1) L.31

$\sum_{n=1}^{+\infty} (-1)^n \ln\left(1+\frac{1}{n}\right)$ — CONV. PER LEIBNIZ

$a_n = \ln\left(1+\frac{1}{n}\right)$

$a_n \searrow 0$? (sì)

$\frac{1}{n}$ decresce

$\ln$ è crescente

$\sum (-1)^n \ln\left(1+\frac{1}{n}\right)$ NO

$\sum (-1) \frac{1}{\ln\left(1+\frac{1}{n}\right)}$

2) L.31

$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n + \sin\left(\frac{\pi n}{100}\right)}$ $a_n > 0$

$a_n \to 0$

$a_n = \frac{1}{n+\sin n}$ $\searrow 0$ DEF.

$f(x) = x + \sin x$ CRESC.

$f'(x) = 1 + \cos x \geq 0$

$a_n \not\to 0$

$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n+\frac{1}{2}\sin n}$ $f(x) = x + \frac{1}{2}\sin x$

Analisi Matematica 1- Lista n. 30

Studio del carattere di Serie a Termini Positivi

www.problemisvolti.it

Titolo nota

Studiare il carattere delle seguenti serie:

1) $\sum_{n=1}^{+\infty} \frac{1}{n^3+1}$

2) $\sum_{n=0}^{+\infty} \frac{1}{2^n+3^n}$

3) $\sum_{n=1}^{+\infty} \frac{n^2}{n^3+n^2}$

4) $\sum_{n=1}^{+\infty} \ln\left(1+\frac{1}{n}\right)$

5) $\sum_{n=0}^{+\infty} e^{\frac{1}{n!}}$

6) $\sum_{n=1}^{+\infty} \frac{1}{n^n}$

7) $\sum_{n=0}^{+\infty} \frac{e^{\sin n}}{1+n^2}$

8) $\sum_{n=1}^{+\infty} \frac{1}{n!n}$

9) $\sum_{n=1}^{+\infty} \frac{e^{\cos n}}{\sqrt{n}}$

10) $\sum_{n=1}^{+\infty} \ln\left(1+\frac{1}{2n}\right)^{\frac{n}{2}}$

11) $\sum_{n=1}^{+\infty} \frac{1}{(1+n)\cdot\ln(1+n^2)}$

12) $\sum_{n=1}^{+\infty} \frac{1}{n\cdot\ln^2(1+\sqrt{n})}$

13) $\sum_{n=0}^{+\infty} \frac{(n!)^2}{(2n)!}$

14) $\sum_{n=1}^{+\infty} \frac{(3n)!}{n^{3n}}$

15) $\sum_{n=1}^{+\infty} \frac{(n+1)^{n-1}}{n^{n+1}}$

16) $\sum_{n=3}^{+\infty} \left(\frac{\ln(\ln n)}{\ln n}\right)^{\ln n}$

17) $\sum_{n=2}^{+\infty} \frac{1}{\ln(n!)}$

18) $\sum_{n=2}^{+\infty} \frac{1}{(\ln n)^{\ln n}}$

19) $\sum_{n=3}^{+\infty} \frac{1}{(\ln n)\ln(\ln n)}$

20) $\sum_{n=3}^{+\infty} \frac{1}{(\ln n)\ln(\ln n)}$

21) $\sum_{n=3}^{+\infty} n^{1-\frac{1}{2\ln n}}$

22) $\sum_{n=1}^{+\infty} n^{\frac{1}{\ln n}}$

23) $\sum_{n=1}^{+\infty} \frac{\cos(\ln n)}{n\ln n}$

24) $\sum_{n=1}^{+\infty} \left(\sin(\sin(n))\right)^n$

25) $\sum_{n=1}^{+\infty} \left(\arctan\frac{1}{\sqrt{n}} - \sin\frac{1}{\sqrt{n}}\right)$

26) $\sum_{n=1}^{+\infty} \left(\sqrt{1-\frac{1}{n}} - \cos\frac{1}{\sqrt{n}}\right)$

27) $\sum_{n=1}^{+\infty} \frac{|n!|}{(\sqrt{n})^n}$

28) $\sum_{n=1}^{+\infty} \frac{1}{\sqrt[n]{(n!)^3}}$

29) $\sum_{n=1}^{+\infty} \frac{(n!)!}{n^{n!}}$

30) $\sum_{n=1}^{+\infty} \frac{(n!)^{n!}}{(n^n)!}$

31) $\sum_{n=1}^{+\infty} \frac{4+\cos n}{n}$

32) $\sum_{n=1}^{+\infty} \left(e - \sum_{k=0}^{n} \frac{1}{k!}\right)$

33) $\sum_{n=1}^{+\infty} \left(\sum_{k=n}^{2n} \frac{1}{k^2}\right)$

34) $\sum_{n=1}^{+\infty} \left(\sum_{k=1}^{n^2} \frac{1}{k^2}\right)$

Studiare al variare del parametro reale $a>0$, il carattere delle seguenti serie:

35) $\sum_{n=1}^{+\infty} \left(1-\cos\frac{1}{n}\right)^a$

36) $\sum_{n=1}^{+\infty} \left(\frac{\pi}{2} - \arctan(n^a)\right)$

37) $\sum_{n=1}^{+\infty} \left(\ln\left(\cos\frac{1}{\sqrt{n}}\right) + \frac{1}{2n} + \frac{a}{n}\right)$

38) $\sum_{n=1}^{+\infty} \left(\cos\left(\sin\frac{1}{\sqrt{n}}\right) - 1 + \frac{1}{2n} - \frac{a}{n}\right)$

39) $\sum_{n=1}^{+\infty} \frac{1}{a^{\ln n}}$

40) $\sum_{n=2}^{+\infty} \left(\frac{\log_2 n}{\log_3 n^2}\right)^a$

41) $\sum_{n=1}^{+\infty} \left(1-\frac{1}{n^2}\right)^{n^a}$

42) $\sum_{n=1}^{+\infty} \frac{n!}{n^n} a^n$

43) $\sum_{n=3}^{+\infty} \frac{1}{n\cdot\ln n\cdot(\ln(\ln n))^a}$

44) $\sum_{n=1}^{+\infty} \left(e - \left(1+\frac{1}{n}\right)^n\right)^a$

45) $\sum_{n=1}^{+\infty} \left(e - \sum_{k=0}^{n} \frac{1}{k!}\right)^a$

46) $\sum_{n=1}^{+\infty} \frac{1}{\sqrt[n]{(n!)^n}}$

47) $\sum_{n=1}^{+\infty} \frac{1}{n^2}\left(\frac{n+2}{n+1}\right)^{n^2} a^n \ln n$

48) $\sum_{n=1}^{+\infty} \left(\frac{n^2+n-1}{n^2+3n+5}\right)^{n^2} a^n$

49) $\sum_{n=1}^{+\infty} \frac{n!\cdot n^{n+1}}{(2n)!} a^n$

50) $\sum_{n=1}^{+\infty} \frac{n^{2n}}{(2n)!} a^n$

Analisi Matematica 1- Lista n. 31

Studio del carattere di Serie a Termini di segno qualsiasi

www.problemisvolti.it

Titolo nota

Studiare convergenza semplice e assoluta delle seguenti serie:

1) $\sum_{n=1}^{+\infty} (-1)^n \tan\frac{1}{n}$

2) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n+\sin n}$

3) $\sum_{n=2}^{+\infty} (-1)^n \left(1+\frac{1}{n}\right)^n \ln n$?

4) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+\sin n}$

5) $\sum_{n=1}^{+\infty} (-1)^n \frac{n^3+6n+3}{n^3+2n^2+7n}$

6) $\sum_{n=1}^{+\infty} \frac{\cos\left(\frac{\pi}{n}\right)}{n}$

7) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{\ln n + e^{-n}}$

8) $\sin\left(\pi n + \frac{\pi}{n}\right) = \sin\left(\pi n + \frac{\pi}{2n}\right) = \sin\left(\pi n + \frac{\pi}{2}\right) \cdot \cos\frac{\pi}{2n} = (-1)^n \cdot \frac{\pi}{2n}$

9) $\sum_{n=2}^{+\infty} \frac{(-1)^n n}{n\ln n + 1}$

10) $\sum_{n=1}^{+\infty} \left(e^{\frac{1}{n^2}} - 1\right)$

11) $\sum_{n=1}^{+\infty} \ln\left(1+\frac{(-1)^n n}{\sqrt{n}}\right)$

12) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+e^{(-1)^n}n}$

13) $\sin\left(\pi n^2 + \frac{1}{n}\right)$ $\left(-1^n \cdot \frac{\pi}{n}\right)$

14) $\sin\left(\pi n + \frac{1}{n}\right)$ DIV.

15) $\sum_{n=1}^{+\infty} \frac{\sqrt{2}\cdot\sin\left((2n+1)\frac{\pi}{4}\right)}{n^{2+(-1)^n}}$

16) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{\sqrt{n}+\sin n}$

17) $\sum_{n=2}^{+\infty} \ln\left(\frac{n^2+2(-1)^n}{n^2+1}\right)$

18) $\sum_{n=2}^{+\infty} \ln\left(\frac{n+2(-1)^n}{n+1}\right)$

19) $\sum_{n=1}^{+\infty} (-1)^n \left(e^{\frac{1}{\sqrt{n}}} - \cos\frac{2}{\sqrt{n}}\right)$

20) $\sum_{n=1}^{+\infty} \left(\sin\frac{(-1)^n}{\sqrt{n}} - \frac{(-1)^n}{\sqrt{n}} + \cos\frac{(-1)^n}{\sqrt{n}}\right)$

21) $\sum_{n=1}^{+\infty} (-1)^n \left(2e^{\frac{1}{n^2}} - 2\cos\frac{1}{n} + \ln\left(1+\frac{1}{n^2}\right)\right)$

22) $\sum_{n=1}^{+\infty} \left(2e^{\frac{1}{n^2}} - 2\cos\frac{(-1)^n}{\sqrt{n}} + \ln\left(n-2\frac{(-1)^n}{\sqrt{n}}\right)\right)$

23) $\sum_{n=1}^{+\infty} \frac{\sin(\pi\cdot\log_2 n^n)}{n}$

24) $\sum_{n=1}^{+\infty} \frac{\sin n}{n}$

Studiare, al variare di $a>0$, convergenza semplice e assoluta delle seguenti serie:

29) $\sum_{n=1}^{+\infty} (-1)^n \left(\frac{1}{n} - \arctan\frac{1}{n}\right)^a$

26) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n^2+1}\left(1+\frac{1}{n}\right)^n$

29) $\sum_{n=2}^{+\infty} \frac{(-1)^n}{n(\ln n)^a}\left(1+\frac{1}{n}\right)^n$

28) $\sum_{n=2}^{+\infty} (-1)^n \frac{(n+1)^a}{n^a\cdot\ln n}$

29) $\sum_{n=1}^{+\infty} (-1)^n n^{an} \sin\frac{1}{(n+100)!}$

30) $\sum_{n=1}^{+\infty} \frac{1}{1+\left(1+\frac{(-1)^n}{n^a}\right)^{n^2 a}}$

31) $\sum_{n=2}^{+\infty} \frac{(-1)^n}{n+a\sin n}$

32) $\sum_{n=1}^{+\infty} \left(\sqrt{1+\frac{(-1)^n}{n^a}} - 1\right)$

33) $\sum_{n=2}^{+\infty} \frac{(-1)^n}{n^a + \cos(n\pi)}$

34) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n^a + \cos n}$

Data $f(x) = 2\arctan x + \ln(1-2x) + 2x^2$, studiare, al variare di $a>0$, convergenza semplice e assoluta delle serie:

35) $\sum_{n=1}^{+\infty} (-1)^n f\left(\frac{1}{n^a}\right)$

36) $\sum_{n=1}^{+\infty} f\left(\frac{(-1)^n}{n^a}\right)$

Data $f(x) = \sin(3x) + 2\ln(1-x) + x^2$, studiare, al variare di $a>0$, convergenza semplice e assoluta delle serie:

35) $\sum_{n=1}^{+\infty} (-1)^n f\left(\frac{1}{n^a}\right)$

36) $\sum_{n=1}^{+\infty} f\left(\frac{(-1)^n}{n^a}\right)$

Per ogni $n \in \mathbb{N}-\{0\}$ definiamo $a_n = \begin{cases} \frac{2}{\sqrt{2n}} & \text{se } n \text{ è un quadrato perfetto} \\ -\frac{1}{\sqrt{2n}} & \text{altrimenti.} \end{cases}$

Studiare convergenza semplice e assoluta di:

38) $\sum_{n=1}^{+\infty} a_n$

38) $\sum_{n=1}^{+\infty} (a_n)^3$

39) $\sum_{n=1}^{+\infty} (a_n)^3$

40) $\sum_{n=1}^{+\infty} \arctan(a_n)$

$a=1$

$\sum_{n=1}^{+\infty} \frac{1}{n+\sin(an)}$ b_n

$a_n = \frac{1}{b_n} \searrow 0$ (??)

b_n CRESCENTE ?? $f(x) = x + \left(1+\frac{1}{100}\right)\sin x$

$b_{n+1} \geq b_n$?

$n+1+\left(1+\frac{1}{100}\right)\sin((n+1)\cdot 1) \geq n + \left(1+\frac{1}{100}\right)\sin n$??

$\left(1+\frac{1}{100}\right)\left(\sin(n+1) - \sin n\right) \geq -1$

$\sin(n+1) - \sin n \geq -\frac{100}{101}$

$\sin(n+1) - \sin n \geq -2\sin\frac{1}{2} > -\frac{100}{101}$

$\sin\frac{1}{2} < \frac{\sqrt{2}}{2} < \frac{100}{101}$

$\sum_{n=1}^{+\infty} (-1)^n \frac{1}{n+\sin(an)}$

$\sum_{n=1}^{+\infty} (-1)^n \left(\frac{1}{n+\sin(an)} - \frac{1}{n}\right)$

$\sum_{n=1}^{+\infty} (-1)^n \frac{-\sin(an)}{n^2+n\sin(an)}$

$\left[\sum_{n=1}^{+\infty} (-1)^n \frac{-\sin(an)}{n^2+n\sin(an)}\right]$

$\sum_{n=2}^{+\infty} \frac{\frac{1}{2}\sin(an)}{(n^2+n\sin(an))}$ $\geq \ln\left(n+\sin(an)\right)$

$0 \leq \theta \leq \frac{1}{n^2+n\sin(an)} \leq \frac{1}{n^2}$

Es.3. Lista 3?

$\sum_{n=1}^{+\infty} (-1)^n \left(1-\frac{1}{n}\right)^{n\ln n}$ (*)

BELLA COPIA

PROPONGO 2 MODI PER RISOLVERLO

I° MODO

SICCOME, GRAZIE AL CR. DI LEIBNIZ LA SERIE $\sum_{n=2}^{+\infty} \frac{(-1)^n}{\ln n}$ CONVERGE,

IL CARATTERE DI (*) E' UGUALE A QUELLO DI:

$(\star) \quad \sum_{n=2}^{+\infty} \left((-1)^n \left(1-\frac{1}{n}\right)^{n\ln n} - \frac{(-1)^n}{n} \right)$

CIOE' DI:

$\sum_{n=2}^{+\infty} (-1)^n \left(\left(1-\frac{1}{n}\right)^{n\ln n} - \frac{1}{n} \right)$

MA SI OSSERVI CHE

$\left(1-\frac{1}{n}\right)^{n\ln n} = e^{n\ln n \cdot \ln\left(1-\frac{1}{n}\right)} =$

$= e^{n\ln n \cdot \left(-\frac{1}{n} - \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right)\right)} =$

$= e^{-\ln n - \frac{1}{2n} + o\left(\frac{1}{n^2}\right)} =$

$= e^{-\ln n} \cdot e^{-\frac{1}{2n} + o\left(\frac{1}{n^2}\right)} =$

$= \frac{1}{n} \cdot e^{-\frac{1}{2n} + o\left(\frac{1}{n^2}\right)} =$

$= \frac{1}{n}\left(1 - \frac{1}{2n} + o\left(\frac{1}{n^2}\right)\right)$

QUINDI

$\left(1-\frac{1}{n}\right)^{n\ln n} - \frac{1}{n} = -\frac{\ln n}{2n^2} + o\left(\frac{\ln n}{n^2}\right) \approx -\frac{\ln n}{2n^2}$

MA ALLORA

$\left| (-1)^n \left(\left(1-\frac{1}{n}\right)^{n\ln n} - \frac{1}{n} \right) \right| \approx \frac{\ln n}{2n^2}$

QUINDI DAL FATTO CHE $\sum \frac{\ln n}{2n^2}$ CONVERGE,

SEGUE, GRAZIE AL CR. DEL CONF. ASINTOTICO, CHE

CONVERGE ANCHE:

$\sum_{n=2}^{+\infty} \left| (-1)^n \left(\left(1-\frac{1}{n}\right)^{n\ln n} - \frac{1}{n} \right) \right|$

DI CONSEGUENZA, GRAZIE AL CR. DELLA ASSOLUTA CONVERGENZA CONVERGE ANCHE (*), CHE HA LO STESSO CARATTERE DELLA NOSTRA SERIE (*), CHE QUINDI CONVERGE.

II° MODO

SE SI VUOLE USARE IL CR. DI LEIBNIZ BISOGNA MOSTRARE CHE

$\left(1-\frac{1}{n}\right)^{n\ln n} \to 0$ DECRESCENDO

CHE TENDA A ZERO E' OVVIO PERCHE'

$\left(1-\frac{1}{n}\right)^{n\ln n} = \left(\left(1-\frac{1}{n}\right)^n\right)^{\ln n} \to \left(\frac{1}{e}\right)^{+\infty} = 0$

PER MOSTRARE MONOTONIA SI OSSERVI CHE:

$\left(1-\frac{1}{n}\right)^{n\ln n} = e^{n\ln n \cdot \ln\left(1-\frac{1}{n}\right)} = e^{-\left[n\cdot\ln n\cdot\left(\ln n - \ln(n-1)\right)\right]}$

QUINDI BASTA MOSTRARE CHE

(**) $n\cdot\ln n\cdot(\ln n - \ln(n-1))$ E' DEFINITIVAMENTE CRESCENTE.

CIOE' CHE POSTA

$f(x) = x\cdot\ln x\cdot(\ln x - \ln(x-1))$

SI OTTIENE $f'(x) > 0$ SE x E' SUFFICIENTEMENTE GRANDE. INFATTI:

$f'(x) = \left(x\cdot\ln x\cdot(\ln x - \ln(x-1))\right)' =$

$= \ln x\cdot\left(\ln x - \ln(x-1)\right) + x\cdot\frac{1}{x}\left(\ln x - \ln(x-1)\right) + x\ln x\cdot\left(\frac{1}{x} - \frac{1}{x-1}\right) =$

$= -\ln x\cdot\ln\left(1-\frac{1}{x}\right) - \ln\left(1-\frac{1}{x}\right) + \frac{1}{x}\ln x\cdot\frac{-1}{x(x-1)} =$

$= \ln x\cdot\left(-\ln\left(1-\frac{1}{x}\right) - \frac{1}{x-1}\right) - \ln\left(1-\frac{1}{x}\right) =$

$= \ln x\cdot\left(\frac{1}{x} + o\left(\frac{1}{x}\right) - \frac{1}{x-1}\right) - \ln\left(1-\frac{1}{x}\right) =$

$= \ln x\cdot\left(\frac{1}{x} + o\left(\frac{1}{x}\right) - \frac{1}{x-1}\right) + \frac{1}{x} + o\left(\frac{1}{x}\right) =$

$= \ln x\cdot o\left(\frac{1}{x}\right) - \left(-\frac{1}{x} + o\left(\frac{1}{x}\right)\right) = \frac{1}{x} + o\left(\frac{1}{x}\right) = \frac{1}{x}\left(1 + o(1)\right) > 0$

$\frac{1}{x} - \frac{1}{x-1} = \frac{-1}{x^2-x} \sim -\frac{1}{x^2} = o\left(\frac{1}{x}\right)$

DEFINITIVAMENTE PER $x \to +\infty$

QUINDI VALE (**) E PERCIO' SI PUO' APPLICARE IL CRIT. DI CONFRONTO

5) $\sum (-1)^n \frac{n^3+6n+8}{n^3+2n^2+7n}$ $\frac{x^3+6x+8}{x^3+2x^2+7x} \leftarrow f(x)$ $f'(x) = \frac{D(x)}{Q(x)}$

$\frac{1}{n}\left(\frac{n^3+6n+8}{n^2+2n+9}\right)$

$\frac{1}{n}\left(\frac{n^3+2n^2+7n}{n^2+2n+9}\right)$

$\frac{n^2+2n+7}{n^2+2n+9}$

$\frac{2n+1}{n^3+2n^2+7n}$

$\frac{1}{n}\left(1+\frac{2n+2}{n^3+2n^2+7n}\right)$

$\frac{1}{n}\left(1+\frac{2n+1}{n^2+2n+9}\right)$

$\frac{1}{n}$ DIC

$\frac{1}{n}\left(1+\frac{2n+1}{n^2+2n+9}\right) = \frac{1}{n} + \frac{2n+1}{n^3+2n^2+7n} =$

$\frac{2n+9-3}{n^3+2n^2+7n} =$

$\frac{2n+4}{n^3+2n^2+7n} - \frac{3}{n^3+2n^2+7n}$

$\frac{1}{n}\left(1+\left(\frac{n+2}{n^3+2n^2+7n} - \frac{3}{n^3+2n^2+7n}\right)\right) =$

$\frac{1}{n} + \frac{n+2}{n^4+2n^3+7n^2} - \frac{3}{n^4+2n^3+7n^2}$

$\frac{1}{n} + \frac{(n+2)^2}{(n^2+n+7)n^2} - \frac{3}{(n^2+n+7)n^2}$

$\sum_{n=2}^{+\infty} \frac{(-1)^n}{\ln n + e^{-n}}$ $\sum (-1)^n a_n$

$\ln n + e^{-n} \to 0$

$f(x) = \ln x + e^{-x}$

$f'(x) = \frac{1}{x} - e^{-x} = \frac{1}{x}\left(1 - \frac{x}{e^x}\right) > 0$ DEF. IN 0

$\sum_{n=1}^{+\infty} \frac{\cos\left(2n\cdot\frac{\pi}{2}\right)}{n}$

$\sum_{k=1}^{+\infty} \frac{\cos\left(2k\cdot\frac{\pi}{2}\right)}{2k} = \sum_{k=1}^{+\infty} \frac{(-1)^k}{2k} = \frac{1}{2}\sum_{k=1}^{+\infty} \frac{(-1)^k}{k} =$ CONV.

$n = 2k$

$\cos\left(2n\cdot\frac{\pi}{2}\right) = \cos(k\pi)$

$n = 2k+1$

$\cos\left((2k+1)\frac{\pi}{2}\right) = \cos\left(k\pi + \frac{\pi}{2}\right) = 0$

$\sum_{n=2}^{+\infty} \frac{(-1)^n/n}{(n\ln n + 1)^{a_n}}$

$\theta_n = \frac{1}{(2n+\frac{\pi}{2})^{a_n}} \searrow 0$

$f(x) = \ln x + \frac{1}{x}$

$f'(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{1}{x}\left(1-\frac{1}{x}\right) > 0$

$\sum_{n=2}^{+\infty} \ln\left(\frac{n^2+2(-1)^n}{n^2+1}\right) =$

$\sum_{n=2}^{+\infty} \ln\left(1+\frac{-1+2(-1)^n}{n^2+1}\right)$ CONV. PER CONV. ASS.

CONV. PER IL CONFRONTO

$\left|\ln\left(1+\frac{-1+2(-1)^n}{n^2+1}\right)\right| \asymp \frac{|-1+2(-1)^n|}{n^2+1}$ ② CONV. PER CONF.

$0 \leq \theta \leq \frac{3}{n^2-1} \leq \frac{3}{n^2}$

$\sum_{n=2}^{+\infty} \ln\left(\frac{n+2(-1)^n}{n+1}\right) =$

$\sum_{n=2}^{+\infty} \ln\left(1+\frac{-1+2(-1)^n}{n+1}\right)$

$\sum_{k=2}^{+\infty} \ln\left(\frac{2k+2}{2k+1}\right) + \ln\left(\frac{2k-1}{2k+1}\right) =$

$= \sum_{k=2}^{+\infty} \ln\left(\frac{2k+2}{2k-1}\cdot\frac{2k-1}{2k+1}\right) =$

$\frac{2k+2}{2k-1} = 1 + \frac{3}{2k-1}$

$\frac{2k-1}{2k+1} = 1 - \frac{2}{2k+1}$

$\sum_{k=2}^{+\infty} \frac{1}{2k+1} \sim \sum \frac{1}{k}$

$\frac{2k+1-2}{2k+1} = 1 - \frac{2}{2k+1}$