Analisi Matematica 2 - A.A. 21/22 - Callegari

Lezione 9

23 Marzo 2022

1) CRIT. CONFRONTO ASINTOTICO

2) ALTRI LIMITI NOTEVOLI

A) $\int_0^1 \frac{1}{x^\alpha}$

B) $\int_0^{\frac{1}{2}} \frac{1}{x^\alpha |\ln x|^\beta}$

C) $\int_0^{+\infty} A^x\,dx$

④ $\int_0^{+\infty} \frac{\cos(\cos x)}{\ln(|\sin x|+|\cos x|+e^x)}\,dx$ ⑤ $\int_1^{+\infty} \left(1-\frac{1}{\sqrt{t}}\right)^t dt$

⑤ $\int_0^{+\infty} \frac{\left(1+\frac{1}{x}\right)^x}{\log_2(1+x^2)}\,dx$ ⑥ $\int_0^{+\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\ln^3(1+\sqrt{x})}\,dx$

⑦ $\int_1^{+\infty} \left(1+\frac{\sin x}{x}\right)^x dx$ $\int_0^{+\infty} \left(1.1+\frac{\sin x}{x}\right)^x dx$ $\int_0^{+\infty} \left(0.9+\frac{\sin x}{x}\right)^x dx$

$$\int_1^{+\infty} \left(1-\frac{1}{\sqrt{t}}\right)^t dt$$

$\frac{(\sqrt{t})^4}{2^{\sqrt{t}}} \to 0$

DEF. PROSSIMA

$$\left(\left(1-\frac{1}{\sqrt{t}}\right)^{\sqrt{t}}\right)^{\sqrt{t}} < \left(\frac{1}{2}\right)^{\sqrt{t}} < \frac{1}{(\sqrt{t})^4} = \frac{1}{t^2}$$

$\forall t > t_0$ $\left(1-\frac{1}{\sqrt{t}}\right)^t < \frac{1}{t^2}$

$$\int_{t_0}^{+\infty} \frac{1}{t^2}\,dt$$

$\boxed{T.1}$ (CR. CONF. ASINTOTICO)

DATE $f,g:[a,b)\to(0,+\infty)$ t.c. $f,g\in\mathcal{R}([a,c])$ $\forall c\in[a,b)$

T.C. $f(x)\simeq g(x)$ PER $x\to b^-$.

ALLORA $\int_a^b f(x)\,dx$ E $\int_a^b g(x)\,dx$ HANNO STESSO CARATTERE

$\boxed{DIM}$

$$\frac{f(x)}{g(x)} \to 1 \quad \text{PER } x\to b^-$$

$$\Downarrow$$

$$\exists x_0\in[a,b) \text{ t.c. } x\geq x_0 \Rightarrow \frac{1}{2} < \frac{f(x)}{g(x)} < \frac{3}{2}$$

$$\Updownarrow$$

$$\boxed{\frac{1}{2}g(x) < f(x) < \frac{3}{2}g(x)}$$

$\boxed{\int_a^b f(x)\,dx}$ $\boxed{\int_{x_0}^b f(x)\,dx}$

$\boxed{\int_a^b g(x)\,dx}$ $\boxed{\int_{x_0}^b g(x)\,dx}$

T.CONF.

$\int_{x_0}^b g(x)\,dx$ conv. $\Rightarrow \int_{x_0}^b f(x)\,dx$ CONV ?

$\int_{x_0}^b g(x)\,dx$ CONV. $\Rightarrow \int_{x_0}^b \frac{3}{2}g(x)\,dx \Rightarrow \int_{x_0}^b f(x)\,dx$ CONV

CONV.

TRA POCO

$\int_{x_0}^b f(x)\,dx$ CONV. $\Rightarrow \int_{x_0}^b \frac{1}{2}g(x)\,dx$ CONV $\Rightarrow \int_{x_0}^b g(x)\,dx$

$\boxed{OSS.1}$ (PER FARE PASS.)

DATE $f,g:[a,b)\to\mathbb{R}$ t.c. $f,g\in\mathcal{R}([a,c])$ con $c\in[a,b)$

SE $\int_a^b f(x)\,dx$ E $\int_a^b g(x)\,dx$ CONVERG.

ALLORA $\forall\alpha,\beta\in\mathbb{R}$ $\int_a^b \alpha f(x)+\beta g(x)\,dx$ CONV.

$\boxed{DIM}$

$$\int_a^b \alpha f+\beta g = \lim_{c\to b^-} \int_a^c \alpha f(x)+\beta g(x)\,dx = \lim_{c\to b^-}\left(\alpha\int_a^c f(x)\,dx + \beta\int_a^c g(x)\,dx\right) =$$

$$= \alpha\int_a^b f(x)\,dx + \beta\int_a^b g(x)\,dx$$

$\boxed{OSS.2}$ DATE $f,g:[a,b)\to\mathbb{R}$ t.c. $f,g\in\mathcal{R}([a,c])$ $\forall c\in[a,b)$

E T.C. $\int_a^b g(x)\,dx$ CONV.

ALLORA

$\int_a^b f(x)\,dx$ $\boxed{\int_a^b f(x)+g(x)\,dx}$ HANNO ST. CARATTERE

$\boxed{DIM}$

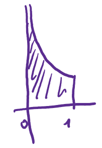
$$\lim_{c\to b^-} \int_a^c f(x)+g(x)\,dx =$$

$$= \lim_{c\to b^-}\left(\int_a^c f(x)\,dx + \underbrace{\int_a^c g(x)\,dx}_{\lambda\in\mathbb{R}}\right) = \begin{cases} \text{ES. FINITO} \\ \text{ES. INFINITO} \\ \text{NON ESISTE} \end{cases} \text{A SECONDA DI CIÒ CHE FA}$$

$\boxed{ES.1}$

$$\int_0^1 \frac{1}{x^\alpha}\,dx = \begin{cases} \text{CONV. } \frac{1}{1-\alpha} & \alpha<1 \\ \text{DIV. } +\infty & \alpha\geq 1 \end{cases}$$

$$\int_0^1 \frac{1}{x}\,dx = \lim_{c\to 0^+}\int_c^1 \frac{1}{x}\,dx =$$



$$= \lim_{c\to 0^+}\left[\ln x\right]_c^1 =$$

$$= \lim_{c\to 0^+}(\ln 1 - \ln c) = +\infty$$

$$\int_0^1 \frac{1}{x^\alpha}\,dx = \lim_{c\to 0^+}\int_c^1 x^{-\alpha}\,dx = \lim_{c\to 0^+}\left[\frac{1}{-\alpha+1}x^{-\alpha+1}\right]_c^1 =$$

$$= \lim_{c\to 0^+}\left(\frac{1}{1-\alpha} - \frac{1}{1-\alpha}\cdot c^{1-\alpha}\right) = \begin{cases} \frac{1}{1-\alpha} & \alpha<1 \\ +\infty & \alpha>1 \end{cases}$$

$\boxed{ES.2}$

$$\int_0^{\frac{1}{2}} \frac{1}{x^\alpha|\ln x|^\beta}\,dx = \begin{cases} \alpha>1 & \text{DIV.} \\ \alpha=1 & \begin{cases} \beta>1 & \text{CONV} \\ \beta\leq 1 & \text{DIV.} \end{cases} \\ \alpha<1 & \text{CONV} \end{cases}$$

$\alpha=1$

$$\int_0^{\frac{1}{2}} \frac{1}{|\ln x|^\beta}\cdot(\ln x)'\,dx = \lim_{c\to 0^+}\int_c^{\frac{1}{2}} \frac{1}{|\ln x|^\beta}(\ln x)'\,dx =$$

$$= \lim_{c\to 0^+}\int_{\ln c}^{\ln\frac{1}{2}} \frac{1}{|y|^\beta}\,dy =$$

$$= \lim_{c\to 0^+}\int_{\ln c}^{\ln\frac{1}{2}} \frac{1}{(-y)^\beta}\,dy = \lim_{c\to 0^+}\int_{-\ln c}^{-\ln\frac{1}{2}} \frac{1}{t^\beta}(-1)\,dt =$$

$$= \lim_{c\to 0^+}\int_{\ln 2}^{\ln\frac{1}{c}} \frac{1}{t^\beta}\,dt =$$

$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx$ $= \int_{\ln 2}^{+\infty} \frac{1}{t^\beta}\,dt$ = ST. CAR $\int_1^{+\infty} \frac{1}{t^\beta}\,dt$ CONV. $\beta>1$ / DIV. $\beta\leq 1$

$\boxed{\alpha>1}$ $\alpha=1+2\delta$ $\delta>0$

$$\boxed{\int_0^{\frac{1}{2}} \frac{1}{x^\alpha|\ln x|^\beta}\,dx}$$

$\int_0^{x_0}$ $\int_0^{x_0}$

$$\frac{1}{x^{1+2\delta}\cdot|\ln x|^\beta} = \frac{1}{x^{1+\delta}}\cdot\frac{1}{x^\delta|\ln x|^\beta} > \frac{1}{x^{1+\delta}}\cdot 1$$

$+\infty$

DEF. PER $x\to 0^+$

$$|\ln x| = \left|\ln\left(\frac{1}{x}\right)^{-1}\right| = \left|\ln\frac{1}{x}\right|$$

$$\frac{\left(\frac{1}{x}\right)^\delta}{\left|-\ln\frac{1}{x}\right|^\beta} \qquad \frac{y^\delta}{(\ln y)^\beta} \quad y\to+\infty \quad \to +\infty$$

0 x_0 $\frac{1}{2}$

12:30

$$\int_0^{\frac{1}{2}} \frac{1}{x^{1-2\delta}\cdot|\ln x|^\beta}\,dx$$

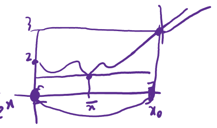
$$\frac{1}{x^{1-\delta}\cdot x^{-\delta}|\ln x|^\beta} = \frac{1}{x^{1-\delta}}\cdot\frac{x^\delta}{|\ln x|^\beta} < \frac{1}{x^{1-\delta}}$$

$\to 0$

$[0,x_0)$ $x_0\leq\frac{1}{2}$

$\int_0^{x_0}$ CONV

PERCHE $\int_0^{x_0}$ CONV



$\overbrace{|\sin x|+|\cos x|+e^x}^{f(x)}$

$\geq f(\bar{x})$ $c = |\sin\bar{x}|+|\cos\bar{x}|+e^{\bar{x}} > 1$

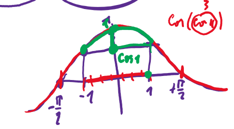
$$\int_0^{+\infty} \frac{\cos(\cos x)}{\ln(|\sin x|+|\cos x|+e^x)}\,dx$$

$\frac{\pi}{4} \geq c > 0$

$\boxed{|\sin x|+|\cos x|+e^x}$ $-1<\cos x<1$ $\cos 1 \leq \cos(\cos x) \leq 1$

>1

$\geq c > 1$



$\int_1^{+\infty} \frac{1}{x}\,dx$ $\int_1^{\infty} \frac{k}{x}\,dx$

$$\int_0^{+\infty} \frac{\cos(\cos x)}{\ln(|\cos x|+|\sin x|+e^x)}\,dx$$

$$\int_1^{+\infty} \qquad dx$$

$\cos 1 \leq \cos(\cos x) \leq 1$

$$\frac{\cos(\cos x)}{\ln(|\cos x|+|\sin x|+e^x)} \geq \cos(\cos 1)\cdot\frac{1}{\ln(|\cos x|+|\sin x|+e^x)}$$

$$\int_1^{+\infty} \frac{1}{\ln(|\cos x|+|\sin x|+e^x)}\,dx \qquad \int_1^{+\infty} \frac{1}{x}\,dx$$

$$\ln(|\cos x|+|\sin x|+e^x) = \ln\left(e^x\cdot\left(\frac{|\cos x|+|\sin x|}{e^x}+1\right)\right) =$$

$\sim\frac{1}{x}$

$$= x + \ln\left(1+\underbrace{\frac{|\cos x|+|\sin x|}{e^x}}_{0}\right) \approx x$$