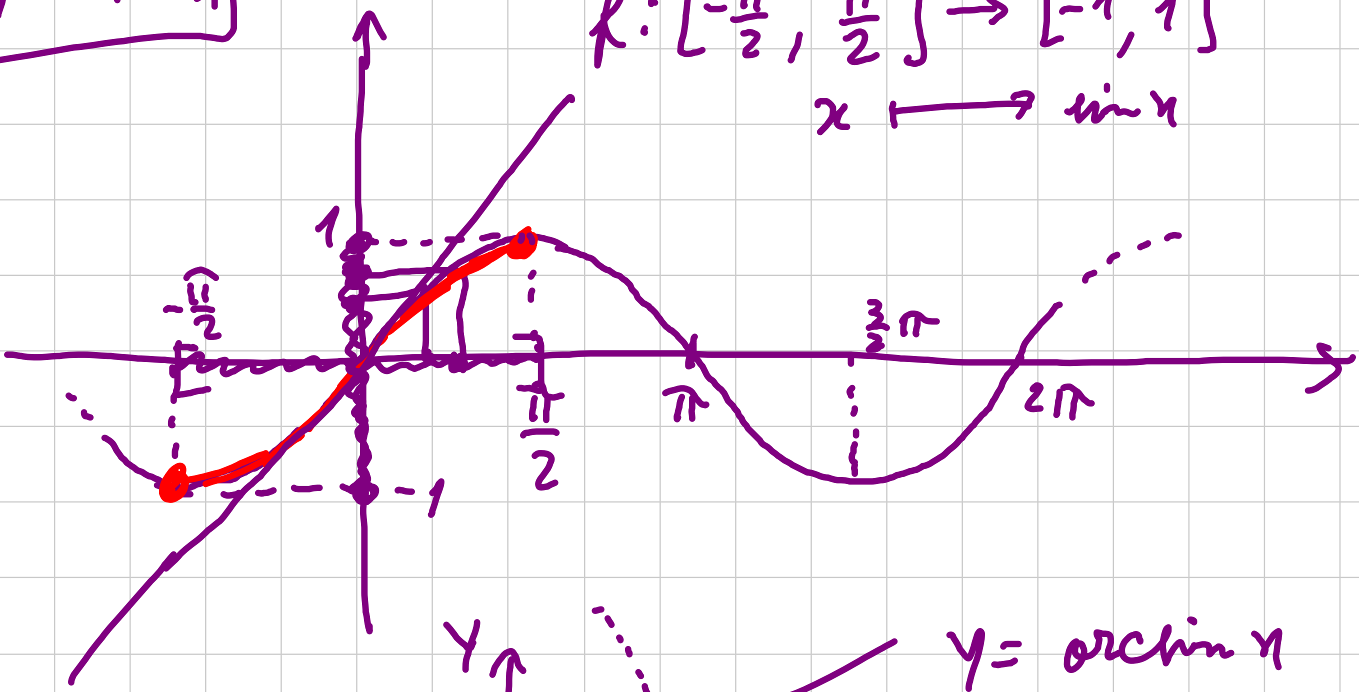
$\ln y$ = " esponente da dare ad e per ottenere y "



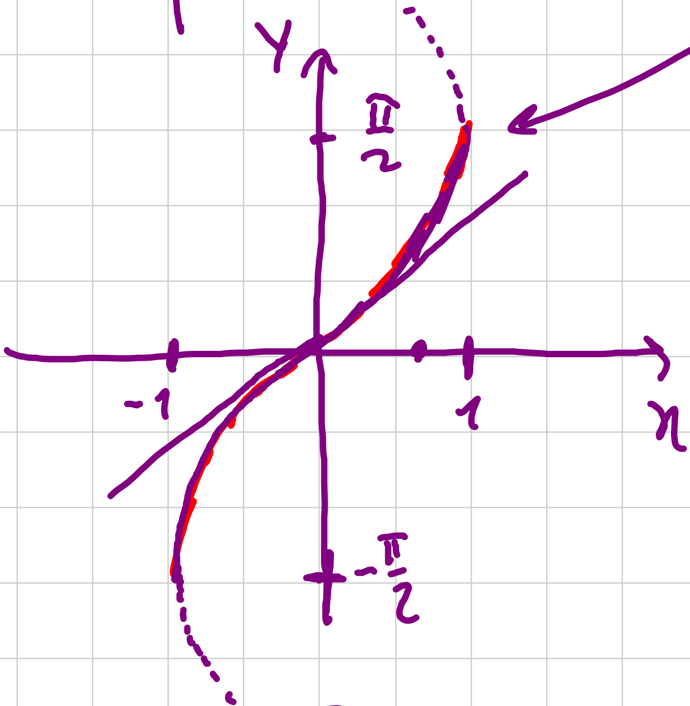
$\boxed{y = \sin x}$

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$

$x \longmapsto \sin x$



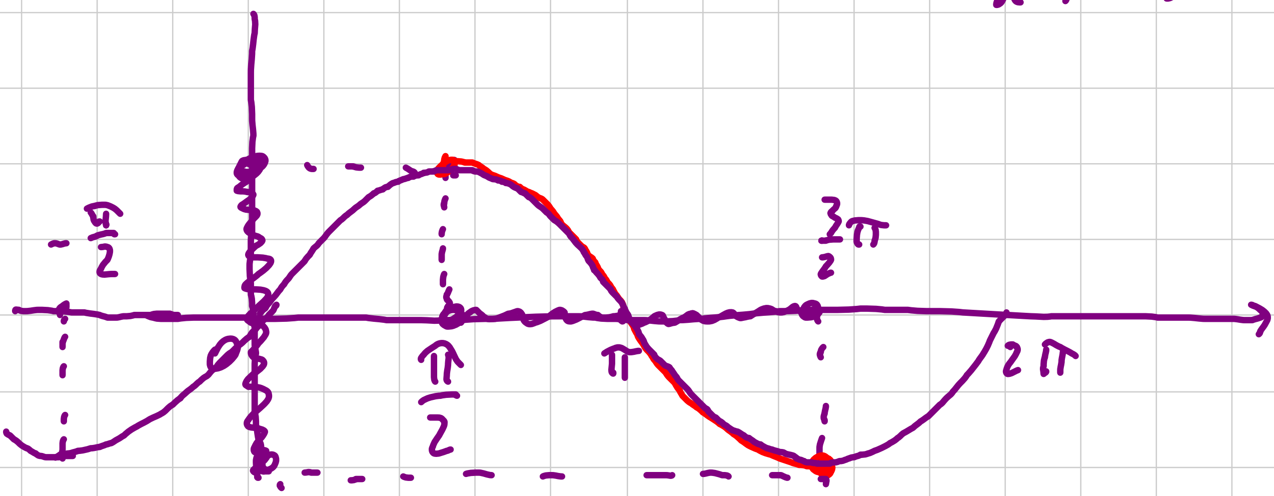
$y = \arcsin x$

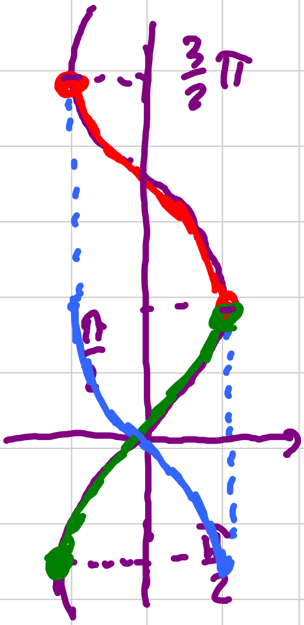


$\boxed{\text{Esercizio facilitato}}$

Trovare inversa di: $f : \left[\frac{\pi}{2}, \frac{3}{2}\pi\right] \to [-1, 1]$

$x \longmapsto \sin x$



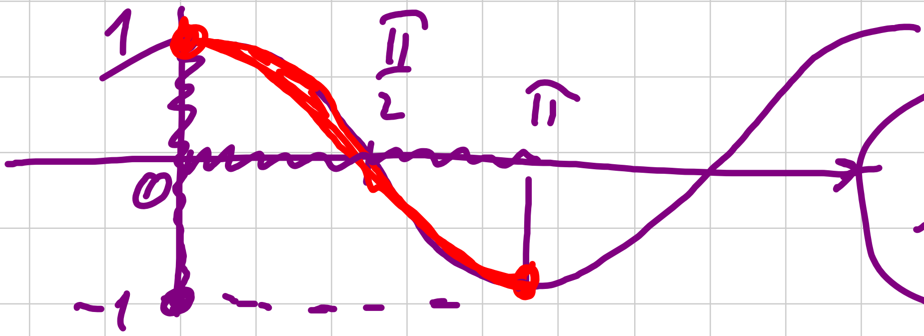


$$\boxed{f^{-1}(x) = \pi - \arcsin x}$$

Def. di arccos.

è la funzione inversa

$$\boxed{\begin{aligned} f: [0, \pi] &\to [-1, 1] \\ x &\mapsto \cos x \end{aligned}}$$

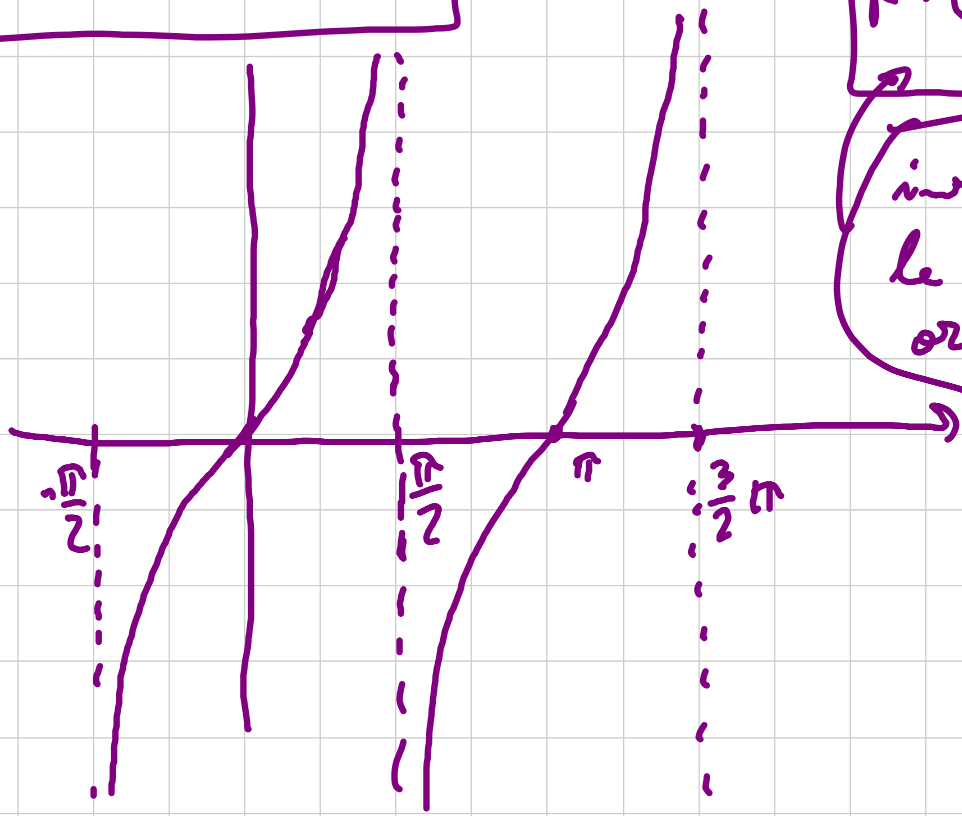


arcoseno è funzione inversa di questa

Def di arctan.

$$\boxed{\begin{aligned} f: \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right) &\to \mathbb{R} \\ x &\mapsto \tan x \end{aligned}}$$

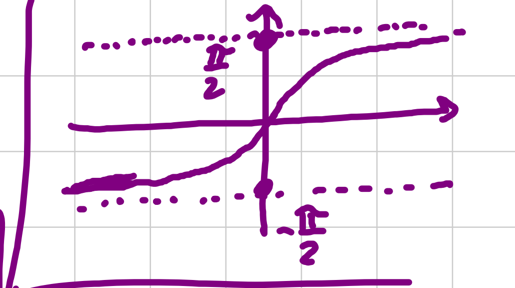
inversa di questa la chiamiamo arcotangente



$\boxed{\text{P.B. 34.3}}$

$$f(x) = \arctan \frac{1}{x}$$

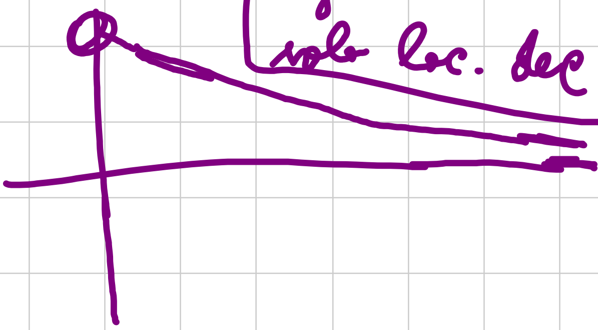
non monotona globalmente
solo loc. dec



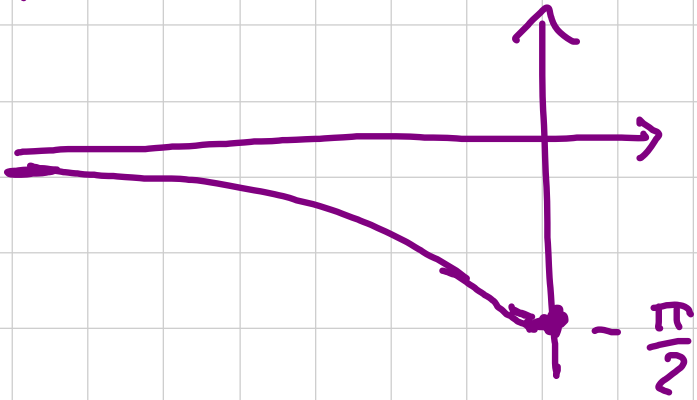
$y = \arctan x$

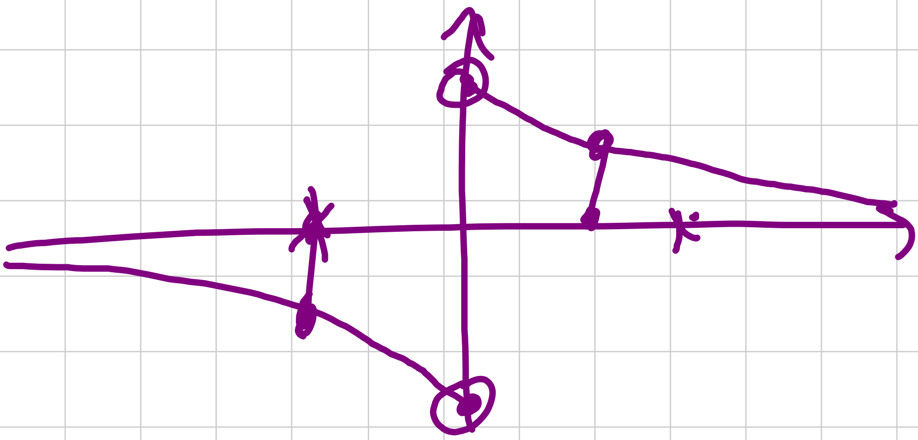
$\boxed{x>0}$

decrescente



$\boxed{x<0}$





Periodicità No perché è iniettiva

Disparità: $\boxed{\text{Sì}}$

$$f(-x) = \arctan\left(\frac{1}{-x}\right) = \arctan\left(-\frac{1}{x}\right) = -\arctan\left(\frac{1}{x}\right) = -f(x)$$

$\boxed{\text{P.B. } 36.3}$

$$f(x) = \arctan x + \arctan \frac{1}{x} = \begin{cases} \frac{\pi}{2} & \text{se } x > 0 \\ -\frac{\pi}{2} & \text{se } x < 0 \end{cases}$$

$\boxed{x > 0}$