AM2 - EXE (17-05-2023)

DIFFERENZIABILITÀ

STUDIARE PROLUNGABILITÀ A TUTTO $\mathbb{R}^2$ E EVENTUALE DIFFERENZIABILITÀ DI:

$\boxed{1}$ $f(x,y) = \dfrac{x \cdot |y|^\alpha}{x^4 + y^4}$ $\quad \alpha \geq 0$

$\boxed{2}$ $f(x,y) = \dfrac{x^3 \cdot |y|^\alpha}{x^4 + y^{20}}$ $\quad \alpha \geq 0$ $\qquad \left(f(x,y) = \dfrac{|x|^\alpha \cdot y^5}{x^4 + y^{20}} \quad \alpha \geq 0 \right)$

$\boxed{3}$ $f(x,y) = \dfrac{x^{32}\, y^5}{x^{48} + y^{12}}$

$\boxed{4}$ $f(x,y) = \dfrac{x^{33}\, y^4}{x^{48} + y^{12}}$

$\boxed{5}$ $f(x,y) = \dfrac{\sin x - \sin y}{x - y}$

$\boxed{6}$ $f(x,y) = (x+y)^2 \sin \dfrac{1}{x+y}$

CALCOLARE LE DERIVATE PARZIALI DI

$\boxed{7}$ $f(x,y) = (x\, y)^{100}$

$\boxed{8}$ $f(x,y) = x^y$

$\boxed{9}$ $f(x,y) = \displaystyle\int_x^y e^{t^2}\, dt$

QUESITI: $\boxed{10}$ FUNZ. SEPARATAMENTE CONTINUE [...]

$\boxed{11}$ $C^1 \Leftrightarrow$ LIP. SU APERTI CONNESSI

$$f(x,y) = \frac{x \cdot |y|^{\alpha}}{x^4 + y^4}$$

$\boxed{\alpha \geq 0}$

$\boxed{\alpha > 3}$ $\quad \delta > 0$

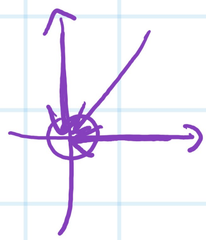
$$\lim_{(x,y)\to(0,0)} \frac{x^1 \cdot |y|^{3+\delta}}{x^4+y^4} =$$

$$\left[\boxed{\frac{x \cdot |y|^3}{x^4+y^4}} \cdot \underbrace{|y|^{\delta}}_{\downarrow\, 0} \right] \to 0$$

Lim.

$\boxed{\alpha > 3}$

$$f(x,y) = \begin{cases} 0 & (x,y)=(0,0) \\ \boxed{\dfrac{x \cdot |y|^{\alpha}}{x^4+y^4}} & \text{ALTR.} \end{cases}$$



$\boxed{\alpha \leq 3}$

$$\lim_{(x,y)\to(0,0)} \frac{x|y|^{\alpha}}{x^4+y^4} = \lim_{x\to 0^+} \frac{x|x|^{\alpha}}{2x^4} = \begin{cases} +\infty & \alpha < 3 \\ \frac{1}{2} & \alpha = 3 \end{cases}$$

$(x,y) \in \Gamma = \{(x,x) \mid x > 0\}$

$f(x,y) \equiv 0$ su $(x,0)$ $\forall x \in \mathbb{R}$

$(y,0)$ $\forall y \in \mathbb{R}$

$$\begin{cases} f(0,0) = 0 \\ f_x(0,0) = 0 \\ f_y(0,0) = 0 \end{cases}$$

$$\lim_{t\to 0} \frac{f(t,0) - f(0,0)}{t} = \lim_{t\to 0} \frac{0-0}{t} = 0$$

$$f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$$

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y) - \boxed{\text{P.T.}}}{\sqrt{x^2+y^2}} = 0$$

lim

$$\lim_{(x,y)\to(0,0)} \boxed{\frac{x^1\cdot|y|^\alpha}{(x^4+y^4)\cdot\sqrt{x^2+y^2}}} = 0$$

$\boxed{\alpha>4}$ $\alpha = 4+\delta$ $\delta>0$

$$\boxed{\underbrace{\frac{x\cdot|y|^3}{x^4+y^4}}_{\text{LIM.}}\cdot\underbrace{\frac{|y|^1}{\sqrt{x^2+y^2}}}_{\text{LIM.}}\cdot\underbrace{|y|^\delta}_{\to 0}} \longrightarrow 0$$

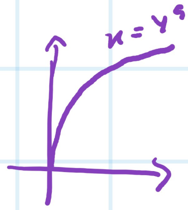
$$\Gamma = \{(x,y) \mid x=y,\ x>0\}$$

$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in\Gamma}} \boxed{} = \lim_{x\to0^+}\frac{x\cdot|x|^\alpha}{2\sqrt{2}\,x^4\cdot|x|} = \begin{cases}\frac{1}{2\sqrt{2}} & \alpha=4\\ +\infty & \alpha<4\end{cases}$$

$$f(x,y) = \frac{x^3\cdot|y|^\alpha}{x^4+y^{20}} \qquad \boxed{\alpha\geq0}$$

$$\lim_{(x,y)\to(0,0)}\frac{x^3\cdot|y|^\alpha}{x^4+|y|^{20}} = 0$$

$\Leftarrow$ $\boxed{\alpha>5}$ $\boxed{\delta>0}$

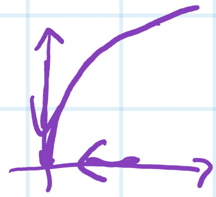


$$\underbrace{\boxed{\frac{x^3\cdot|y|^5}{x^4+(|y|^5)^4}}}_{\text{LIM.}}\cdot\underbrace{|y|^\delta}_{\to 0}$$

$$\boxed{x=|y|^5}$$

$$\left(\frac{x}{\left(x^4+\left(|y|^5\right)^4\right)^{\frac{1}{4}}}\right)^3 \cdot \frac{|y|^5}{\left(x^4+\left(|y|^5\right)^4\right)^{\frac{1}{4}}} \le 1$$

≤ 1



$$\lim_{\substack{(x,y)\to(0,0)\\ (x,y)\in\Gamma}} \frac{x^3|y|^\alpha}{x^4+y^{20}} = \qquad \left\{(x,y) \,\middle|\, x=y^5,\ \begin{matrix} x>0 \\ y>0\end{matrix}\right\} = \Gamma$$

$$= \lim_{y\to0^+} \frac{y^{15}\cdot y^\alpha}{y^{20}+y^{20}} = \lim_{y\to0^+} \frac{y^{15+\alpha}}{2y^{20}} = \begin{cases} \frac{1}{2} & \alpha=5 \\ +\infty & \alpha<5 \end{cases}$$

$$f(x,y) = \frac{x^3|y|^\alpha}{x^4+y^{20}}$$

$$\left.\begin{matrix} f(0,0) = 0 \\ f_x(0,0) = 0 \\ f_y(0,0) = 0 \end{matrix}\right\} \Rightarrow \text{piano tangente è } 0$$

$$\lim_{(x,y)\to(0,0)} \frac{\boxed{x^3}|y|^\alpha}{(x^4+y^{20})\cdot\sqrt{x^2+y^2}} = 0 \ ?$$

$\boxed{\alpha > 6}$

$$\frac{x^3(|y|^5)^1}{\underbrace{x^4+y^{20}}_{(|y|^5)^4}} \cdot \frac{|y|^1}{\sqrt{x^2+y^2}} \cdot |y|^5$$

$\boxed{\alpha > 6}$ $\alpha = 6+\delta$ $\delta > 0$

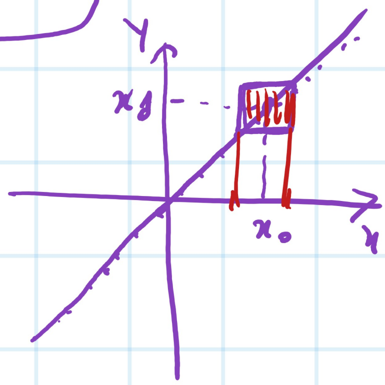
$$\underbrace{\frac{x^3\cdot|y|^5}{x^4+(|y|^5)^4}}_{LIM.} \cdot \underbrace{\frac{|y|}{\sqrt{x^2+y^2}}}_{LIM} \cdot \underbrace{|y|^\delta}_{\to 0} \to 0$$

$\sqrt{y^2}$

$$\Gamma = \{(x,y) \mid x = y^5,\ x>0,\ y>0\}$$

$$\lim_{y\to0^+} \frac{y^{15+\alpha}}{2y^{20}\cdot\sqrt{y^{10}+y^2}} = \lim_{y\to0^+} \frac{y^{15+\alpha}}{2y^{21}\cdot\sqrt{y^8+1}} = \begin{cases} \frac{1}{2} & \alpha = 6 \\ +\infty & \alpha < 6 \end{cases}$$

$$f(x,y) = \boxed{\frac{\sin y - \sin x}{y - x}}$$

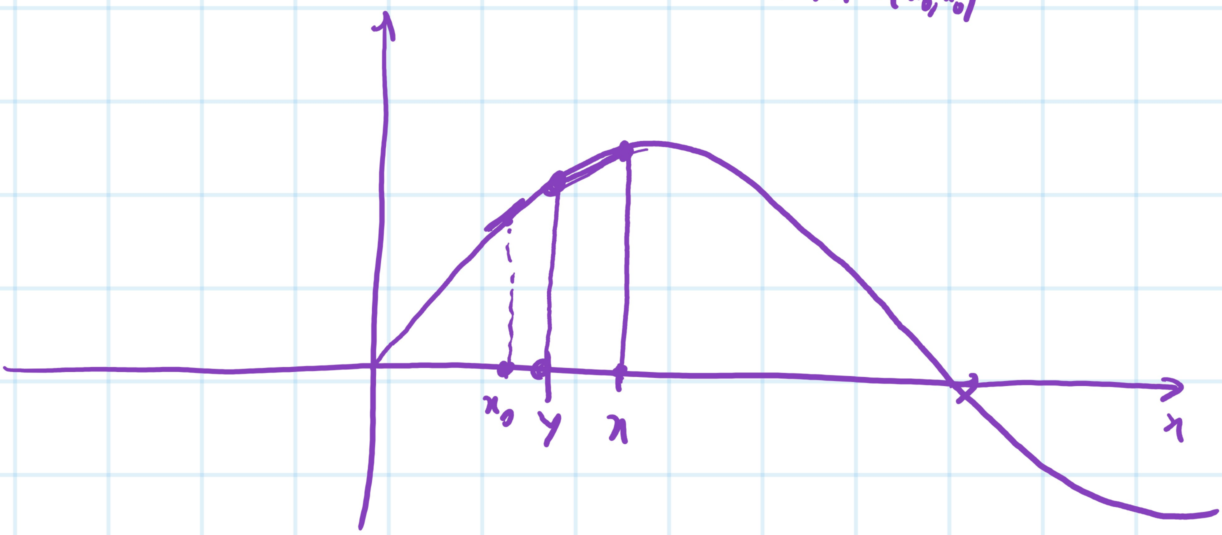


$\forall \varepsilon > 0 \quad \exists \delta > 0$ t.c.

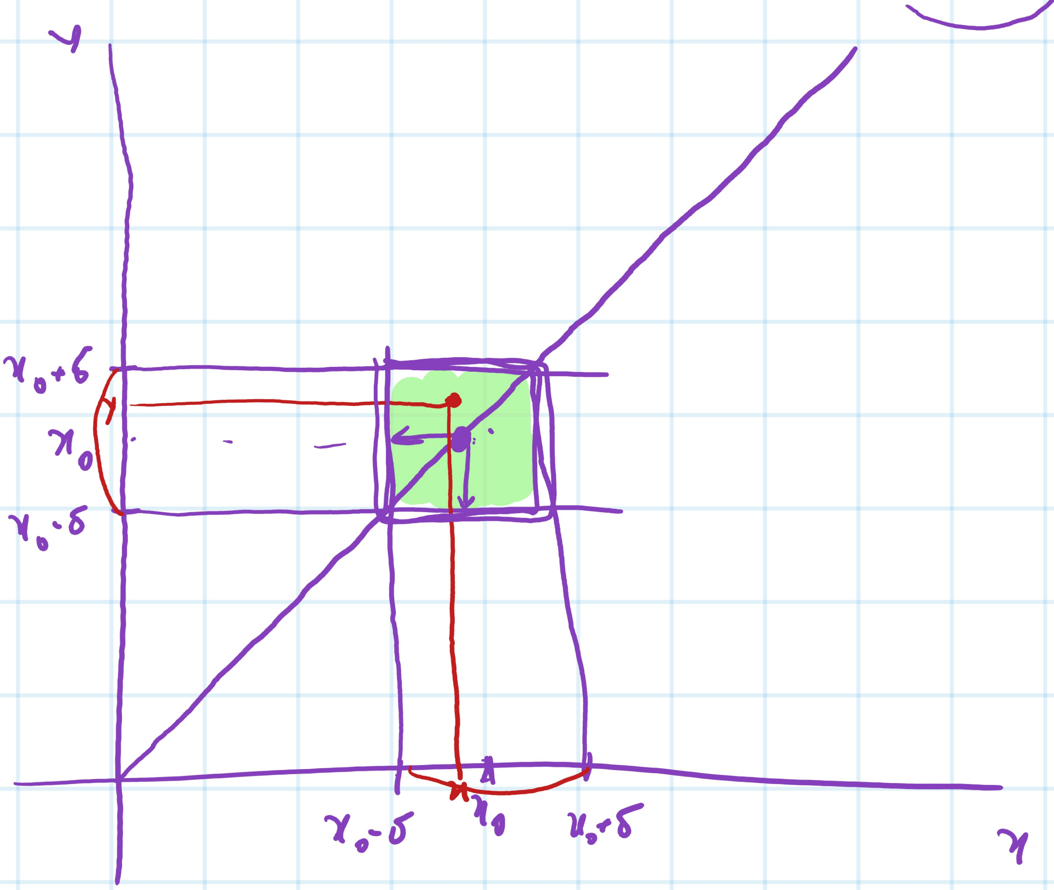
$$f(x,y) = \begin{cases} \boxed{\dfrac{\sin y - \sin x}{y - x}} & x \neq y \\ \cos x & (x,y) = (x,x) \end{cases}$$

$f(x,y) \to \cos x_0$

$(x,y) \to (x_0, x_0)$



$$\forall \varepsilon>0 \quad \exists \delta>0 \quad t.c. \quad (x,y)\in I_\delta(x_0,x_0) \Rightarrow \left|\frac{\sin y-\sin x}{y-x}-\cos x_0\right|<\varepsilon$$

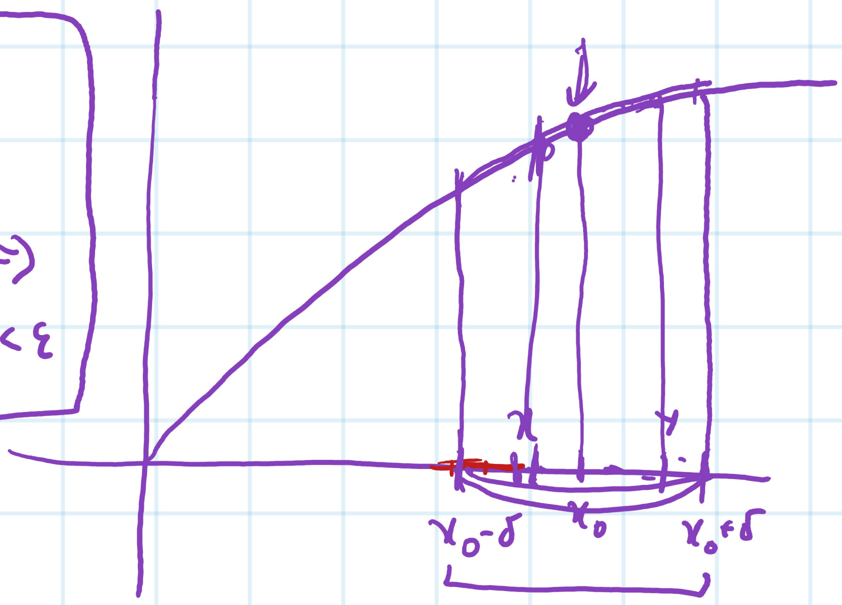


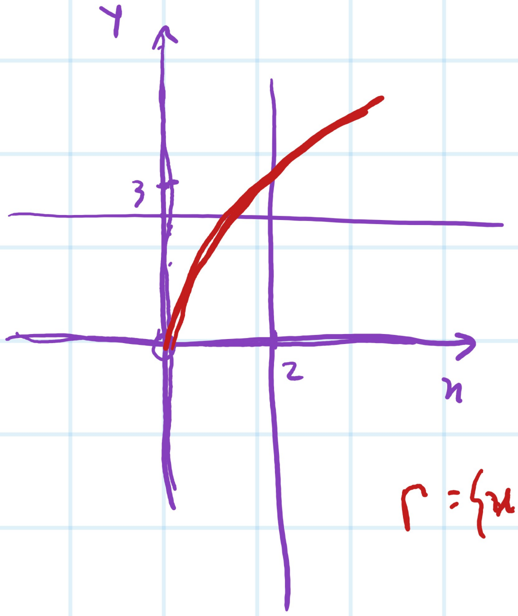
$\forall \varepsilon>0$

$\delta>0$ t.c.

$|x-x_0|<\delta \Rightarrow$

$\Rightarrow |\cos x-\cos x_0|<\varepsilon$

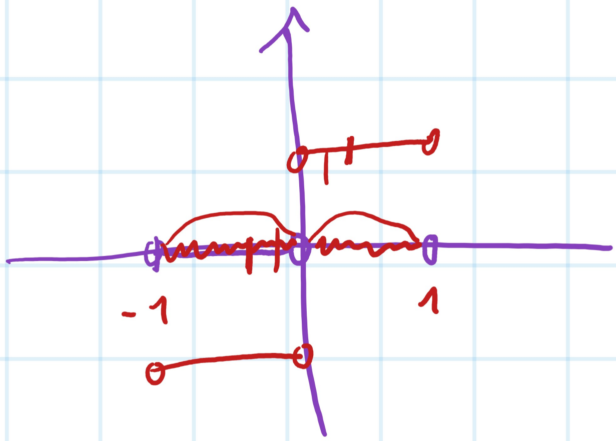




$\Gamma = \{x = y^2\}$

$\boxed{\dfrac{y^4}{2y^4}}$ $\quad \dfrac{1}{2}$

$$\begin{cases} \dfrac{x \cdot y^2}{x^2 + (y^2)^2} & (x,y) \neq (0,0) \\[2ex] 0 & (x,y) = (0,0) \end{cases}$$



$\Omega \subset \mathbb{R}^2$ $\boxed{\text{aperto connesso}}$ $f \in C^1(\Omega)$

con f_x f_y LIMITATE