Analisi Matematica 2 - A.A. 21/22 - Callegari

Lezione 6

16 Marzo 2022

INDICE

1) $\int \frac{1}{x^n}\,dx$ 2) $\int \frac{x}{(1+x^2)^n}\,dx$ 3) $\int \frac{1}{(1+x^2)^n}\,dx$

4) $\int_{-1}^{\sqrt{3}-1} \frac{1}{x^2+2x+10}\,dx$ 5) $\int_{-1}^{0} \frac{4x^2+7x+7}{(x^2+2x+2)^2}\,dx$

6) $\int_{1}^{\sqrt[5]{2}} \frac{28}{x^{11}+2x}\,dx$ 7) $\int_0^1 \frac{x^{11}+2x^6+2x^5+2}{x^6+1}\,dx$

8) TROVARE ORDINE PER $x\to 0^+$ DI $F(x)$ NEI CASI:

A) $F(x)=\int_0^x \sqrt{e^t-1}\,dx$ B) $F(x)=\int_0^{x^2} \sqrt{e^t-1}\,dt$

C) $F(x)=\int_{x+x^3}^{x+x^2} \sqrt{e^t-1}\,dt$ D) $F(x)=\int_0^x \sin\frac{1}{t}\,dt$

9) $\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \sqrt[9]{\sin t}\,dt$

10) SUP E INF PER $x>0$ DI $F(x)=\int_{\ln x}^{\ln(1+x)} \sqrt[3]{1+e^{3t}}\,dt$

$$\boxed{G(y)=\int_c^y f(t)\,dt}$$

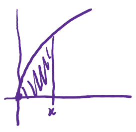
$$\boxed{F(x)=\int_{h(x)}^{g(x)} f(t)\,dt} = \int_c^{g(x)} f(t)\,dt - \int_c^{h(x)} f(t)\,dt = G(g(x)) - G(h(x))$$

$$\boxed{F'(x)} \overset{?}{=} \big(G(g(x)) - G(h(x))\big)' =$$

$$= G'(g(x))\cdot g'(x) - G'(h(x))\cdot h'(x) =$$

$$= \underline{f(g(x))\cdot g'(x) - f(h(x))h'(x)}$$

$\boxed{8.A}$ $F(x)=\int_0^x \sqrt{e^t-1}\,dt$

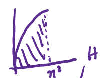


$$\lim_{x\to 0^+} \frac{F(x)}{x^\alpha} \overset{H}{=} \lim_{x\to 0^+} \frac{F'(x)}{\alpha x^{\alpha-1}} = \lim_{x\to 0^+} \frac{\sqrt{e^x-1}}{\alpha x^{\alpha-1}} = \lim_{x\to 0^+} \frac{\sqrt{x}}{\alpha x^{\alpha-1}} =$$

$\alpha-1=\frac{1}{2}$ $\alpha=\frac{3}{2}$

$$F(x) \approx C x^{\frac{3}{2}}$$

$\boxed{8.B}$ $F(x)=\int_0^{x^2} \sqrt[3]{\sin t}\,dt = G(x^2)$ $G(y)=\int_0^y \sqrt[3]{\sin t}\,dt$



$$\lim_{x\to 0^+} \frac{F(x)}{x^\alpha} \overset{H}{=} \lim_{x\to 0^+} \frac{F'(x)}{\alpha x^{\alpha-1}} = \lim_{x\to 0^+} \frac{(G(x^2))'}{\alpha x^{\alpha-1}} =$$

$$= \lim_{x\to 0^+} \frac{G'(x^2)\cdot 2x}{\alpha x^{\alpha-1}} = \lim_{x\to 0^+} \frac{\overbrace{\sqrt[3]{\sin(x^2)}\cdot 2x}^{\approx 2x^{\frac{5}{3}}}}{\alpha x^{\alpha-1}} =$$

$=$

$\alpha-1=\frac{5}{3}$ $\boxed{\alpha=\frac{8}{3}}$

$\boxed{8.C}$ $F(x)=\int_{x+x^3}^{x+x^2} \sqrt{\tan t}\,dt =$

$$\lim_{x\to 0^+} \frac{F(x)}{x^\alpha} \overset{H}{=} \lim_{x\to 0^+} \frac{F'(x)}{\alpha x^{\alpha-1}} =$$

$$= \lim_{x\to 0^+} \frac{\overbrace{\sqrt{\tan(x+x^2)}\cdot(1+2x) - \sqrt{\tan(x+x^3)}\cdot(1+3x^2)}^{(1)}}{\alpha x^{\alpha-1}} = (\bullet)$$

$$(1) = \overbrace{\sqrt{\tan(x+x^2)} - \sqrt{\tan(x+x^3)}}^{\frac{x\sqrt{x}}{2}+o(x\sqrt{x})} + \overbrace{2x\sqrt{\tan(x+x^2)}}^{2x\sqrt{x}+o(x\sqrt{x})} - \overbrace{3x^2\sqrt{\tan(x+x^3)}}^{o(x\sqrt{x})}$$

$$\frac{\overbrace{\tan(x+x^2)-\tan(x+x^3)}^{\cancel{x}+x^2+O(x^3)-(\cancel{x}+x^3)+O(x^3)}}{\sqrt{x}\left(\sqrt{\frac{\tan(x+x^2)}{x}}+\sqrt{\frac{\tan(x+x^3)}{x}}\right)} = \frac{x^2+O(x^3)}{2\sqrt{x}+o(\sqrt{x})} \approx \frac{x\sqrt{x}}{2}$$

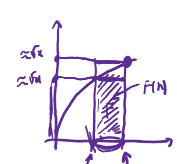
$$2x\sqrt{\tan(x+x^2)} \approx 2x\sqrt{x}$$

$$= 2x\sqrt{x}+o(x\sqrt{x})$$

$$(1) \approx \frac{5}{2}x\sqrt{x}$$

$$(\bullet) = \lim_{x\to 0^+} \frac{\frac{5}{2}x\sqrt{x}}{\alpha x^{\alpha-1}}$$

$\alpha-1=\frac{3}{2}$ $\alpha=\frac{5}{2}$



$\sqrt{\tan t}$

$(x+x^2)-(x+x^3) = x^2-x^3 \approx x^2$

$b(x)\cdot h(x)$
x^2 $\sqrt{x}$

$\frac{5}{2}$

$$\sqrt{\tan(x+x^3)}\cdot(\cancel{x}+x^2-\cancel{x}-x^3) \le F(x) \le (\cancel{x}+x^2-\cancel{x}-x^3)\cdot\sqrt{\tan(x+x^2)}$$

$$(\sqrt{x}+o(\sqrt{x}))(x^2+o(x^2)) \le F(x) \le (x^2+o(x^2))(\sqrt{x}+o(\sqrt{x}))$$

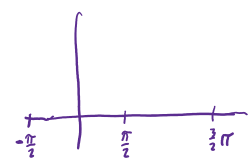
$$x^{\frac{5}{2}}+o(x^{\frac{5}{2}}) \le F(x) \le x^{\frac{5}{2}}+o(x^{\frac{5}{2}})$$

$$\underbrace{1+o(1)}_{\downarrow\,1} \le \underbrace{\frac{F(x)}{x^{\frac{5}{2}}}}_{\downarrow\,1} \le \underbrace{1+o(1)}_{\downarrow\,1}$$

(9) $\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \sqrt[9]{\sin t} = 0$

$$\boxed{\int_{-a}^{a} \sqrt[9]{\sin t}\,dt = 0}$$

$\sqrt[3]{\sin t}$ $\sin(f(t))$ $f(\sin t)$



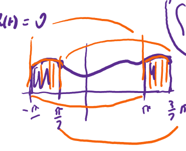
$$\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \sqrt[3]{\sin t}\,dt = \int_{-\frac{\pi}{2}}^{\frac{3}{2}\pi} \sqrt[3]{\sin t}\,dt = \int_{-\pi}^{\pi} \sqrt[3]{\sin t}\,dt = 0 \quad t=2\pi+u$$

$\int_b^b f(t)\,dt = \int_b^b f(u)\,du$

$\int_{\pi}^{\frac{3}{2}\pi} f(t)\,dt = \int_{-\pi}^{-\frac{\pi}{2}} f(2\pi+u)\,du = \int_{-\pi}^{-\frac{3}{2}\pi} f(u)\,du$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(t) = 0$

$\int_{-\frac{\pi}{2}}^{\frac{3}{2}\pi} = \int_{-\frac{\pi}{2}}^{\pi} + \boxed{\int_{\pi}^{\frac{3}{2}\pi}} = \int_{-\pi}^{-\frac{\pi}{2}} + \int_{-\pi}^{\pi}$



(1, 2, 3) $\int_a^b \frac{1}{x^n}\,dx$

$$\int_a^b \frac{x}{(1+x^2)^n}\,dx =$$

$n=1$: $\frac{1}{2}\int_a^b \frac{2x}{1+x^2}\,dx = \frac{1}{2}\left[\ln(1+x^2)\right]_a^b$

$n\ge 2$: $\frac{1}{2}\int_a^b \frac{2x}{(1+x^2)^n}\,dx = \frac{1}{2}\int_a^b \frac{1}{(1+x^2)^n}\cdot(1+x^2)'\,dx =$

$$= \frac{1}{2}\int_{1+a^2}^{1+b^2} \frac{1}{y^n}\,dy$$

$$\boxed{\int_a^b \frac{1}{(1+x^2)^n}}\,dx$$

$$\boxed{I_n = \int_a^b \frac{1}{(1+x^2)^n}\,dx}$$

$$I_1 = \int_a^b \frac{1}{1+x^2}\,dx = \left[\arctan x\right]_a^b = \arctan b - \arctan a$$

$$I_2 = \int_a^b \frac{1+x^2-x^2}{(1+x^2)^2}\,dx =$$

$$= \int_a^b \frac{1}{1+x^2}\,dx - \int_a^b \frac{x^2}{(1+x^2)^2}\,dx =$$

$$= I_1 + \frac{1}{2}\int_a^b x\left(\frac{-2x}{(1+x^2)^2}\right)dx =$$

$$= I_1 + \frac{1}{2}\int_a^b x\cdot\left(\frac{1}{1+x^2}\right)'dx =$$

$$= I_1 + \frac{1}{2}\left(\left[x\cdot\frac{1}{1+x^2}\right]_a^b - \int_a^b \frac{1}{1+x^2}\,dx\right) =$$

$$= I_1 + \frac{1}{2}\left[\frac{x}{1+x^2}\right]_a^b - \frac{1}{2}I_1 =$$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

$$I_2 = \dots\dots = \boxed{\frac{1}{2}I_1 + \frac{1}{2}\left[\frac{x}{1+x^2}\right]_a^b}$$

$$\left(\frac{1}{(1+x^2)^{n-1}}\right)' = \left((1+x^2)^{1-n}\right)' =$$
$$= (1-n)\cdot(1+x^2)^{-n}\cdot 2x =$$
$$= (1-n)\frac{2x}{(1+x^2)^n} =$$
$$= (n-1)\frac{-2x}{(1+x^2)^n}$$

$$I_n = \int_a^b \frac{1+x^2-x^2}{(1+x^2)^n}\,dx =$$

$$= \int_a^b \frac{1}{(1+x^2)^{n-1}}\,dx + \frac{1}{2(n-1)}\int_a^b x\cdot\left(\frac{-2x}{(1+x^2)^n}(n-1)\right)dx =$$

$$= I_{n-1} + \frac{1}{2(n-1)}\cdot\int_a^b x\cdot\left(\frac{1}{(1+x^2)^{n-1}}\right)'dx =$$

$$= I_{n-1} + \frac{1}{2(n-1)}\left(\left[\frac{x}{(1+x^2)^{n-1}}\right]_a^b - \int_a^b \frac{1}{(1+x^2)^{n-1}}\,dx\right) =$$

$$\frac{2n-2-1}{2(n-1)} = \frac{2n-3}{2n-2}$$

$$= I_{n-1} + \frac{1}{2(n-1)}\cdot\left(\left[\frac{x}{(1+x^2)^{n-1}}\right]_a^b - I_{n-1}\right) =$$

$$I_n = \dots\dots = \frac{2n-3}{2n-2}I_{n-1} + \frac{1}{2n-2}\left[\frac{x}{(1+x^2)^{n-1}}\right]_a^b$$