LEZ 12 - (30/03/2022)

ESERCIZI SU INTEGRALI IMPROPRI

STUDIARE CONVERGENZA DI:

1) $$\int_1^{+\infty} \frac{\sin(\sin x)}{\ln(3+x^2)}\,dx$$

2) $$\int_1^{+\infty} \frac{\sin(x^2)}{5+x^\alpha}\,dx$$

FARE 2 CASI: $\alpha = 2$, $\alpha = 1$

3) $$\int_1^{+\infty} \frac{\sin x}{\sqrt{x}+\sin x}\,dx$$

4) $$\int_1^{+\infty} \frac{\cos(x+e^{-x})}{\sqrt{1+x^2}}\,dx$$

5) $$\int_1^{+\infty} \left|\lambda + \frac{\sin x}{4}\right|^x dx$$

FARE 4 CASI: $\lambda = 0$, $\lambda = 2$, $\lambda = 1$, $\lambda = \frac{3}{4}$

1. $$\int_1^{+\infty} \frac{\sin(\sin x)}{\ln(3+x^2)}\,dx = \int_1^{+\infty} \frac{1}{\ln(3+x^2)} \cdot \boxed{\sin(\sin x)}\,dx = \underline{\text{CONVERGE}}$$

$$|\circledcirc| \leq$$

$f(x)$

$f(x) \cdot g(x)$

$T = 2\pi$

$$\int_{-\pi}^{\pi} \sin(\sin x)\,dx = 0$$

LIMITATA $\to$ $G(x) = \int_0^x \sin(\sin t)\,dt$

$$\boxed{G(x+T)} = \int_0^{x+T} \sin(\sin t)\,dt =$$

$$= \int_0^x \sin(\sin t)\,dt + \int_x^{x+T} \sin(\sin t)\,dt =$$

$$= \boxed{G(x)} + 0 =$$

$$=$$

2. $$\int_1^{+\infty} \frac{\sin(x^2)}{5+x^2}\,dx$$

$$\int_1^{+\infty}$$

$$0 \leq |\bullet| \leq \frac{1}{5+x^2} \leq \boxed{\frac{1}{x^2}}$$

$$\int_1^{+\infty} |\bullet|\,dx$$

2 bis

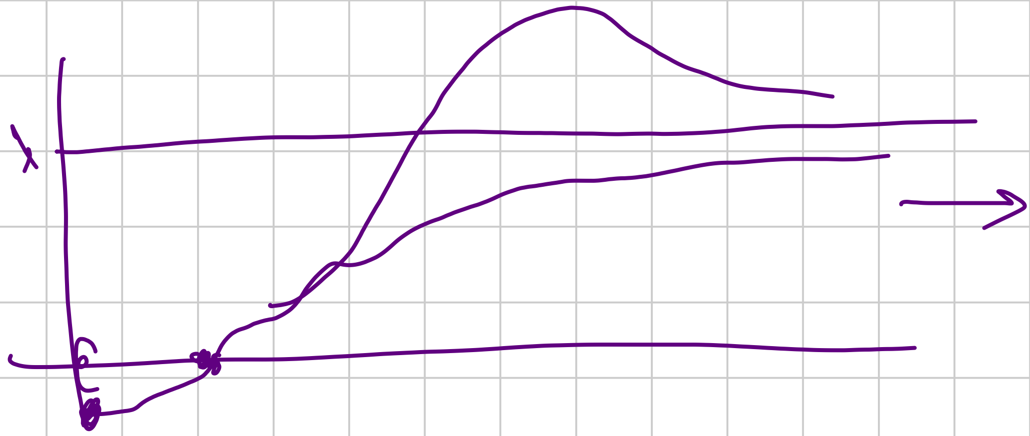
$$\int_1^{+\infty} \frac{\sin(x^2)}{5+x}\,dx = \int_1^{+\infty} \frac{1}{5+x}\cdot\boxed{(\sin(x^2))}\,dx$$

$$\boxed{G(x) = \int_1^{x} \sin(t^2)\,dt}$$

$$\int_1^{+\infty} \sin(x^2)\,dx$$

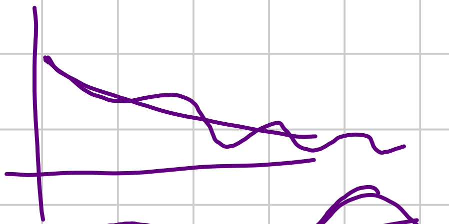
$$\int_1^{+\infty} \frac{\sin t}{\sqrt{t}}\,dt$$

$$\lim_{c\to+\infty} \underbrace{\int_1^{c} \sin(x^2)\,dx}_{G(c)} = \text{ESISTE FINITO}$$

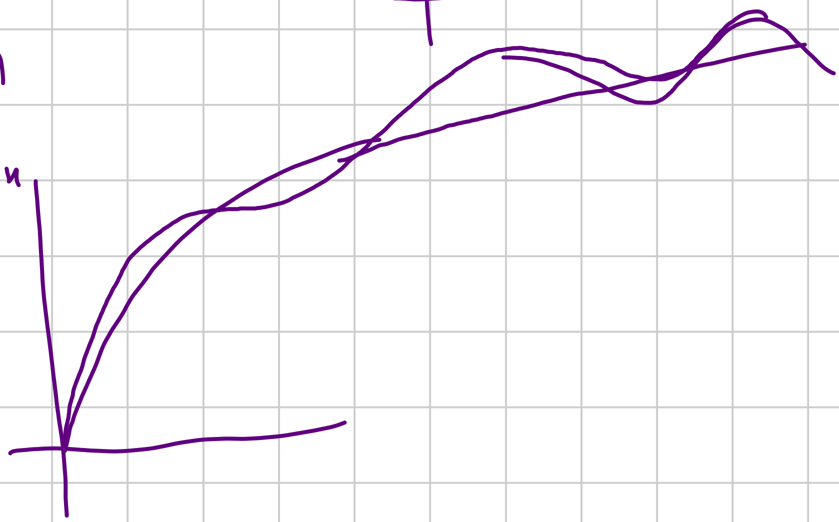


3

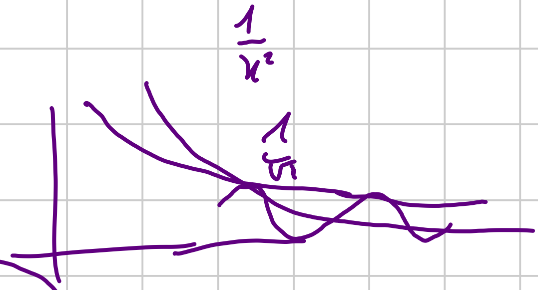
$$\int_1^{+\infty} \frac{\sin x}{\sqrt{x}+\sin x}\,dx =$$



$$= \int_1^{+\infty} \frac{1}{\sqrt{x}+\sin x}\cdot \underbrace{\sin x}_{g(x)}\,dx$$



$$0 \le | \,\cdot\, | = \frac{1}{\sqrt{x}+\sin x}\cdot|\sin x| \le$$



$$\int_1^{+\infty}\frac{\sin x}{\sqrt{x}}\,dx \quad \text{CONV.}$$

$$\underbrace{\int_1^{+\infty}\frac{\sin x}{\sqrt{x}+\sin x}}\,dx$$

$$\int_1^{+\infty}\left(\frac{\sin x}{\sqrt{x}+\sin x}-\frac{\sin x}{\sqrt{x}}\right)dx$$

$$= \int_1^{+\infty}\sin x\,\frac{\cancel{\sqrt{x}}-(\cancel{\sqrt{x}}+\sin x)}{(\sqrt{x}+\sin x)\cdot\sqrt{x}}\,dx =$$

$$= -\int_1^{+\infty}\frac{+\sin^2 x}{x+\sqrt{x}\sin x}\,dx =$$

$$\frac{1}{2}x \le x+\sqrt{x}\sin x \le 2x$$

$$\frac{1}{2x} \le \frac{1}{x+\sqrt{x}\sin x} \le \frac{2}{x}$$

$$\frac{\sin^2 x}{2x} \le \frac{\sin^2 x}{x+\sqrt{x}\sin x} \le \frac{2\sin^2 x}{x}$$

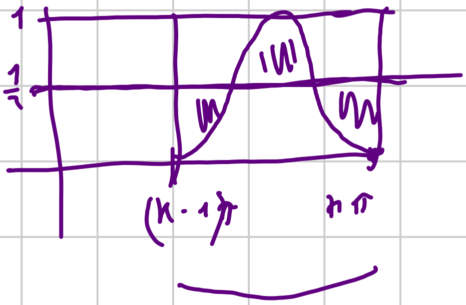
$$\int_1^{+\infty}\frac{\sin^2 x}{x}\,dx$$

$$\int_{\pi}^{+\infty} \frac{\sin^2 x}{x}\,dx$$

$$\lim_{\ell \to +\infty} \int_{\pi}^{\ell} \frac{\sin^2 x}{x}\,dx$$

$$\lim_{n \to +\infty} \int_{\pi}^{n\pi} \frac{\sin^2 x}{x}\,dx =$$

$$= \lim_{n \to +\infty} \Bigg(\underbrace{\int_{\pi}^{2\pi} \cdot}_{\frac{1}{2}} + \underbrace{\int_{2\pi}^{3\pi} \cdot}_{\frac{1}{4}} + \dots + \underbrace{\int_{(n-1)\pi}^{n\pi} \cdot}_{\frac{1}{2n}} \Bigg) \geq$$



$$[(k-1)\pi, k\pi]$$

$$\frac{\sin^2 x}{x} \geq \frac{\sin^2 x}{k\pi}$$

$$\int_{(k-1)\pi}^{k\pi} \frac{\sin^2 x}{x}\,dx \geq \int_{(k-1)\pi}^{k\pi} \frac{\sin^2 x}{k\pi}\,dx =$$

$$= \frac{1}{k\pi} \int_{(k-1)\pi}^{k\pi} \sin^2 x\,dx =$$

$$= \frac{1}{2k}$$

$$\int_0^{\pi} \sin^2 x\,dx = \frac{\pi}{2}$$

$$\lim_{n \to +\infty} \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots + \frac{1}{2n} \right) =$$

$$= \lim_{n \to +\infty}$$

$$\sum \frac{1}{n}$$

$$\frac{1}{2}\left(1 + \frac{1}{2} + \dots + \frac{1}{n} \right) \geq$$

$$\geq \frac{1}{2}\left(\ln\left(1+\frac{1}{1}\right) + \ln\left(1+\frac{1}{2}\right) + \dots \ln\left(1+\frac{1}{n}\right) \right)$$

$$= \frac{1}{2}\left(\cancel{\ln 2} - \ln 1 + \cancel{\ln 3} - \cancel{\ln 2} + \cancel{\ln 4} - \cancel{\ln 3}, \dots + \ln n+1 - \cancel{\ln n}\right)$$

$$= \frac{1}{2} \ln(n+1) \to +\infty$$

$$\int_0^{+\infty} \frac{\cos(f(x))}{\sqrt{1+x^2}}\,dx$$

$\to$ 1) $f(x) = x$

2) $f(x) = x + e^{-x}$

$$\int_0^{+\infty} \frac{\cos x}{\sqrt{1+x^2}}\,dx$$

$\boxed{\text{CONV. PER CR. INT. OSC.}}$

$$\int_0^{+\infty} \frac{\cos(x+e^{-x})}{\sqrt{1+x^2}}\,dx$$

$$\int_0^{+\infty} \left(\frac{\cos(x+e^{-x})}{\sqrt{1+x^2}} - \frac{\cos x}{\sqrt{1+x^2}}\right) dx =$$

$$= \int_0^{+\infty} \frac{\cos(x+e^{-x}) - \cos x}{\sqrt{1+x^2}}\,dx =$$

$$0 \le |\ \ | = \frac{|\cos(x+e^{-x}) - \cos x|}{\sqrt{1+x^2}} \le$$

$$\left|\cos(x+e^{-x}) - \cos x\right| \leq \left|x+e^{-x}-x\right| = e^{-x}$$

$$\left|\cos(x)-\cos(y)\right| \leq |x-y|$$

$$\leq \frac{e^{-x}}{\sqrt{1+x^2}} = \frac{1}{e^{x}\cdot\sqrt{1+x^2}} \leq \frac{1}{x^2}$$

$$\int_1^{+\infty} \left|\lambda + \frac{\sin x}{4}\right|^x dx$$

~~$\lambda = 0$~~

$\lambda = 2$

$\lambda = 1$

$\lambda = \frac{3}{4}$ (Recherche)

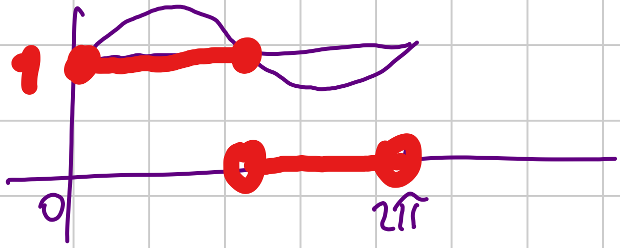
$$\int_1^{+\infty} \left|\frac{\sin x}{4}\right|^x dx$$

$$0 \leq \left|\frac{\sin x}{4}\right|^x \leq \left(\frac{1}{4}\right)^x \leq \frac{1}{x^2}$$

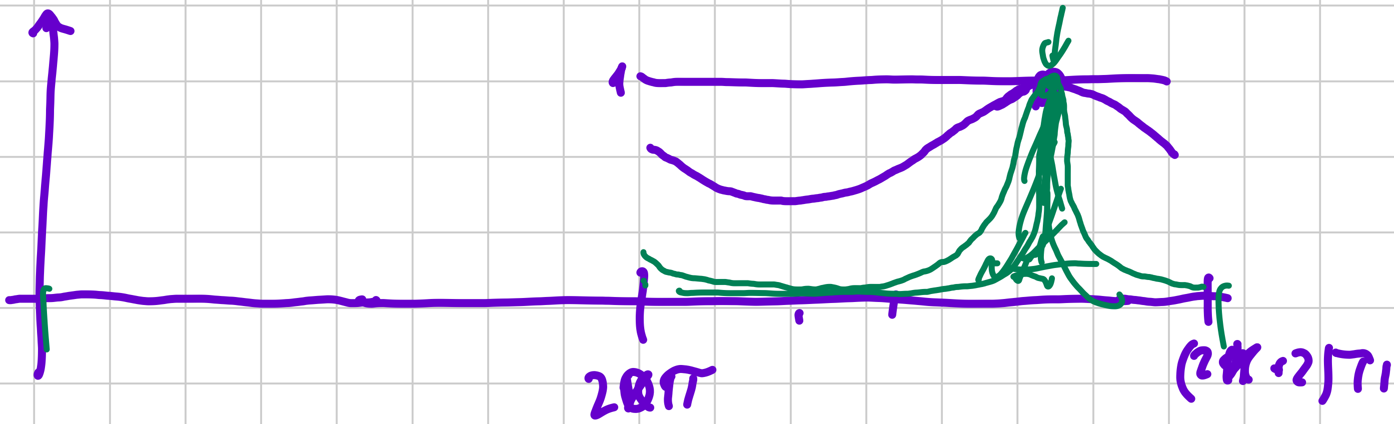
$$\int_1^{+\infty} \left|2 + \frac{\sin x}{4}\right|^x dx$$

$$1 \leq \left(\frac{7}{4}\right)^x \leq \left|2 + \frac{\sin x}{4}\right|^x$$

$$\left|1 + \frac{\sin x}{4}\right|^x$$



1
0
2π



1
2πn
(2n+2)π