14 MARZO 2023

A.M.2 - LEZ. 4 - INTEGRALE DI RIEMANN IV

1) $\boxed{\text{DERIVATA DI } \int_{g(x)}^{h(x)} f(t)\,dt}$

2) DATA $F(x)=\int_x^{x^2} \frac{e^{\cos t}}{1+t^2}\,dt$

a) CALCOLARE $\lim_{x\to+\infty} F(x)$ (SE $\ell=0$ TROVARE ORDINE)

b) DIRE SE ESISTE $\lim_{x\to-\infty} F(x)$

c) DIRE SE $F(x)$ HA PUNTI DI MAX E MIN (REL O ASS)

3) DATA $F(x)=\int_x^{x+1} \sqrt[4]{1+t^2+t^4}\,dt$

a) TROVARE EVENTUALI ASINTOTI

b) TROVARE PUNTI DI ESTREMO

c) TROVARE EVENTUALI SIMMETRIE

4) DATA $F(x)=\int_0^x \left(t-\lfloor t\rfloor+\alpha\right)dt$ TROVARE $\lim_{x\to+\infty} F(x)$ AL VARIARE DI $\alpha\in\mathbb{R}$

5) PER OGNI $x\in(0,1)$ DEFINIAMO $F(x)=\int_x^1 \left\lfloor \frac{1}{t}\right\rfloor dt$, QUANTE SONO LE SOLUZIONI DI $F(x)=100$?

6) DIRE SE LE SEGUENTI FUNZIONI SONO INFINITESIME PER $x\to 0^+$ E, IN CASO AFFERMATIVO DETERMINARNE L'ORDINE:

a) $F(x)=\int_0^{x^2} \sin(\sin t)\,dt$ b) $G(x)=\int_x^{x+x^2} \sin(\sin t)\,dt$ c) $H(x)=\int_{x^2}^{x^2+x} \sin(\sin t)$

7) SIANO $f,g\in C([0,1])$ TALI CHE $\forall \varphi\in C_0((0,1))$ SI ABBIA $\int_0^1 f(x)\varphi(x)\,dx=\int_0^1 g(x)\varphi(x)\,dx$. MOSTRARE CHE $f(x)=g(x)$ $\forall x\in[a,b]$.

$\boxed{\text{T.}}$ DATE $[a,b] \subset \mathbb{R}$ $f \in C([a,b])$ $g, h : \mathbb{R} \to [a,b]$

$\boxed{g, h \text{ DERIV.}}$

$$G(x) = \int_{g(x)}^{h(x)} f(t)\, dt$$

ALLORA G È DERIVABILE E

$$G'(x) = f(h(x)) \cdot h'(x) - f(g(x)) \cdot g'(x)$$

$\boxed{\text{DIM}}$

$$\left(G(x)\right)' = \left(\int_{g(x)}^{h(x)} f(t)\, dt\right)' = \left(\underbrace{\int_a^{h(x)} f(t)\, dt}_{} - \int_a^{g(x)} f(t)\, dt\right)' =$$

$$F(s) = \int_a^s f(t)\, dt$$

$$= \left(F(h(x)) - F(g(x))\right)' =$$

$$= F'(h(x)) \cdot h'(x) - F'(g(x)) \cdot g'(x) =$$

$$= \underline{f(h(x))\, h'(x) - f(g(x))\, g'(x)}$$

2 $F(x)=\displaystyle\int_x^{x^2} \frac{e^{\cos t}}{1+t^2}\,dt$

$\displaystyle\lim_{x\to+\infty} F(x) = 0$

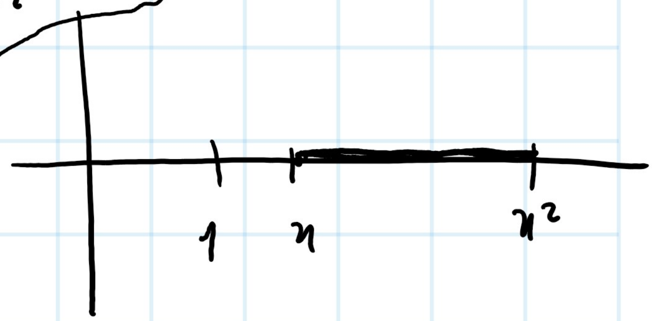
$\displaystyle\lim_{x\to-\infty} F(x)$ ∃ finito?

DIRE SE ∃$^{\text{no}}$ ESTR. rel. ass.

$$0 < \boxed{\int_x^{x^2} \frac{e^{\cos t}}{1+t^2}\,dt} \le \boxed{\int_x^{x^2} \frac{e}{1+t^2}\,dt}$$

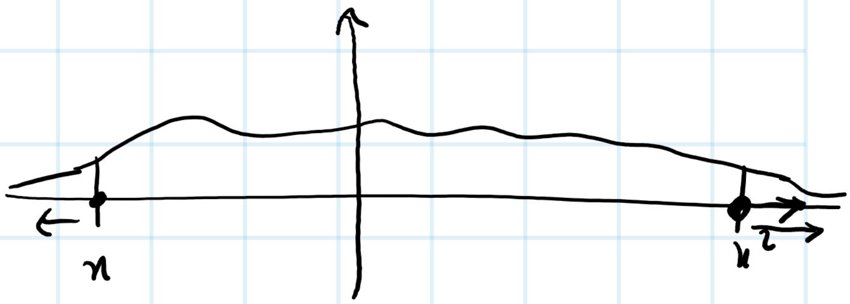
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$0 \qquad 0 \qquad 0$$



$$\int_x^{x^2} \frac{e}{1+t^2}\,dt = \Big[e \arctan t\Big]_x^{x^2} = e\cdot\underbrace{(\arctan x^2 - \arctan x)}_{\downarrow\; e\cdot 0 = 0}$$

$$\lim_{x\to-\infty} \int_x^{x^2} \frac{e^{\cos t}}{1+t^2}\,dt$$



$$x_1 < x_2 < 0$$

$$[x_1, x_1^2] \supset [x_2, x_2^2]$$

$$F(x_1) = \int_{x_1}^{x_1^2} f(t)\,dt \;\geq\; F(x_2) = \int_{x_2}^{x_2^2} f(t)\,dt$$

$$\lim_{x \to -\infty} F(x) \text{ ESISTE}$$

$\exists M > 0$ t.c. $\forall x \in (-\infty, 0)$ $\boxed{F(x)} \leq M$

$$\lim_{x \to -\infty} F(x) = l \leq M$$

$$\boxed{\frac{e^{\cos t}}{1+t^2}} \leq \boxed{\frac{e}{1+t^2}}$$

$$\boxed{\int_x^{x^2} \frac{e}{1+t^2}\,dt} < M$$

$$= \left[e \arctan t\right]_x^{x^2} = e \cdot (\arctan x^2 - \arctan x)$$

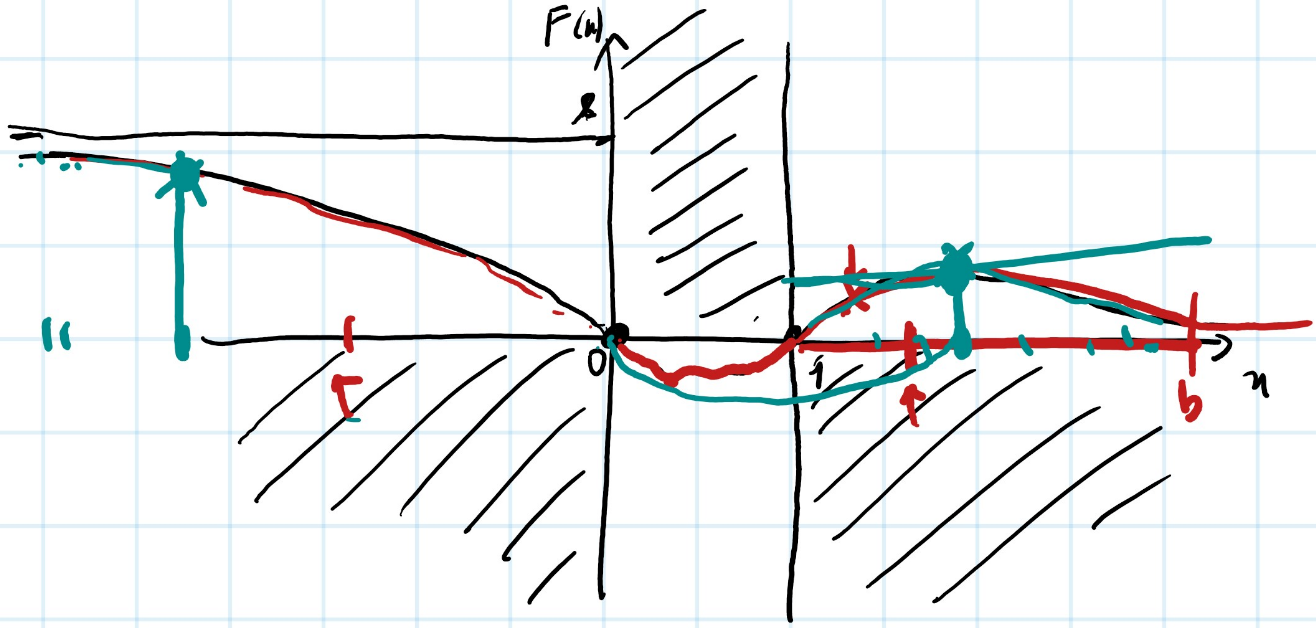
$$\leq e \cdot \left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \boxed{\pi e}$$

$$F(x) = \int_x^{x^2} \frac{e^{\cos t}}{1+t^2}\, dt$$

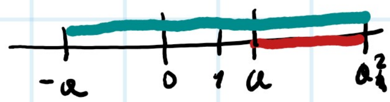
$F(x) > 0$ $\quad$ $F(x) < 0$

$x^2 > x$ $\quad$ $x^2 < x$

$x^2 - x > 0$ $\quad$ $(x-1)x > 0$



$\boxed{a > 1}$

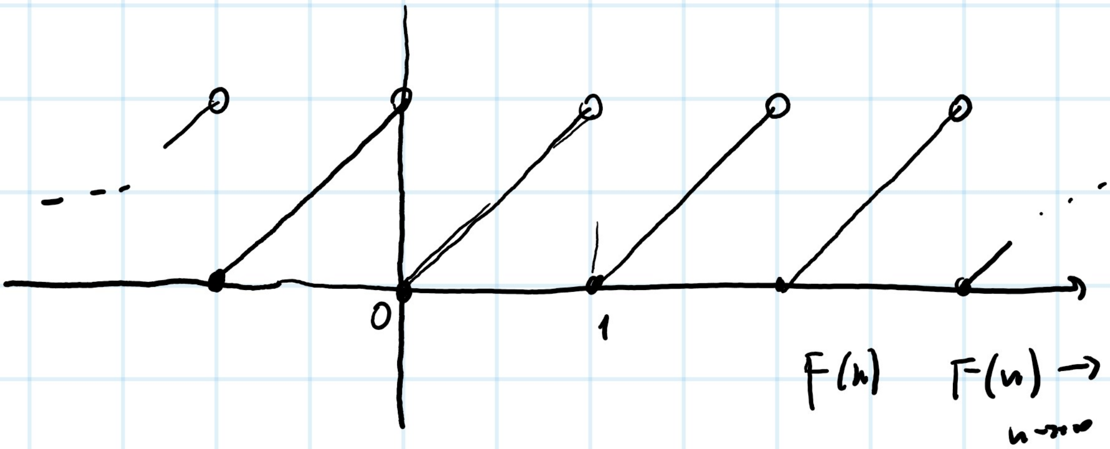


$F(a)$ $\quad$ $F(-a)$

$$\boxed{\int_a^{a^2} f(t)\,dt} \qquad \boxed{\int_{-a}^{a^2} f(t)\,dt}$$

$F(x)=\int_0^x \underbrace{(t-\lfloor t\rfloor+\alpha)}_{f}\,dt$

$\lim\limits_{x\to+\infty} F(x) = ?$



$F(x)$ $\quad F(x) \xrightarrow[n\to+\infty]{}$

$\boxed{F(n)} = \int_0^n f(t)\,dt = n\cdot\int_0^1 f(t)\,dt = \boxed{n\cdot\frac{1}{2}}$

$F(x)$ cresce

$F'(x) = \underline{f(x)}$

$f(t)\geq 0$

$F(x) = \int_a^x f(t)\,dt$

$x_1 < x_2$

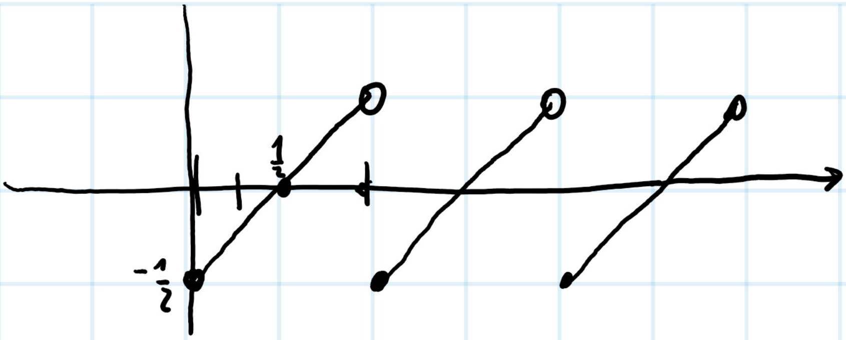
$F(x_2)-F(x_1) = \int_a^{x_2} f(t)\,dt - \int_a^{x_1} f(t)\,dt =$

$= \int_{x_1}^{x_2} f(t)\,dt \geq 0$

$\alpha = -\frac{1}{2}$ $\qquad \int_0^1 \left(t-\lfloor t\rfloor-\frac{1}{2}\right)dt$

$F(1) =$

$= \int_0^1 t - \frac{1}{2} = 0$



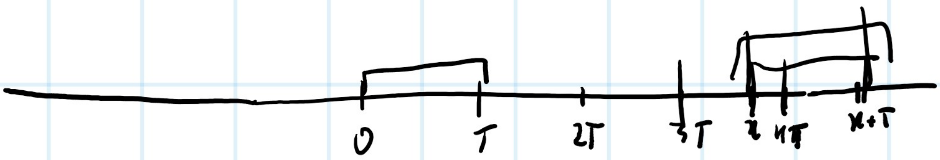
$t \qquad t+n$

$\int_x^{x+1} f(t)\,dt$

$\boxed{T \text{ periodo}}$

$\forall x \in \mathbb{R}$

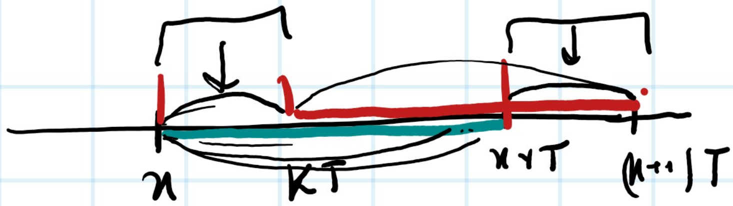
$$\underbrace{\int_0^T f(t)\,dt}_{} = \underbrace{\int_x^{x+T} f(t)\,dt}_{}$$



$$\int_x^{x+T} f(t)\,dt = \int_x^{kT} + \int_{kT}^{x+T} =$$

$$= \int_{x+T}^{(k+1)T} + \int_{kT}^{x+T} =$$

$$= \int_{kT}^{(k+1)T} f(t)\,dt$$



$$\boxed{\int_x^{kT} f(t)\,dt} \overset{?}{=} \boxed{\int_{x+T}^{(k+1)T} f(t)\,dt} =$$

$u = t - T$

$$= \int_x^{kT} f(u+T)\,du = \boxed{\int_x^{kT} f(u)\,du}$$

$f(t)$ t.c. $\boxed{\int_0^T f(t)\,dt} = 0$

$\Downarrow$

$F(x) = \int_0^x f(t)\,dt$ È PERIODICA di PER T.

$\forall x \qquad F(x+T) = F(x)$?

$$\int_0^{x+T} f(t)\,dt \overset{?}{=} \int_0^x f(t)\,dt$$

$$\int_0^{x+T} f(t)\,dt - \int_0^x f(t)\,dt \overset{?}{=} 0$$

$$\int_x^{x+T} f(t)\,dt \overset{?}{=} 0$$

$$\| $$

$$\int_0^T f(t)\,dt = 0$$

⑥ $F(x) = \int_0^{x^2} \sin(\sin t)\, dt$ Trovare ordine di infinitesimo per $x \to 0$

$\lim_{x \to 0^+} F(x) = 0$?

$\int_a^{g(x)}$

$$\underset{\downarrow\, 0}{0} \le \underbrace{\int_0^{x^2} \sin(\sin t)\, dt}_{\downarrow\, 0} \le \int_0^{x^2} t\, dt = \left[\frac{t^2}{2}\right]_0^{x^2} = \underset{\downarrow\, 0}{\frac{x^4}{2}}$$

$\alpha > 0$

$$\lim_{x \to 0^+} \frac{F(x)}{x^\alpha} \overset{H}{=} \lim_{x \to 0^+} \frac{\sin(\sin(x^2)) \cdot 2x}{\alpha x^{\alpha-1}} =$$

$$= \lim_{x \to 0^+} \frac{2x^3}{\alpha x^{\alpha-1}}$$

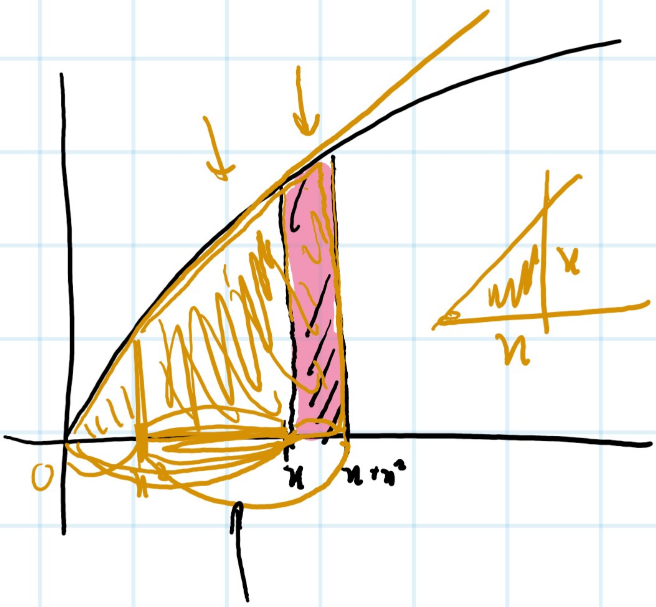
$\boxed{x \to 0^+}$

$O(x^3) = \boxed{G(x) = \int_x^{x+x^2} \sin(\sin t)\, dt}$

$H(x) = \boxed{\int_{x^2}^{x^2+x} \sin(\sin t)\, dt}$

$\boxed{O(x^2)}$

$\int_0^x$

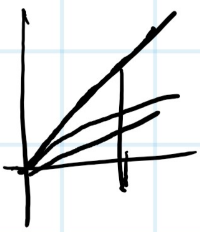


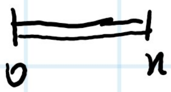
$$\lim_{x\to0^+} H(x) = \lim_{x\to0} \frac{\int_{x^2}^{x^2+x} \sin(\sin t)\,dt}{x^\alpha} \overset{H}{=}$$

$$= \lim_{x\to0} \frac{\sin(\sin(x^2+x))\cdot(2x+1) - \sin(\sin(x^2))\cdot 2x}{\alpha x^{\alpha-1}} =$$

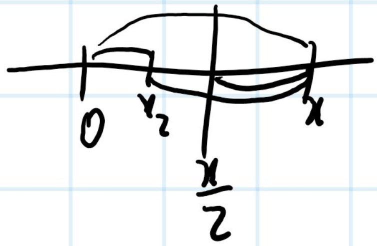
$$= \lim_{x\to0^+} \frac{2x\cdot\left(\sin(\sin(x^2+x)) - \sin(\sin(x^2))\right) + \overbrace{\sin(\sin(x^2+x))}^{\theta(x)}}{\alpha x^{\alpha-1}}$$

$$= \lim_{x\to0^+} \frac{\qquad}{\alpha x^{\alpha-1}}$$





$\sin(\sin t)$



$$H(x) = \int_{x^2}^{x^2+x} f(t)\,dt =$$

$$\int_{\frac{x}{2}}^{x} \frac{t}{2}\,dt \le \int_{\frac{x}{2}}^{x} f(t)\,dt \le \int_{x^2}^{x} f(t)\,dt \le \boxed{H(x)} \le \int_0^{x^2+x} f(t)\,dt \le \int_0^{2x} f(t)\,dt < \int_0^{2x} t\,dt = \boxed{2x^2}$$

$$\left[\frac{t^2}{4}\right]_{\frac{x}{2}}^{x} = \cdot x^2$$