ESERCIZI SU LIMITI DI FUNZIONI

ES 1

$$\lim_{x\to 0} \frac{3^x-2^x}{x} = \lim_{x\to 0} \frac{(3^x-1)+(1-2^x)}{x} =$$

$$= \lim_{x\to 0} \left(\frac{e^{(\ln 3)\cdot x}-1}{x} - \frac{e^{(\ln 2)\cdot x}-1}{x} \right) =$$

$$= \lim_{x\to 0} \left(\frac{\cancel{x}\cdot \ln 3}{\cancel{x}} - \frac{\cancel{x}\cdot \ln 2}{\cancel{x}} \right) = \boxed{\ln \frac{3}{2}}$$

ES. 2

$$\lim_{x\to +\infty} \frac{\frac{\pi}{2} - \arctan x}{1-\cos\frac{1}{\sqrt{x}}} =$$

$$= \lim_{x\to +\infty} \frac{\boxed{\arctan\left(\frac{1}{x}\right)}}{\frac{1}{2}\cdot\left(\frac{1}{\sqrt{x}}\right)^2} =$$

$$\left[\begin{array}{l} \frac{\pi}{2} - \arctan x \overset{?}{=} \arctan \frac{1}{x} \\ \\ \arctan x + \arctan \frac{1}{x} \overset{?}{=} \frac{\pi}{2} \end{array} \right]$$

$$= \lim_{x \to +\infty} \frac{\boxed{\frac{1}{x}}}{\frac{1}{2}\left(\frac{1}{\sqrt{x}}\right)^2} =$$

$$= \lim_{x \to +\infty} \frac{\frac{\cancel{1}}{\cancel{x}}}{\frac{1}{2} \cdot \frac{\cancel{1}}{\cancel{x}}} = 2$$

INTERMEZZO

$$\boxed{\lim_{x \to 0} \frac{\arctan x}{x} = 1} \quad \boxed{?}$$

$$= \lim_{y \to 0} \frac{y}{\tan y} = 1$$

$\boxed{y = \arctan x} \iff \tan y = x$

Se poi $f(x) \to 0$ per $x \to x_0$

allora

$$\boxed{\lim_{x \to x_0} \frac{\arctan(f(x))}{f(x)} = 1}$$

$$\Updownarrow$$

$$\boxed{\arctan(f(x)) \simeq f(x) \quad \text{per } x \to x_0}$$

$$\Updownarrow$$

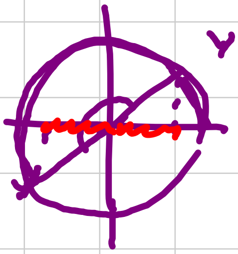
$$\boxed{\arctan(f(x)) = f(x) + o(f(x)) \quad \text{per } x \to x_0}$$

ES. 3

$$\lim_{x\to\pi} \frac{1+\cos x}{(\pi-x)\tan x} =$$

$$\boxed{y = x-\pi}$$

$$= \lim_{x\to\pi} \frac{1+\cos(\pi+\boxed{x-\pi})}{\boxed{(\pi-x)}\tan(\pi+\boxed{x-\pi})} =$$



$$= \lim_{y\to 0} \frac{1+\cos(\pi+y)}{-y\cdot\tan(\pi+y)} =$$

$$= \lim_{y\to 0} \frac{1-\cos y}{-y\cdot\tan y} =$$

$$= \lim_{y\to 0} \frac{\frac{\cancel{y^2}}{2}}{-\cancel{y}\cdot\cancel{y}} = -\frac{1}{2}$$

ES. 4

$$\boxed{\lim_{x\to 0^+} x^x = 1}$$

Limite notevole

$$= \lim_{x\to 0^+} e^{\ln(x^x)} =$$

$$= \lim_{x\to 0^+} e^{\boxed{x\ln(x)}} = e^0 = 1$$

$$\boxed{y = \frac{1}{x}}$$

$$\lim_{x\to 0^+} -x\ln(x^{-1}) = \lim_{x\to 0^+} -\frac{\ln\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \lim_{y\to +\infty} -\frac{\ln y}{y} = 0$$

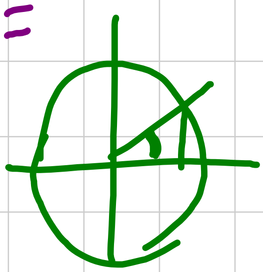
ES 5

$$\lim_{x\to 0^+} \frac{x^x-1}{x} = \lim_{x\to 0^+} \frac{e^{\boxed{x\ln x}}-1}{x} = \lim_{x\to 0^+} \frac{\cancel{x}\ln x}{\cancel{x}} = -\infty$$

$\left(\text{perché so che } x\ln x \to 0 \text{ per } x\to 0^+\right)$

ES 6

$$\lim_{x\to 0^+} \frac{(\sin x)^{\sin x}-1}{x} = \lim_{x\to 0^+} \frac{e^{\boxed{\sin x\cdot \ln(\sin x)}}-1}{x} = \lim_{x\to 0^+} \frac{\sin x\cdot \ln(\sin x)}{x} =$$



$$= \lim_{x\to 0^+} \left(\underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \underbrace{\ln(\sin x)}_{\to -\infty} \right) = -\infty$$

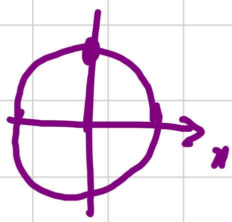
$$\lim_{x\to 0^+} \sin x \cdot \ln(\sin x) = \lim_{y\to 0^+} y\ln y = 0$$

$y=\sin x$

FATTO PRIMA

ES 7

$$\lim_{x\to \frac{\pi}{2}^+} (\sin x)^{\frac{1}{\cos x}} =$$



$$\boxed{y = x - \frac{\pi}{2}}$$

$$= \lim_{x \to \frac{\pi}{2}^+} \left(\sin\left(\frac{\pi}{2} + \left(x - \frac{\pi}{2}\right)\right)\right)^{\frac{1}{\cos\left(\frac{\pi}{2} + \left(x - \frac{\pi}{2}\right)\right)}} =$$

$$= \lim_{y \to 0^+} \left(\sin\left(\frac{\pi}{2} + y\right)\right)^{\frac{1}{\cos\left(\frac{\pi}{2} + y\right)}} =$$

$$= \lim_{y \to 0^+} \left(\cos(-y) \right)^{\frac{1}{\sin(-y)}} =$$

$$= \lim_{y \to 0^+} \left(\cos y \right)^{-\frac{1}{\sin y}} =$$

$$\boxed{\lim_{x \to 0} (1 + \boxed{x})^{\frac{1}{x}} = e}$$

$$= \lim_{y \to 0^+} \left(1 + \boxed{(\cos y - 1)} \right)^{-\frac{1}{\sin y}} =$$

$$= \lim_{y \to 0^+} \Bigg(\underbrace{\left(1 + (\cos y - 1)\right)^{\frac{1}{\cos y - 1}}}_{e} \Bigg)^{\overbrace{\frac{-\cos y + 1}{\sin y}}^{\to 0}} = e^0 = 1$$

$\boxed{\text{ES. 8}}$

$$\lim_{x \to -\infty} \frac{\overset{\sigma(x)}{e^x} - x}{\sqrt{1 + x^2} - x} =$$

$$-x$$

$$= \lim_{x \to -\infty} \frac{-x}{|x|\sqrt{\frac{1}{x^2} + 1} - x} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{-x}}{\cancel{-x}\left(\sqrt{1+\frac{1}{x^2}}+1\right)} = \frac{1}{2}$$

$\boxed{\text{ES. 9}}$

$$\lim_{x \to 0} \frac{(1+3x)^{8x}-1}{(1+4x)^{6x}-1} =$$

$$= \lim_{x \to 0} \frac{e^{\boxed{8x \cdot \ln(1+3x)}}-1}{e^{6x \cdot \ln(1+4x)}-1} =$$

$$= \lim_{x \to 0} \frac{8\cancel{x} \cdot \ln(1+3x)}{6\cancel{x} \cdot \ln(1+4x)} =$$

$$= \lim_{x \to 0} \frac{\cancel{8 \cdot 3x}}{\cancel{6 \cdot 4x}} = 1$$

$\boxed{\text{ES. 10}}$

$$\lim_{x \to +\infty} \left(1+\frac{1}{x^x}\right)^{(x+1)^x} =$$

$$= \lim_{x \to +\infty} \left(\left(1+\frac{1}{x^x}\right)^{x^x}\right)^{\frac{(x+1)^x}{x^x}} =$$

$$= \lim_{x \to +\infty} \left(\underbrace{\boxed{\left(1+\frac{1}{x^x}\right)^{x^x}}}_{e}\right)^{\overbrace{\boxed{\left(1+\frac{1}{x}\right)^x}}^{e}} = e^e$$

$\boxed{\text{ES. 11}}$

$$\lim_{x\to 0} \frac{\tan x - x}{x^2} = 0 \qquad ??$$

$$\boxed{\lim_{x\to 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}}$$

$\boxed{\text{Oss.}}$ x $|x|<\frac{\pi}{2}$

$$\boxed{|\tan x - x| < |\tan x - \sin x|}$$

$0 < x < \frac{\pi}{2}$

$0 < \sin x < x < \tan x$

$$\lim_{x\to 0} \frac{\tan x - x}{x^2} =$$

funziona anche se $\alpha = 3-\varepsilon$

$$= \lim_{x\to 0} \underbrace{\frac{\tan x - x}{\tan x - \sin x}}_{\text{limitata}} \cdot \frac{\tan x - \sin x}{x^2} = 0$$

$\tan x - \sin x \to 0$, $\sim \frac{x^3}{2}$

$\boxed{\text{se } \alpha = 3}$

$$\lim_{x\to 0} \frac{\tan x - x}{x^3}$$

non posso dire che vale 0

in un intorno di 0

$$\left|\frac{\tan x - x}{x^3}\right| < K$$

per qualche costante $K > 0$

Ciao

$$\tan x - x = O(x^3) \quad \text{per } x \to 0$$

$$\boxed{\tan x = x + O(x^3)}$$

$$\tan x = x + o(x)$$