Analisi Matematica 2 - A.A. 21/22 - Callegari

Lezione 7

18 Marzo 2022

LEZ. PREC.

4) $\boxed{\displaystyle\int_{-1}^{\sqrt{3}-1} \frac{1}{x^2+2x+10}\,dx}$ 5) $\displaystyle\int_{-1}^{0} \frac{4x^2+7x+7}{(x^2+2x+2)^2}\,dx$

6) $\boxed{\displaystyle\int_{1}^{\sqrt[5]{2}} \frac{20}{x^{11}+2x}\,dx}$ 7) $\displaystyle\int_{0}^{1} \frac{x^{11}+2x^6+2x^5+2}{x^6+1}\,dx$

$\boxed{F(x)=\int_0^x \sin\frac{1}{t}}$ $(F'(0)=?)$

$\boxed{\text{TEO INT. F. DISC.}}$

$\left|\frac{S_n+s_n}{2}-S\right| = O\left(\frac{1}{n^2}\right)$

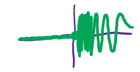
$$\int_1^{\sqrt[5]{2}} \frac{20}{x^{11}+2x}\,dx = 2\int_1^{\sqrt[5]{2}} \frac{10x^9}{x^{10}(x^{10}+2)}\,dx = 2\int_1^{\sqrt[5]{2}} \frac{1}{x^{10}\cdot(x^{10}+2)}\cdot(x^{10})'\,dx$$

$(x^{10})'$

$y = x^{10}$

$$= 2\int_1^{\sqrt[5]{2}} \frac{1}{y(y+2)}\,dy = \int_1^{4} \frac{2+y-y}{y(y+2)}\,dy = \int_1^4 \frac{1}{y} - \frac{1}{y+2}\,dy =$$

$$f(t)=\begin{cases} 0 & \text{se } t\le 0 \\ \sin\frac{1}{t} & \text{se } t>0 \end{cases}$$



$F(x)=\int_{-1}^x f(t)\,dt$

$F'(0) = ??$

$$F'(0)=\lim_{x\to 0^+}\frac{F(x)-F(0)}{x-0} = \lim_{x\to 0^+}\frac{F(x)}{x} \overset{H}{=} \lim_{x\to 0^+}\frac{F'(x)}{1} = \lim_{x\to 0^+}\sin\frac{1}{x} = \text{NON ESISTE}$$

NO

$$\lim_{x\to 0^+}\frac{\int_{-1}^x f(t)\,dt}{x} = \lim_{x\to 0^+}\frac{\boxed{\int_{-1}^0 0\,dt} + \boxed{\int_0^{x^2}\sin\frac{1}{t}\,dt} + \int_{x^2}^x \sin\frac{1}{t}\,dt}{x} =$$

$0 =$ (*) $o(x)$

(*) $$\left|\int_0^{x^2}\sin\frac{1}{t}\,dt\right| \le \int_0^{x^2}\left|\sin\frac{1}{t}\right|dt \le \int_0^{x^2} 1\,dt = x^2$$

$$= \lim_{x\to 0^+}\frac{\int_{x^2}^x \sin\frac{1}{t}\,dt}{x} \overset{t=\frac{1}{u}}{=} \lim_{x\to 0^+}\frac{\int_{\frac{1}{x^2}}^{\frac{1}{x}} \sin u\cdot\left(-\frac{1}{u^2}\right)du}{x} =$$

$$= \lim_{x\to 0^+}\frac{\int_{\frac{1}{x}}^{\frac{1}{x^2}} \frac{\sin u}{u^2}\,du}{x} = \lim_{x\to 0^+}\frac{o(x)}{x} = 0$$

$$\int_{\frac{1}{x}}^{\frac{1}{x^2}} \frac{1}{u^2}\cdot(-\cos u)'\,du = \left[\frac{-\cos u}{u^2}\right]_{\frac{1}{x}}^{\frac{1}{x^2}} + \underbrace{\int_{\frac{1}{x}}^{\frac{1}{x^2}} -2\cdot\frac{\cos u}{u^3}\,du}_{(*)} =$$

$$= \underbrace{\left(-\cos\frac{1}{x^2}\right)x^4 + \left(\cos\frac{1}{x}\right)\cdot x^2}_{o(x)} + o(x) = o(x)$$

$$|*| \le \int_{\frac{1}{x}}^{\frac{1}{x^2}} \frac{2}{u^3}\,du = \left[-\frac{1}{u^2}\right]_{\frac{1}{x}}^{\frac{1}{x^2}} = -x^4 + x^2 = x^2 + o(x^2) = o(x)$$

$\boxed{T.1}$ DATI $[a,b]\subset\mathbb{R}$, $\bar{x}\in[a,b]$, $f:[a,b]\to\mathbb{R}$ LIMITATA E CONTINUA $\forall x\ne x_0$. ALLORA $f\in\mathcal{R}([a,b])$.

$M = osc(f,[a,b])$

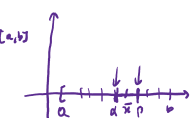
$\boxed{\text{DIM}}$ $(\forall\varepsilon>0)$

PRENDO α E β SU $[a,b]$

IN MODO CHE

$a<\alpha<\bar{x}<\beta<b$

E CHE $\beta-\alpha<\frac{\varepsilon}{3M}$



$\beta-\alpha<\frac{\varepsilon}{3M}$

DOVE $M = osc(f,[a,b])$

PRENDO $P_1=\{x_0,\dots,x_n\}$ ($x_n = \alpha$) PART. DI $[a,\alpha]$ t.c.

$$\sum_{i=1}^n (x_i-x_{i-1})\cdot osc(f,[x_{i-1},x_i]) < \frac{\varepsilon}{3}$$

(POSSO FARLO PERCHÉ $f\in C([a,\alpha])$ E QUINDI $f\in\mathcal{R}([a,\alpha])$)

ANALOG. PRENDO $P_2=\{x_{n+1},\dots,x_m\}$ PART. DI $[\beta,b]$ t.c.

$$\sum_{i=n+2}^m (x_i-x_{i-1})\cdot osc(f,[x_{i-1},x_i]) < \frac{\varepsilon}{3}$$

ALLORA $P=P_1\cup P_2$ È PART. DI $[a,b]$ E SI HA

$$\sum_{i=1}^m (x_i-x_{i-1})\,osc(f,[x_{i-1},x_i]) =$$

$$= \underbrace{\left(\sum_{i=1}^n (x_i-x_{i-1})\,osc(f,[x_{i-1},x_i])\right)}_{<\frac{\varepsilon}{3}} + \underbrace{\overbrace{(x_{n+1}-x_n)}^{\beta-\alpha<\frac{\varepsilon}{3M}}\,\overbrace{osc(f,[x_n,x_{n+1}])}^{\le osc(f,[a,b])=M}}_{<\frac{\varepsilon}{3M}\cdot M=\frac{\varepsilon}{3}} + \underbrace{\sum_{i=n+2}^m (x_i-x_{i-1})\,osc(f,[x_{i-1},x_i])}_{<\frac{\varepsilon}{3}} <$$

$$< \frac{\varepsilon}{3}+\frac{\varepsilon}{3}+\frac{\varepsilon}{3} = \varepsilon$$

$$\int_{-1}^{-1+\sqrt{3}} \frac{1}{x^2+2x+10}\,dx =$$

$x^2+2x+10 =$
$x^2+2x+1+9 =$
$= (x+1)^2+9 =$
$= \boxed{9\left(\left(\frac{x+1}{3}\right)^2+1\right)}$

$t=\frac{x+1}{3}$

$$= \frac{1}{3}\int_{-1}^{-1+\sqrt{3}} \frac{1}{\left(\frac{x+1}{3}\right)^2+1}\cdot\left(\frac{x+1}{3}\right)'dx = \frac{1}{3}\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{t^2+1}\,dt = \frac{1}{3}\arctan\frac{1}{\sqrt{3}} = \frac{\pi}{18}$$

$x = \boxed{t\sqrt{2}-1}$, $x=\varphi(t)$

$$\int_{-1}^{0} \frac{4x^2+7x+7}{(x^2+2x+2)^2}\,dx = \int_{-1}^{0} \frac{1}{4}\,\frac{4x^2+7x+7}{\left(\left(\frac{x+1}{\sqrt{2}}\right)^2+1\right)^2}\,dx =$$

$(x^2+2x+2)^2$

$4\left(\left(\frac{x+1}{\sqrt{2}}\right)^2+1\right)^2$

$$= \int_0^{\frac{1}{\sqrt{2}}} \frac{1}{4}\,\frac{4(t\sqrt{2}-1)^2+7(t\sqrt{2}-1)+7}{(t^2+1)^2}\cdot\sqrt{2}\,dt$$

$$= \frac{1}{2\sqrt{2}}\int_0^{\frac{1}{\sqrt{2}}} \frac{8t^2-8\sqrt{2}t+4+7\sqrt{2}t-\cancel{7+7}}{(t^2+1)^2}\,dt =$$

$$= \frac{1}{2\sqrt{2}}\int_0^{\frac{1}{\sqrt{2}}} \boxed{\frac{8t^2-t\sqrt{2}+4}{(t^2+1)^2}}\,dt =$$

$$\frac{8t^2+8-4-t\sqrt{2}}{(t^2+1)^2} = \frac{8(t^2+1)-t\sqrt{2}-4}{(t^2+1)^2} =$$

$$= \boxed{8\cdot\frac{1}{t^2+1} - \sqrt{2}\cdot\frac{t}{(t^2+1)^2} - 4\cdot\frac{1}{(t^2+1)^2}}$$

$$= 2\sqrt{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{1+t^2}\,dt - \frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{t}{(1+t^2)^2}\,dt - \sqrt{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1+t^2)^2}\,dt =$$

$\boxed{\frac{P(x)}{(x^2+1)^n}} =$ SE $g(P(x))\ge 2n$

$P(x) : (x^2+1)^n = Q(x)$
$R(x)$

$P(x) = \boxed{Q(x)\cdot(x^2+1)^n + R(x)}$

$$\frac{Q(x)\,(x^2+1)^n + R(x)}{(x^2+1)^n} = Q(x) + \frac{R(x)}{(x^2+1)^n} = \bullet$$

$g(R(x))>1$

$R(x) : (x^2+1) = A(x)$
$\quad$ resto $= B(x) = ax+b$

$R(x) = \boxed{A(x)\cdot(x^2+1) + ax+b}$

$$\bullet = Q(x) + \frac{A(x)(x^2+1)+ax+b}{(x^2+1)^n} =$$

$$= Q(x) + \frac{a_1x+b_1}{(x^2+1)^n} + \frac{A(x)}{(x^2+1)^{n-1}}$$

$$+ \frac{a_{n-1}x+b_{n-1}}{(x^2+1)^{n-1}} + \frac{\quad}{(x^2+1)^{n-2}} + \frac{\quad}{\quad} + \frac{\quad}{\quad} \cdots + \frac{\quad}{(x^2+1)}$$

$$\frac{A(x)}{(ax^2+bx+c)^{d_1}\cdot(\alpha x^2+\beta x+\gamma)^{d_2}\cdots} =$$

$p(x)$, $q(x)$ COPRIMI

$\exists\ \alpha(x), \beta(x)$

$\alpha(x)p(x) + \beta(x)\cdot q(x) = A(x)$