-1) CR. ABEL **0)** ESEMPI **0A** $\sum_{n=1}^{+\infty}\frac{\cos\left(\frac{2n\pi}{3}\right)}{\sqrt{n}}$ **0B** $\sum_{n=1}^{+\infty}\frac{\cos n}{n}$ **0C** $\sum_{n=1}^{+\infty}\frac{\cos^3 n}{n}$ **0D** $\sum_{n=1}^{+\infty}\frac{\cos^2 n}{n}$

Analisi Matematica 1 - Lista n. 30

Studio del carattere di Serie a Termini Positivi

Titolo nota

www.problemisvolti.it

Studiare il carattere delle seguenti serie:

1) $\sum_{n=1}^{+\infty}\frac{1}{n^4+1}$ 2) $\sum_{n=0}^{+\infty}\frac{1}{2^n+3^n}$

3) $\sum_{n=1}^{+\infty}\frac{n^4}{n^2+n^4}$ 4) $\sum_{n=1}^{+\infty}\ln\left(1+\frac{1}{n}\right)$

5) $\sum_{n=0}^{+\infty}e^{-\sqrt{n}}$ 6) $\sum_{n=1}^{+\infty}\frac{1}{n^n}$

7) $\sum_{n=0}^{+\infty}\frac{e^{\sin n}}{1+n^2}$ 8) $\sum_{n=1}^{+\infty}\frac{1}{n^{\ln n}}$

9) $\sum_{n=1}^{+\infty}\frac{e^{\cos n}}{n}$ 10) $\sum_{n=1}^{+\infty}\ln\left(1+\frac{1}{n^2}\right)^{\sqrt{n}}$

11) $\sum_{n=1}^{+\infty}\frac{1}{(1+n)\cdot\sqrt{\ln(1+n^4)}}$ 12) $\sum_{n=1}^{+\infty}\frac{1}{n\cdot\ln^2(1+\sqrt{n})}$

13) $\sum_{n=0}^{+\infty}\frac{(n!)^2}{(2n)!}$ 14) $\sum_{n=1}^{+\infty}\frac{(3n)!}{n^{4n}}$

15) $\sum_{n=1}^{+\infty}\frac{(n+1)^{n-1}}{n^{n+1}}$ 16) $\sum_{n=3}^{+\infty}\left(\frac{\ln(\ln n)}{\ln n}\right)^{\sqrt{n}}$

17) $\sum_{n=2}^{+\infty}\frac{1}{\ln(n!)}$ 18) $\sum_{n=2}^{+\infty}\frac{1}{(\ln n)^{\ln n}}$

19) $\sum_{n=3}^{+\infty}\frac{1}{(\ln(\ln n))^{\ln n}}$ 20) $\sum_{n=3}^{+\infty}\frac{1}{(\ln n)^{\ln(\ln n)}}$

21) $\sum_{n=3}^{+\infty}n^{-1-\frac{1}{\ln(\ln n)}}$ 22) $\sum_{n=1}^{+\infty}n^{-\sqrt[n]{n}}$

23) $\sum_{n=1}^{+\infty}\frac{\cos(\cos n)}{n\cdot\ln n}$ 24) $\sum_{n=1}^{+\infty}(\sin(\sin n))^n$

25) $\sum_{n=1}^{+\infty}\left(\arctan\frac{1}{\sqrt[4]{n}}-\sin\frac{1}{\sqrt[4]{n}}\right)$ 26) $\sum_{n=1}^{+\infty}\left(\sqrt{1-\frac{1}{\sqrt{n}}}-\cos\frac{1}{\sqrt[4]{n}}\right)$

27) $\sum_{n=1}^{+\infty}\frac{\sqrt{n!}}{(\sqrt{n})^n}$ 28) $\sum_{n=1}^{+\infty}\frac{1}{\sqrt[n]{(n!)^2}}$

29) $\sum_{n=1}^{+\infty}\frac{(n!)!}{n^{n^n}}$ 30) $\sum_{n=1}^{+\infty}\frac{(n!)^{n!}}{(n^n)!}$

31) $\sum_{n=1}^{+\infty}\frac{1+\cos n}{n}$ 32) $\sum_{n=1}^{+\infty}\left(e-\sum_{k=0}^{n}\frac{1}{k!}\right)$

33) $\sum_{n=1}^{+\infty}\left(\sum_{k=n}^{+\infty}\frac{1}{k^k}\right)$ 34) $\sum_{n=1}^{+\infty}\left(\sum_{k=n}^{+\infty}\frac{1}{k^2}\right)$

Studiare, al variare del parametro reale $a>0$, il carattere delle seguenti serie:

35) $\sum_{n=1}^{+\infty}\left(1-\cos\frac{1}{n}\right)^a$ 36) $\sum_{n=1}^{+\infty}\left(\frac{\pi}{2}-\arctan(n^a)\right)$

37) $\sum_{n=1}^{+\infty}\left(\ln\left(\cos\frac{1}{\sqrt[4]{n}}\right)+\frac{1}{2\sqrt{n}}+\frac{a}{n}\right)$ 38) $\sum_{n=1}^{+\infty}\left(\cos\left(\sin\frac{1}{\sqrt[4]{n}}\right)-1+\frac{1}{2\sqrt{n}}-\frac{a}{n}\right)$

39) $\sum_{n=1}^{+\infty}\frac{1}{a^{\ln n}}$ 40) $\sum_{n=2}^{+\infty}\left(\frac{\log_{2^n}4}{\log_4 n^n}\right)^a$

41) $\sum_{n=1}^{+\infty}\left(1-\frac{1}{n^3}\right)^{n^a}$ 42) $\sum_{n=1}^{+\infty}\frac{n!}{n^n}a^n$

43) $\sum_{n=3}^{+\infty}\frac{1}{n\cdot\ln n\cdot(\ln(\ln n))^a}$ 44) $\sum_{n=1}^{+\infty}\left(e-\left(1+\frac{1}{n}\right)^n\right)^a$

45) $\sum_{n=1}^{+\infty}\left(e-\sum_{k=0}^{n}\frac{1}{k!}\right)^a$ 46) $\sum_{n=1}^{+\infty}\frac{1}{\sqrt[n]{(n!)^a}}$

47) $\sum_{n=1}^{+\infty}\frac{1}{n^2}\left(\frac{n+2}{n+1}\right)^{an\ln n}$ 48) $\sum_{n=1}^{+\infty}\left(\frac{n^2+n-1}{n^2+3n+5}\right)^{n^a}$

49) $\sum_{n=1}^{+\infty}\frac{n!\cdot n^{n+1}}{(2n)!}a^n$ 50) $\sum_{n=1}^{+\infty}\frac{n^{2n}}{(2n)!}a^n$

Analisi Matematica 1 - Lista n. 31

Studio del carattere di Serie a Termini di segno qualsiasi

Titolo nota

www.problemisvolti.it

Studiare convergenza semplice e assoluta delle seguenti serie:

1) $\sum_{n=1}^{+\infty}(-1)^n\tan\frac{1}{n}$ 2) $\sum_{n=1}^{+\infty}\frac{(-1)^n}{n+\sin n}$

3) $\sum_{n=2}^{+\infty}(-1)^n\left(1-\frac{1}{n}\right)^{n\ln n}$ 4) $\sum_{n=1}^{+\infty}\frac{(-1)^n}{2n+\sin n}$

5) $\sum_{n=1}^{+\infty}(-1)^n\frac{n^2+4n+8}{n^3+2n^2+7n}$ 6) $\sum_{n=1}^{+\infty}\frac{\cos(\frac{\pi}{2}n)}{n}$

7) $\sum_{n=1}^{+\infty}\frac{(-1)^n}{\ln n+e^{-n}}$ 8) $\sum_{n=1}^{+\infty}\sin\left(\pi n+\frac{1}{n^2}\right)$

9) $\sum_{n=1}^{+\infty}\frac{(-1)^n n}{n\ln n+1}$ 10) $\sum_{n=1}^{+\infty}\left(e^{\frac{(-1)^n}{\sqrt{n}}}-1\right)$

11) $\sum_{n=1}^{+\infty}\ln\left(1+\frac{(-1)^{n+1}}{\sqrt[3]{n}}\right)$ 12) $\sum_{n=1}^{+\infty}\frac{(-1)^n}{2n+(-1)^n n}$

13) $\sum_{n=1}^{+\infty}\sin\left(\pi n^2+\frac{1}{n}\right)$ 14) $\sum_{n=1}^{+\infty}\sin\left(\pi n!+\frac{1}{n}\right)$

15) $\sum_{n=1}^{+\infty}\frac{\sqrt{2}\cdot\sin\left((2n+1)\frac{\pi}{4}\right)}{n^{2+(-1)^n}}$ 16) $\sum_{n=1}^{+\infty}\frac{(-1)^n}{\sqrt{n}+\sin n}$

17) $\sum_{n=2}^{+\infty}\ln\left(\frac{n^2+2-(-1)^n}{n^2+1}\right)$ 18) $\sum_{n=2}^{+\infty}\ln\left(\frac{n+2\cdot(-1)^n}{n+1}\right)$

19) $\sum_{n=1}^{+\infty}(-1)^n\left(e^{-\frac{2}{\sqrt{n}}}-\cos\frac{2}{\sqrt[4]{n}}\right)$ 20) $\sum_{n=1}^{+\infty}\left(\sin\frac{(-1)^n}{\sqrt[5]{n}}-\frac{(-1)^n}{\sqrt[5]{n}}\cdot\cos\frac{(-1)^n}{\sqrt{3}\cdot\sqrt[5]{n}}\right)$

21) $\sum_{n=1}^{+\infty}(-1)^n\left(2\cdot e^{\frac{1}{\sqrt{n}}}-2\cdot\cos\frac{1}{\sqrt[4]{n}}+\ln\left(1-\frac{2}{\sqrt{n}}\right)\right)$ 22) $\sum_{n=1}^{+\infty}\left(2\cdot e^{\frac{(-1)^n}{\sqrt{n}}}-2\cdot\cos\frac{(-1)^n}{\sqrt[3]{n}}+\ln\left(1-2\cdot\frac{(-1)^n}{\sqrt[3]{n}}\right)\right)$

23) $\sum_{n=1}^{+\infty}\frac{\sin(\pi\cdot\log_3 n)}{n}$ 24) $\sum_{n=1}^{+\infty}\frac{\sin n}{n}$

Studiare, al variare di $a>0$, convergenza semplice e assoluta delle seguenti serie:

25) $\sum_{n=1}^{+\infty}(-1)^n\left(\frac{1}{n}-\arctan\frac{1}{n}\right)^a$ 26) $\sum_{n=1}^{+\infty}\frac{(-1)^n}{n^a+1}\left(1+\frac{1}{n}\right)^n$

27) $\sum_{n=2}^{+\infty}\frac{(-1)^n}{n\cdot(\ln n)^a}\left(1+\frac{1}{n}\right)^n$ 28) $\sum_{n=2}^{+\infty}(-1)^n\frac{(n+1)^{an}}{n^n\cdot\ln n}$

29) $\sum_{n=1}^{+\infty}(-1)^n n^{an}\sin\frac{1}{(n+100)!}$ 30) $\sum_{n=1}^{+\infty}\frac{1}{1+\left(-1-\frac{1}{n}\right)^n\cdot n^a}$

31) $\sum_{n=\lfloor a\rfloor+1}^{+\infty}\frac{(-1)^n}{n+a\sin n}$ 32) $\sum_{n=1}^{+\infty}\left(\sqrt{1+\frac{(-1)^n}{n^a}}-1\right)$

33) $\sum_{n=2}^{+\infty}\frac{(-1)^n}{n^a+\cos(n\pi)}$ 34) $\sum_{n=1}^{+\infty}\frac{(-1)^n}{n^a+\cos n}$

Data $f(x)=2\arctan x+\ln(1-2x)+2x^2$, studiare, al variare di $a>0$, convergenza semplice e assoluta delle serie:

35) $\sum_{n=1}^{+\infty}(-1)^n f\left(\frac{1}{n^a}\right)$ 36) $\sum_{n=1}^{+\infty}f\left(\frac{(-1)^n}{n^a}\right)$

Data $f(x)=\sin(2x)+2\ln(1-x)+x^2$, studiare, al variare di $a>0$, convergenza semplice e assoluta delle serie:

35) $\sum_{n=1}^{+\infty}(-1)^n f\left(\frac{1}{n^a}\right)$ 36) $\sum_{n=1}^{+\infty}f\left(\frac{(-1)^n}{n^a}\right)$

Per ogni $n\in\mathbb{N}-\{0\}$ definiamo $a_n=\begin{cases}\frac{2}{\sqrt[4]{n}} & \text{se } n \text{ è un quadrato perfetto}\\ -\frac{1}{\lfloor\sqrt{n}\rfloor^{\frac{5}{3}}} & \text{altrimenti}\end{cases}$ studiare convergenza semplice e assoluta di:

37) $\sum_{n=1}^{+\infty}a_n$ 38) $\sum_{n=1}^{+\infty}(a_n)^2$

39) $\sum_{n=1}^{+\infty}(a_n)^3$ 40) $\sum_{n=1}^{+\infty}\arctan(a_n)$

Analisi Matematica 2 - 11 aprile 2022 - docente: Callegari

Lezione 17

Esercizi sulle Serie

$\boxed{T.1}$ (CR. ABEL)

DATA $\sum a_n b_n$ T.C.

1) $a_n\to 0$ DECR.

2) DETTA (B_n) LA SUCC. T.C. $B_n=b_0+b_1+\dots+b_n$ SI HA (B_n) È LIMITATA, CIOÈ $\exists M>0$ t.c. $\forall n\in\mathbb{N}\ |B_n|\le M$

ALLORA $\boxed{\sum a_n b_n}$ CONV.

$\boxed{DIM}$

I $\overbrace{\sum a_n b_n}^{S_n}$ E $\overbrace{\sum(a_n-a_{n+1})B_n}^{\mathcal{S}_n}$ HANNO ST. CARATTERE

II $\sum(a_n-a_{n+1})B_n$ CONVERGE

I $\boxed{\mathcal{S}_n}=(a_0-a_1)B_0+(a_1-a_2)B_1+(a_2-a_3)B_2+\dots+(a_n-a_{n+1})B_n=$

$=a_0B_0-a_1B_0+a_1B_1-a_2B_1+a_2B_2-a_3B_2+\dots+a_nB_n-a_{n+1}B_n=$

$=a_0b_0+a_1(B_1-B_0)+a_2(B_2-B_1)+a_3(B_3-B_2)+\dots+a_n(B_n-B_{n-1})-a_{n+1}B_n$

$=a_0b_0+a_1b_1+a_2b_2+a_3b_3+\dots+a_nb_n-a_{n+1}B_n=$

$=\boxed{S_n-a_{n+1}B_n}$

$S_n=\mathcal{S}_n+\underbrace{a_{n+1}B_n}_{\to 0}$ PERCHÉ $a_{n+1}\to 0$, B_n LIMITATA

QUINDI SE MOSTRO CHE $\mathcal{S}_n\to\ell$ finito ALLORA ANCHE $S_n\to\ell$

II MOSTRIAMO CHE $\mathcal{S}_n\to\ell$ finito, CIOÈ CHE $\sum(a_n-a_{n+1})\cdot B_n$ CONVERGA.

A TALE SCOPO MOSTRIAMO CHE $\boxed{\sum|(a_n-a_{n+1})B_n|}$ CONV.

$0\le\left|\overbrace{(a_n-a_{n+1})}^{\ge 0\text{ MONOTONIA DI }(a_n)}B_n\right|=(a_n-a_{n+1})\cdot|B_n|\le M\cdot(a_n-a_{n+1})$

MA $\sum_{n=0}^{+\infty}M(a_n-a_{n+1})=M\sum_{n=0}^{+\infty}(a_n-a_{n+1})$

(σ_n) SUCC. SOMME FINITE DI

$\boxed{\sigma_n}=a_0-\cancel{a_1}+\cancel{a_1}-\cancel{a_2}+\cancel{a_2}-\cancel{a_3}+\cancel{a_3}-a_4+\dots+a_n-a_{n+1}=$

$=\boxed{a_0-a_{n+1}}$

$\lim_{n\to+\infty}\sigma_n=\lim_{n\to+\infty}(a_0-a_{n+1})=a_0$

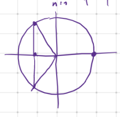
QUINDI LA SERIE IL CUI TERMINE È CONVERGE.

QUINDI $\sum|(a_n-a_{n+1})B_n|$ CONV. PER CONF.

QUINDI $\sum(a_n-a_{n+1})B_n$ CONV. PER CR. ASS. CONV.

1 $\sum_{n=1}^{+\infty}\frac{\overbrace{2\cos\left(\frac{2n\pi}{3}\right)}^{=b_n}}{n}=\sum_{n=1}^{+\infty}b_n\cdot a_n$ $\boxed{a_n=\frac{1}{n}}$

$b_n=2\cos\left(\frac{2n\pi}{3}\right)$



$\boxed{B_n}=b_1+\dots+b_n$

$2\cos(0)=2$

$2\cos\left(\frac{2}{3}\pi\right)=-1$

$2\cos\left(\frac{4}{3}\pi\right)=-1$

$B_1=-1$

$B_2=-1-1=-2$

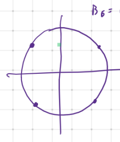
$B_3=-1-1+2=0$

$B_4=-1$

$B_5=-2$

$B_6=0$

1 bis $\sum_{n=1}^{+\infty}\frac{\overbrace{\sqrt{2}\cdot\sin\left(\frac{\pi}{4}+n\frac{\pi}{2}\right)}^{b_n}}{n}$

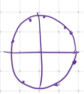


$n=1$ $b_1=1$

$n=2$ $b_2=-1$

$n=3$ $b_3=-1$

$n=4$ $b_4=1$



2 $\sum_{n=1}^{+\infty}\frac{\cos n}{n}=\sum_{n=1}^{+\infty}a_n\cdot b_n$, $a_n=\frac{1}{n}$, $b_n=\cos n$

$|\underbrace{\cos 0+\cos 1+\cos 2+\cos 3+\dots+\cos n}_{z^0,\ z^1,\ z^2,\ \dots,\ z^n}|\le 3$ $\forall n\in\mathbb{N}$

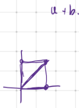
$1+z+z^2+\dots+z^n$

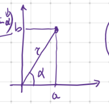
$(a+bi)+(c+di)=$

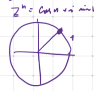
$z=\cos 1+i\sin 1$

$z^n=\cos n+i\sin n$

$a+bi=r(\cos\alpha+i\sin\alpha)$







$a=r\cos\alpha$

$b=r\sin\alpha$

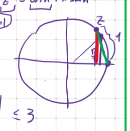
$z=\cos 1+i\sin 1$

$z^n=\cos n+i\sin n$

$|\cos 0+\cos 1+\cos 2+\dots+\cos n|=$

$\boxed{\frac{\sqrt{2}}{2}\le\sin 1\le|z-1|}$

$\boxed{\frac{1}{|z-1|}\le\frac{2}{\sqrt{2}}}$



$=|\Re(1)+\Re(z)+\Re(z^2)+\dots+\Re(z^n)|=$

$=|\Re(1+z+z^2+\dots+z^n)|=\left|\Re\left(\frac{1-z^{n+1}}{1-z}\right)\right|\le 3$

$\left|\frac{1-z^{n+1}}{1-z}\right|=|1-z^{n+1}|\cdot\left|\frac{1}{1-z}\right|\le(|1|+|-z^{n+1}|)\cdot\left|\frac{1}{1-z}\right|\le 2\cdot\left|\frac{1}{1-z}\right|=\frac{2}{|1-z|}\le 2\sqrt{2}\le 3$

$\boxed{OSS}$ $\sum\frac{\sin n}{n}$

$\sin 0+\sin 1+\dots+\sin n$

$\sin k=\Im(z^k)$

$z=\cos 1+i\sin 1$

$\sum\frac{\cos(3n)}{n}$ $\sum\frac{\cos(\alpha n)}{n}$ $\cos\left(\overbrace{\frac{7}{34}\pi}^{\alpha}\cdot n\right)$

$|\cos(3\cdot 0)+\cos(3\cdot 1)+\cos(3\cdot 2)+\dots+\cos(3\cdot n)|=$ $(z^\alpha)^n$

$=|\Re(1)+\Re(z^3)+\Re(z^6)+\dots+\Re(z^{3n})|=$ $\sum\frac{\cos(7n)}{n}$

$=|\Re(1+z^3+z^6+\dots+z^{3n})|=$ $\frac{1}{|z^\alpha-1|}$

$=\left|\Re\left(\frac{1-z^{3n+3}}{1-z^3}\right)\right|$

$\left|\frac{1-z^{3n+3}}{1-z^3}\right|=|1-z^{3n+3}|\cdot\frac{1}{|1-z^3|}\le 2\cdot\frac{1}{\underbrace{|1-z^3|}_{\lambda>0}}=\frac{2}{\lambda}=M$

2 $\sum_{n=1}^{+\infty}\frac{\cos^3 n}{n}=$ $\sum_{n=1}^{+\infty}\frac{\cos^2 n}{n}$

$\boxed{\cos^3 n}=\cos n\cdot(1-\sin^2 n)=$

$=\boxed{\cos n-\cos n\sin^2 n}$

$\boxed{\sin(\alpha n)}$ $\cos(\alpha n)$

$\sum_{n=1}^{+\infty}\left(\frac{\cos n}{n}-\frac{\cos n\cdot\sin^2 n}{n}\right)$

$\boxed{\cos n\cdot\sin^2 n}=$

$=\frac{1}{2}(2\cos n\cdot\sin n)\cdot\sin n=$

$=\frac{1}{2}\sin 2n\cdot\sin n=$

$=\boxed{\frac{1}{2}\cdot\frac{1}{2}(\cos n-\cos 3n)}$

$\boxed{\sum_{n=1}^{+\infty}\frac{\cos n\cdot\sin^2 n}{n}}$

$\sin\alpha\cdot\sin\beta=\dots$

$\to\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\to\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}=\boxed{\sin\alpha\sin\beta}$

$\frac{1}{4}\sum_{n=1}^{+\infty}\left(\frac{\cos n}{n}-\frac{\cos(3n)}{n}\right)$

3 $\sum_{n=1}^{+\infty}\frac{\cos^2 n}{n}=\sum_{n=1}^{+\infty}(1+\cos 2n)\cdot\frac{1}{2n}=\sum_{n=1}^{+\infty}\left(\frac{1}{2n}+\frac{\cos 2n}{2n}\right)$

$\sum\frac{1}{2n}=+\infty$ $\boxed{\cos 2n}=\cos^2 n-\sin^2 n=\boxed{2\cos^2 n-1}$

$\cos^2 n=\boxed{\frac{1+\cos 2n}{2}}$