Analisi Matematica 1 - Lezione 16 (II parte)

Titolo nota 9 Novembre 2015 - docente: Prof. **Emanuele Callegari** - Università di Roma ***Tor Vergata***

O-PICCOLI E LIMITI NOTEVOLI

1

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1}\qquad \boxed{\text{per } x\to 0 \quad \sin x \simeq x}\qquad \boxed{\text{per } x\to 0 \quad \sin x = x+o(x)}$$

$$\boxed{f(x)\to 0 \quad \text{per } x\to x_0}$$

$$\boxed{\lim_{x\to x_0}\frac{\sin f(x)}{f(x)}\underset{Y=f(x)}{=}\lim_{Y\to 0}\frac{\sin Y}{Y}=1}$$

$$\boxed{\begin{array}{l}\text{per } x\to x_0\\ \sin f(x)\simeq f(x)\end{array}}\qquad \boxed{\begin{array}{l}\text{per } x\to x_0\\ \sin f(x) = f(x)+o(f(x))\end{array}}$$

2

$$\boxed{\lim_{x\to 0}\frac{\tan x}{x}=1}\qquad \boxed{\text{per } x\to 0 \quad \tan x\simeq x}\qquad \boxed{\text{per } x\to 0 \quad \tan x = x+o(x)}$$

$$\boxed{\text{Se } f(x)\to 0 \quad \text{per } x\to x_0}$$

$$\boxed{\lim_{x\to x_0}\frac{\tan(f(x))}{f(x)}=1}\qquad \boxed{\text{per } x\to x_0 \quad \tan f(x)\simeq f(x)}\qquad \boxed{\text{per } x\to x_0 \quad \tan(f(x))=f(x)+o(f(x))}$$

3

$$\boxed{\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}}$$

$$\boxed{\lim_{x\to x_0}\frac{1-\cos f(x)}{(f(x))^2}=\frac{1}{2}}\qquad \boxed{\text{per } x\to x_0 \quad 1-\cos(f(x))\simeq \frac{1}{2}(f(x))^2}$$

$$\text{per } x\to x_0 \quad 1-\cos(f(x))=\frac{1}{2}(f(x))^2+o\left((f(x))^2\right)$$

4

$$\boxed{\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}}$$

$$\boxed{\lim_{x\to x_0}\frac{\tan f(x)-\sin f(x)}{(f(x))^3}=\frac{1}{2}}\qquad \boxed{\text{per } x\to x_0 \quad \tan f(x)-\sin f(x)\simeq\frac{1}{2}(f(x))^3}$$

$$\boxed{\text{per } x\to x_0 \quad \tan f(x)-\sin f(x)=\frac{1}{2}(f(x))^3+o\left((f(x))^3\right)}$$

5

$$\boxed{\lim_{x\to 0}\frac{\ln(1+x)}{x}=1}\qquad \boxed{\text{per } x\to x_0 \quad \ln(1+f(x))\simeq f(x)}$$

$$\boxed{\lim_{x\to x_0}\frac{\ln(1+f(x))}{f(x)}=1}\qquad \boxed{\text{per } x\to x_0 \quad \ln(1+f(x))=f(x)+o(f(x))}$$

6

$$\boxed{\lim_{x\to 0} \frac{e^x-1}{x} = 1}$$

$$\boxed{\lim_{x\to x_0} \frac{e^{f(x)}-1}{f(x)} = 1}$$

$$\boxed{\text{per } x\to x_0 \quad e^{f(x)}-1 \approx f(x)}$$

$$\boxed{\text{per } x\to x_0 \quad e^{f(x)}-1 = f(x) + o(f(x))}$$

7

$$\boxed{\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha}$$

$$\boxed{\lim_{x\to x_0} \frac{(1+f(x))^\alpha - 1}{f(x)} = \alpha}$$

$$\boxed{\text{per } x\to x_0 \quad (1+f(x))^\alpha - 1 \approx \alpha f(x)}$$

$$\boxed{\text{per } x\to x_0 \quad (1+f(x))^\alpha - 1 = \alpha f(x) + o(f(x))}$$

ES. 1

$$\lim_{x\to 0} \frac{(1-\cos(\tan x))\cdot(\tan x - \sin x)}{(e^{x^2}-1)\cdot \ln(1+x^3)} =$$

$$= \lim_{x\to 0} \underbrace{\frac{(1-\cos(\tan x))\cdot(\tan x - \sin x)}{x^2\cdot \ln(1+x^3)}}_{??} \cdot \underbrace{\frac{x^2}{e^{x^2}-1}}_{\to 1} = ??$$

$$\lim_{x\to 0} \frac{\overbrace{(1-\cos(\tan x))}^{\approx \frac{1}{2}(\tan x)^2 \approx \frac{x^2}{2}}\cdot(\tan x - \sin x)}{x^2\cdot \ln(1+x^3)} = \ldots\ldots = \lim_{x\to 0} \frac{\frac{x^2}{2}\cdot\frac{x^3}{2}}{x^2\cdot x^3} = \frac{1}{4}$$

REGOLA N. 1

Per calcolare:

$$\lim_{x\to x_0} \frac{f_1(x)\cdot f_2(x)\cdots f_n(x)}{g_1(x)\cdot g_2(x)\cdots g_k(x)}$$

Basta calcolare

$$\lim_{x\to x_0} \frac{F_1(x)\cdot F_2(x)\cdots F_n(x)}{G_1(x)\cdot G_2(x)\cdots G_k(x)}$$

se $F_i(x) \approx f_i(x)$ per $i = 1, \ldots, n$

e $G_i(x) \approx g_i(x)$ per $i = 1, \ldots, k$

ES. 2

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}$$

$$\updownarrow$$

NO $$=\lim_{x\to 0}\frac{x-x}{x^3}=\lim_{x\to 0}\frac{0}{x^3}=0$$

ES 3

$$\lim_{x\to 0}\frac{x+\ln(1+x^3)}{2x+\tan(x^2)}=\lim_{x\to 0}\underbrace{\frac{x}{2x}}_{\frac{1}{2}}\frac{\left(1+\frac{\ln(1+x^3)}{x}\right)}{\left(1+\frac{\tan x^2}{2x}\right)}=\frac{1}{2}$$

$\frac{\ln(1+x^3)}{x}\to 0$, $\left(1+\frac{\ln(1+x^3)}{x}\right)\to 1$, $\frac{\tan x^2}{2x}\to 0$

REGOLA 2

$$\lim_{x\to x_0}\frac{f(x)+o(f(x))}{g(x)+o(g(x))}=\lim_{x\to x_0}\frac{f(x)}{g(x)}$$

ES. 4

$$\lim_{x\to+\infty}\frac{e^{x^2+x}}{e^{x^2}}$$

$x=o(x^2)$

NO $$=\lim_{x\to+\infty}\frac{e^{x^2+\cancel{o(x^2)}}}{e^{x^2}}=\lim_{x\to+\infty}\frac{e^{x^2}}{e^{x^2}}=1$$ NO

NO

$$\updownarrow$$

SI $$=\lim_{x\to+\infty}e^{\cancel{x^2}+x-\cancel{x^2}}=\lim_{x\to+\infty}e^x=+\infty$$

ES 5

$$\lim_{x\to 0}\frac{\sin x+\boxed{1-\cos x}}{\boxed{e^x-1}+\ln(1+x^2)}=$$

$1-\cos x\approx\frac{x^2}{2}=o(x)=o(\sin x)$

$e^x-1=x+o(x)$

$$=\lim_{x\to 0}\frac{\sin x+o(\sin x)}{x+o(x)+\underbrace{x^2}_{o(x)}+\underbrace{o(x^2)}_{o(x)}}=$$

$$=\lim_{x\to 0}\frac{\sin x+o(\sin x)}{x+o(x)}=\lim_{x\to 0}\frac{\sin x}{x}=1$$