17-03-2021 (11-13)
PARTI AGGIUNTE RISPETTO AL 2020

LEZ 7

ECCO GLI ESEMPI FATTI A LEZIONE CHE SONO **ESSENZIALMENTE** DIVERSI DA QUELLI DEL 2020.

ES. I

$$\int_0^1 \frac{x^2}{(1+x^2)^2}\,dx = \int_0^1 \frac{x}{2}\cdot\frac{2x}{(1+x^2)^2}\,dx = \int_0^1 \frac{x}{2}\cdot\left(-\frac{1}{1+x^2}\right)'\,dx =$$

$$= \left[\frac{x}{2}\cdot\left(-\frac{1}{1+x^2}\right)'\right]_0^1 + \frac{1}{2}\int_0^1 \frac{1}{1+x^2}\,dx =$$

$$= -\frac{1}{4} + \frac{1}{2}\left[\arctan x\right]_0^1 = -\frac{1}{4} + \frac{\pi}{8}$$

ES. II

$$\overset{\mathrm{I}}{\boxed{\int_0^\pi e^x\cdot\sin x\,dx}} = \int_0^\pi (e^x)'\cdot\sin x\,dx =$$

$$= \left[e^x\cdot\sin x\right]_0^\pi - \int_0^\pi e^x\cos x\,dx = 0 - \int_0^\pi (e^x)'\cos x\,dx =$$

$$= -\left[e^x\cos x\right]_0^\pi + \int_0^\pi e^x\cdot(-\sin x)\,dx =$$

$$= e^\pi + 1 - \overset{\mathrm{I}}{\boxed{\int_0^\pi e^x\cdot\sin x\,dx}}$$

ABBIAMO QUINDI OTTENUTO:

$$I = e^{\pi} + 1 - I$$

DA CUI SEGUE:

$$I = \frac{e^{\pi}+1}{2}$$

ES. III

$$\overset{I}{\boxed{\int_0^{\pi} x e^{x} \sin x \, dx}} = \int_0^{\pi} (e^{x})' \cdot x \sin x \, dx =$$

$$= \left[e^{x} \cdot x \sin x \right]_0^{\pi} - \int_0^{\pi} e^{x} \cdot (\sin x + x \cos x) \, dx$$

$$= 0 - \boxed{\int_0^{\pi} e^{x} \sin x \, dx} - \int_0^{\pi} e^{x} \cdot x \cdot \cos x \, dx =$$

ES II

$$= \boxed{-\frac{e^{\pi}+1}{2}} - \int_0^{\pi} (e^{x})' x \cos x \, dx =$$

$$= -\frac{e^{\pi}+1}{2} - \left[e^{x} \cdot x \cdot \cos x \right]_0^{\pi} + \int_0^{\pi} e^{x} \cdot (\cos x - x \sin x) \, dx =$$

$$= -\frac{e^{\pi}+1}{2} + \pi e^{\pi} + \boxed{\int_0^{\pi} e^{x} \cos x \, dx} - \int_0^{\pi} x e^{x} \sin x \, dx =$$

$$\int_0^{\pi} (e^{x})' \cos x \, dx = \left[e^{x} \cos x \right]_0^{\pi} + \int_0^{\pi} e^{x} \sin x = -e^{\pi} - 1 + \frac{e^{\pi}+1}{2} = -\frac{e^{\pi}+1}{2}$$

$$= \underbrace{-\frac{e^{\pi}+1}{2} + \pi e^{\pi} \boxed{- \frac{e^{\pi}+1}{2}}}_{(\pi-1)e^{\pi}-1} - \underset{I}{\boxed{\int_0^{\pi} x e^{x} \sin x \, dx}}$$

ABBIAMO QUINDI OTTENUTO: $I = (\pi - 1)e^{\pi} - 1 - I$

DA CUI SEGUE: $I = \left(\frac{\pi}{2} - \frac{1}{2} \right) e^{\pi} - \frac{1}{2}$