A.M.2 - (LEZ.6 / LEZ.7) - INTEGRAZIONE IN SENSO IMPROPRIO II ((20/21) MARZO 2023)

LE PAGINE SI RIFERISCONO A "DISPENSE INTEGRALI IMPROPRI"

→ 0) INTEGRALI IMPROPRI NOTEVOLI (DALLA VOLTA SCORSA) PAG 8−12

1) ESEMPIO: STUDIARE CONVERGENZA DI $\displaystyle\int_0^{+\infty} \frac{2+\sin\frac{1}{x}}{x\ln^2(1+\sqrt[3]{x-1})}\,dx$ (NUOVO)

INTEGRANDA A SEGNO VARIABILE

2) CR. ASSOLUTA CONVERGENZA PAG 13

3) ESEMPIO: $\int f$ CONVERGE MA $\int |f|$ NON CONVERGE PAG 14

→ 4) CR. INTEG. OSCILLANTI + (GENERALIZZAZIONE) + (CASO NON A MEDIA NULLA) PAG 18−20 + (NUOVO)

5) ESEMPIO: STUDIARE CONVERGENZA DI $\displaystyle\int_0^{+\infty} \frac{\sin x}{\ln(1+x^2+\cos x)}\,dx$ (NUOVO)

6) CRITERIO DI CAUCHY. PAG. 30−31

7) ATTENZIONE, NON VALE CONFRONTO ASINTOTICO: $\displaystyle\int^{+\infty} \frac{(-1)^{\lfloor x\rfloor}}{x}\,dx$ E $\displaystyle\int_2^{+\infty} \frac{(-1)^{\lfloor x\rfloor}}{x}+\frac{1}{x\ln x}\,dx$ PAG. 22−24

8) TRUCCO IMPORTANTE: ESEMPIO dx (NUOVO)

① $$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx \quad \begin{cases} \alpha > 1 & \text{conv.} \\ \alpha \leq 1 & \text{DIV.} \end{cases}$$

=

$$\int_1^{+\infty} \frac{1}{x}\,dx = \lim_{b \to +\infty} \int_1^b \frac{1}{x}\,dx = \lim_{b \to +\infty} \left[\ln x\right]_1^b =$$

$$= \lim_{b \to +\infty} (\ln b - \ln 1) = +\infty$$

$\alpha < 1$ $\quad \boxed{x > 1}$ $\quad \boxed{\frac{1}{x^\alpha}} > \boxed{\frac{1}{x}}$

$\boxed{\alpha > 1}$

$$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx = \lim_{b \to +\infty} \int_1^b \frac{1}{x^\alpha}\,dx =$$

$x^{-\alpha}$ $\quad \frac{x^{-\alpha+1}}{-\alpha+1}$

$$= \lim_{b \to +\infty} \left[\frac{x^{-\alpha+1}}{-\alpha+1}\right]_1^b =$$

$$= \lim_{b \to +\infty} \left(\underbrace{\frac{1}{-\alpha+1} \cdot \frac{1}{b^{\alpha-1}}}_{\downarrow\, 0} - \frac{1}{-\alpha+1} \cdot 1\right) = \frac{1}{\alpha - 1}$$

$$\int_2^{+\infty} \frac{1}{x^\alpha (\ln x)^\beta}\,dx = \begin{cases} \alpha > 1 & \text{CONV} \\ \alpha = 1 & \begin{cases} \beta > 1 & \text{CONV.} \\ \beta \leq 1 & \text{DIV.} \end{cases} \\ \alpha < 1 & \text{DIV.} \end{cases}$$

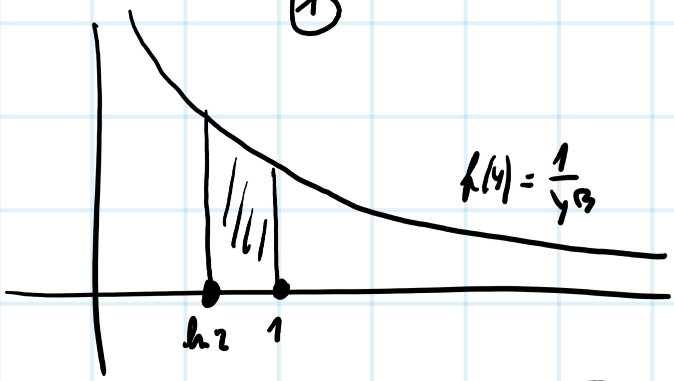
$\boxed{\alpha = 1}$

$$\int_2^{+\infty} \frac{1}{(\ln x)^\beta}\cdot\frac{1}{x}\,dx =$$

$y = \ln x$

$$= \lim_{b\to+\infty}\int_2^b \frac{1}{(\ln x)^\beta}\cdot(\ln x)'\,dx =$$

$$= \lim_{b\to+\infty}\int_{\ln 2}^{\ln b}\frac{1}{y^\beta}\,dy = \int_{\underbrace{\ln 2}_{1}}^{+\infty}\frac{1}{y^\beta}\,dy = \begin{cases}\beta>1 & \text{CONV.}\\ \beta\le 1 & \text{DIV.}\end{cases}$$



$\boxed{\alpha > 1}$

$\alpha = 1+\delta$

$\delta > 0$

$$\int_2^{+\infty}\frac{1}{x^\alpha(\ln x)^\beta}\,dx \quad \text{CONV.}$$

$\beta < 0$ $\quad \dfrac{(\ln x)^{-\beta}}{x^{\frac{\delta}{2}}} \to 0$

PER $x > x_0$

$$\boxed{\frac{1}{x^{1+\delta}(\ln x)^\beta}} = \frac{1}{x^{1+\frac{\delta}{2}}}\cdot\boxed{\frac{1}{x^{\frac{\delta}{2}}(\ln x)^\beta}} \le \boxed{\frac{1}{x^{1+\frac{\delta}{2}}}\cdot 1}$$

$x\to+\infty$ $\quad 0 \quad \forall\beta$

$\int_{x_0}^{+\infty} \bigcirc\,dx$ CONV.

CONV. $\quad \int_{x_0}^{+\infty}\frac{1}{x^{1+\frac{\delta}{2}}}\,dx$

$\boxed{\alpha < 1}$

$$\int_2^{+\infty} \frac{1}{x^\alpha (\ln x)^\beta}\, dx$$

$1 - \delta$

$\delta > 0$

$$\int_{x_0}^{+\infty} \bullet \, dx \quad \boxed{\text{DIV.}}$$

PER $x > x_0$

$$\boxed{\frac{1}{x^{1-\delta} (\ln x)^\beta}} = \frac{1}{x^{1-\frac{\delta}{2}}} \cdot \boxed{\frac{x^{\frac{\delta}{2}}}{(\ln x)^\beta}} \geq \boxed{\frac{1}{x^{1-\frac{\delta}{2}}}} \cdot 1$$

$\int_2^{+\infty} \bullet$ diverge

$+\infty$ $\forall \beta \in \mathbb{R}$

$$\int_0^1 \frac{1}{x^\alpha}\, dx = \begin{cases} \alpha \geq 1 & \text{DIV.} \\ \alpha < 1 & \text{CONV.} \end{cases}$$

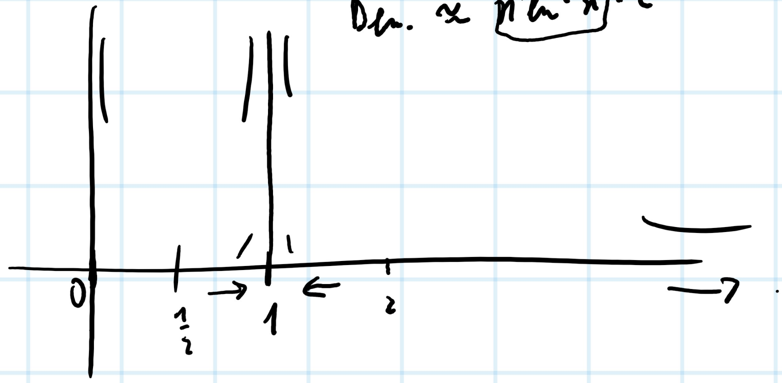
$\frac{1}{x}$ $\qquad$ $\frac{1}{\sqrt{x}}$

$$\boxed{\int_0^{\frac{1}{2}} \frac{1}{x^\alpha |\ln x|^\beta}\, dx = \begin{cases} \alpha > 1 & \text{DIV.} \\ \alpha = 1 & \begin{cases} \beta > 1 & \text{CONV.} \\ \beta \leq 1 & \text{DIV.} \end{cases} \\ \alpha < 1 & \text{CONV} \end{cases}}$$

$$\int_0^{+\infty} \frac{2+\sin\frac{1}{x}}{x\ln^2(1+\sqrt[3]{x-1})}\,dx$$

$x \to +\infty$

Den. $\approx \boxed{x\ln^2 x}\cdot c$



$$\left(\ln\sqrt[3]{x}\right)^2$$

$$\left(\frac{1}{3}\ln x\right)^2$$

$\boxed{\int_0^{\frac{1}{2}} f(x)\,dx}$ $\boxed{\int_{\frac{1}{2}}^{1} f(x)\,dx}$ $\boxed{\int_1^2 f(x)\,dx}$ $\boxed{\int_2^{+\infty} f(x)\,dx}$

CONV.

$x \to +\infty$

$$\text{DEN} \approx \boxed{\frac{1}{9}\,x\ln^2 x}$$

$x \to +\infty$

$$\boxed{f(x)} = \frac{2+\sin\frac{1}{x}}{\boxed{}} \approx \boxed{\frac{18}{x\ln^2 x}}$$

$\int_2^{+\infty} \cdot\, dx$ conv.

$x \to 0^+$

$$f(x) \approx ?$$

$$\|$$

$$\frac{2+\sin\frac{1}{x}}{x\cdot\left(\ln(1+\sqrt[3]{x-1})\right)^2}$$

$x \to 0^+$

$$\ln(1+\sqrt[3]{x-1}) =$$

$$= \ln\left(\boxed{1-\sqrt[3]{1-x}}\right) \approx \ln x$$

$$-\left(\sqrt[3]{1-x}-1\right) \approx -\left(-\frac{1}{3}x\right) = \frac{1}{3}x$$

$\int_0^{\frac{1}{2}} f\,dx$ CONV. $\quad\int_0^{\frac{1}{2}} \, dx$ CONV $\quad\int_0^{\frac{1}{2}} \, dx$ CONV

$$\boxed{f(x)} \leq \boxed{\frac{3}{x\left(\ln\left(1+\sqrt[3]{x-1}\right)\right)^2}} \approx \boxed{\frac{3}{x(\ln x)^2}}$$

$x \to 1^+$

$$f(x) = \frac{2+\sin\frac{1}{x}}{x\left(\ln\left(1+\sqrt[3]{x-1}\right)\right)^2} \approx \frac{2+\sin 1}{\left(\sqrt[3]{x-1}\right)^2} = k \cdot \frac{1}{|x-1|^{\frac{2}{3}}}$$

$2+\sin\frac{1}{x} \to 2+\sin 1$

$\ln\left(1+\sqrt[3]{x-1}\right) \sim \sqrt[3]{x-1}$

$$\int_0^1 \frac{1}{x^\alpha}\,dx \longleftarrow \int_1^2 \frac{1}{(x-1)^{\alpha}}\,dx \quad \text{CONV.}$$

$\frac{2}{3} < 1$

$\boxed{\text{T.}}$ Cr. ASS. Conv.

DATA $f:[a,+\infty) \to \mathbb{R}$ t.c. $f \in \mathcal{R}([a,c])$ $\forall c \in (a,+\infty)$ ALLORA SI HA:

$$\int_a^{+\infty} |f(x)|\,dx \text{ CONV.} \Rightarrow \int_a^{+\infty} f(x)\,dx \text{ CONV.}$$

$\boxed{\text{DIM}}$ f^+, f^-

$$0 \leq f^+ \leq |f|$$

$$0 \leq f^- \leq |f|$$

$\int_a^{+\infty} f^-(x)\,dx$, $\int_a^{+\infty} f^+(x)\,dx$, CONV. (Per. Conf. con $|f|$)

se $h, g : [a, +\infty) \to \mathbb{R}$ sono tali che $\underbrace{\int_a^{+\infty} h(x)\,dx}_{\otimes}$, $\underbrace{\int_a^{+\infty} g(x)\,dx}_{\circledast}$ CONV.

ALLORA $\boxed{\int_a^{+\infty} (\alpha h(x) + \beta g(x))\,dx}$ conv

DIM.

$$\otimes = \lim_{b\to+\infty} \left(\int_a^b \alpha h(x) + \beta g(x)\,dx \right) = \lim_{b\to+\infty} \left(\alpha \int_a^b h(x)\,dx + \beta \int_a^b g(x)\,dx \right) =$$

$$= \alpha \int_a^{+\infty} h(x)\,dx + \beta \int_a^{+\infty} g(x)\,dx$$

$\int f^+$ $\int f^-$ CONV.

$\forall x$ $\quad \underbrace{f(x) = f^+(x) - f^-(x)}$

ES. CATTIVO

(1) $\boxed{\int_1^{+\infty} \frac{\sin x}{x}\,dx \text{ conv.}}$

(2) $\boxed{\int_1^{+\infty} \left| \frac{\sin x}{x} \right| dx \text{ div.}}$

(1) $$\boxed{\lim_{b\to+\infty} \int_1^b \frac{\sin x}{x}\,dx} = \lim_{b\to+\infty} \int_1^b \frac{1}{x} \cdot (-\cos x)'\,dx =$$

$$= \lim_{b\to+\infty} \left(\left[\frac{-\cos x}{x} \right]_1^b - \int_1^b -\frac{1}{x^2} \cdot (-\cos x)\,dx \right) =$$

$$= +\cos 1 - \lim_{b\to+\infty} \int_1^b \frac{\cos x}{x^2}\,dx =$$

$$= \boxed{+\cos 1} - \boxed{\int_1^{+\infty} \frac{\cos x}{x^2}\,dx}$$

$\int_1^{+\infty} \ldots dx$ CONV. PERCHÉ $\int_1^{+\infty} \frac{1}{x^2}\,dx$ CONV

$$\boxed{\left|\frac{\cos x}{x^2}\right|} \leq \boxed{\frac{1}{x^2}}$$

$$\int_1^{+\infty} \left|\frac{\cos x}{x}\right| dx \quad \text{CONV.}$$

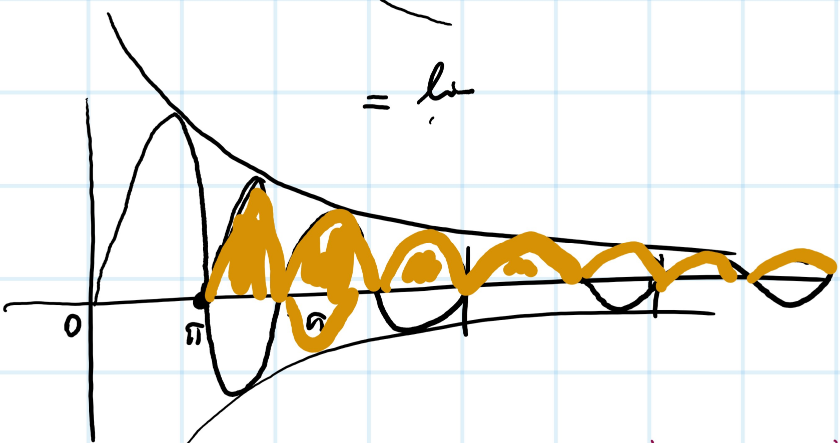
T. Prec.. $\longrightarrow \Downarrow$

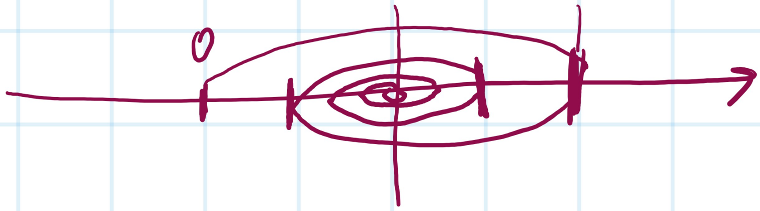
$$\int_1^{+\infty} \frac{\cos x}{x}\,dx \quad \text{CONV.}$$

$$\lim_{b\to+\infty} \boxed{\int_\pi^b \frac{|\sin x|}{x}\,dx} \;\nexists\; +\infty$$

$$\left(\lim_{n\to+\infty} \int_\pi^{n\pi} \frac{|\sin x|}{x} =\right.$$

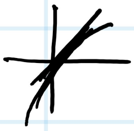
$$= \lim$$





$$\left(\int_{2\pi}^{3\pi}\frac{|\sin x|}{x}\,dx\right) \geq \int_{2\pi}^{3\pi}\frac{|\sin x|}{3\pi}\,dx = \frac{1}{3\pi}\cdot 2$$

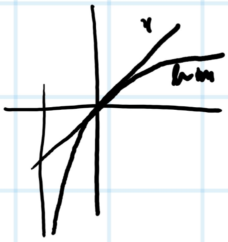
$$\boxed{\lim_{n\to+\infty}\int_{\pi}^{n\pi}\frac{|\sin x|}{x}} = \lim_{n\to+\infty}\left(\int_{\pi}^{2\pi} + \int_{2\pi}^{3\pi} + \dots + \int_{(n-1)\pi}^{n\pi}\right) \geq$$



$$\geq \lim_{n\to+\infty}\left(\frac{1}{2\pi}\cdot 2 + \frac{1}{3\pi}\cdot 2 + \frac{1}{4\pi}\cdot 2 + \dots + \frac{1}{n\pi}\cdot 2\right) =$$

$\ln(1+x) \leq x$

$$= \frac{2}{\pi}\lim_{n\to+\infty}\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots+\frac{1}{n}\right) \geq$$



$$\geq \frac{2}{\pi}\lim_{n\to+\infty}\left(\ln\Big(\overbrace{1+\tfrac{1}{2}}^{\frac{3}{2}}\Big) + \ln\Big(\overbrace{1+\tfrac{1}{3}}^{\frac{4}{3}}\Big) + \ln\Big(\overbrace{1+\tfrac{1}{4}}^{\frac{5}{4}}\Big) + \dots + \ln\Big(\overbrace{1+\tfrac{1}{n}}^{\frac{n+1}{n}}\Big)\right)$$

$$= \frac{2}{\pi}\lim_{n\to+\infty}\ln\left(\frac{\cancel{3}}{2}\cdot\frac{\cancel{4}}{\cancel{3}}\cdot\frac{\cancel{5}}{\cancel{4}}\cdots\frac{n+1}{\cancel{n}}\right) =$$

$$= \frac{2}{\pi}\lim_{n\to+\infty}\ln\left(\frac{n+1}{2}\right) = \boxed{+\infty}$$

INIZIO LEZ. 7

$\boxed{T.}$ DATI $[a,+\infty) \in \mathbb{R}$ $f,g:[a,+\infty)\to\mathbb{R}$ t.c.

1) $g \in C^1([a,+\infty))$ E $g(x)\to 0$ DECRESCENDO PER $x\to+\infty$
2) $f \in C([a,+\infty))$ E $\exists F:[a,+\infty)\to\mathbb{R}$ T.C. $F'(x)=f(x)$ E $F(x)$ È LIMITATA.

ALLORA $\int_a^{+\infty} f(x)\cdot g(x)\,dx$ CONVERGE.

$\boxed{\text{DIMO}}$

$$\int_a^{+\infty} f(x)g(x)\,dx = \lim_{b\to+\infty}\int_a^b f(x)g(x)\,dx =$$

$$= \lim_{b\to+\infty}\int_a^b (F(x))'\,g(x)\,dx =$$

F(x) limitata
$g(x)\to 0$

$$= \lim_{b\to+\infty}\left[F(x)g(x)\right]_a^b - \int_a^b F(x)g'(x)\,dx =$$

$b\to+\infty$

0

$$= \lim_{b\to+\infty} F(b)g(b) - F(a)g(a) - \int_a^b F(x)g'(x)\,dx =$$

$$= -F(a)g(a) - \boxed{\int_a^{+\infty} F(x)g'(x)\,dx}$$ CONVERGE?

⊕

g(x) DECRESCENTE

MOSTRO CHE CONVERGE:

⊕ $\boxed{\int_a^{+\infty} |F(x)\cdot g'(x)|\,dx} = -\int_a^{+\infty} |F(x)|\,g'(x)\,dx$

F(x) È LIMITATA

$$0 \le \boxed{|F(x)|\cdot g'(x)} \le \boxed{M\cdot g'(x)}$$

MA $\int_a^{+\infty} M \cdot g'(x)\,dx = M \cdot \int_a^{+\infty} g'(x)\,dx = M \lim_{b \to +\infty} \int_a^b g'(x)\,dx =$

$= M \lim_{b \to +\infty} \left[g(x) \right]_a^b = M \lim_{b \to +\infty} g(b) - g(a) = M \cdot (-g(a))$

QUINDI $\int_a^{+\infty} M\, g'(x)\,dx$ CONVERGE QUINDI ANCHE $\int_a^{+\infty} |F(x)|\, g'(x)\,dx$

CONVERGE. QUINDI ⊕ CONVERGE.

QUINDI ⊛ CONVERGE

$$\int_2^{+\infty} \frac{\sin x}{x^2 + \cos x}\,dx$$

$\boxed{x \geq 2}$

$$0 \leq \left| \frac{\sin x}{x^2 + \cos x} \right| \leq \frac{1}{|x^2 + \cos x|} \leq \frac{1}{x^2 - 1} \leq \frac{1}{\frac{x^2}{2}} = \boxed{\frac{2}{x^2}}$$

$x^2 - 1 \geq \frac{x^2}{2}$ (?)

$\frac{x^2}{2} \geq 1$ (?)

S

$$\boxed{\int_0^{+\infty} \frac{\boxed{\sin x}}{\ln(2 + x^2 + \cos x)}\,dx}$$

$f(x) = \sin x$

$g(x) = \frac{1}{\ln(2 + x^2 + \cos x)}$

$\to \int_1^{+\infty}$

CR. CAUCHY

DATI $[a,+\infty)\subset\mathbb{R}$ $\quad f:[a,+\infty)\to\mathbb{R}$ T.C. $f\in\mathcal{R}([a,c])$ $\forall c\in(a,+\infty)$.

ALLORA È EQUIVALENTE DIRE CHE:

1) $\int_a^{+\infty} f(x)\,dx$ CONVERGE

2) $\forall\varepsilon>0\ \exists M>a$ t.c. $\forall\alpha,\beta\geq M$ SI HA $\boxed{\left|\int_\alpha^\beta f(x)\,dx\right|<\varepsilon}$

$\boxed{\text{DIM}}$

$\boxed{\text{SIA } F(x)=\int_a^x f(t)\,dt.}$

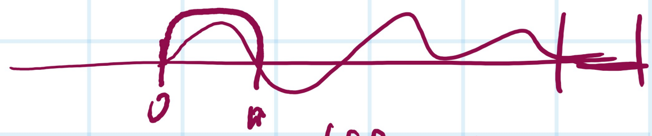
$|F(\alpha)-F(\beta)|<\varepsilon$

(1) $\iff$ $\boxed{\lim_{x\to+\infty} F(x)=\ell\in\mathbb{R}}$

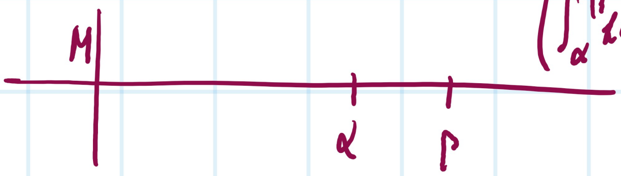
(2) $\iff$ $\boxed{\forall\varepsilon>0\ \exists M>a \text{ t.c. } \forall\alpha,\beta\geq M \text{ SI HA } |F(\alpha)-F(\beta)|<\varepsilon}$

$\int_a^{+\infty}\sqrt[3]{\sin x}\,dx$ NON CONV.

$\forall n\in\mathbb{N}$ $\quad \int_{2n\pi}^{(2n+1)\pi}\sqrt[3]{\sin x}\,dx=\int_0^{\pi}\sqrt[3]{\sin x}\,dx=\boxed{c>0}$



$\left|\int_\alpha^\beta f(x)\,dx\right|\geq c$



$f(x)$ PERIODICA Per. T

$$\int_0^T f(x)\,dx = 0$$

$$\boxed{F(x)} = \int_0^x f(t)\,dt$$

$$F(x+T) = \int_0^{x+T} f(t)\,dt = \overbrace{\int_0^T f(t)\,dt}^{=0} + \overbrace{\int_T^{x+T} f(x)\,dx}^{=\int_0^x} =$$

$$= F(x)$$

$$\boxed{\int_2^{+\infty} \overbrace{\frac{(-1)^{[x]}}{x}}^{f(x)}\,dx}$$

$f(x) \approx g(x)$ PER $x \to +\infty$

$$\boxed{\int_2^{+\infty} \overbrace{\frac{(-1)^{[x]}}{x} + \frac{1}{x\ln x}}^{g(x)}\,dx}$$

$$\int_{2k}^{2k+2} \frac{(-1)^{[x]}}{x} =$$

$$= \int_{2k}^{2k+1} \frac{1}{x}\,dx - \int_{2k+1}^{2k+2} \frac{1}{x}\,dx = \ln(2k+1) - \ln(2k) - \left(\ln(2k+2) - \ln(2k+1)\right) =$$

$$= \boxed{\ln\left(\frac{(2k+1)^2}{(2k+2)\cdot 2k}\right)} =$$

$$= \ln\left(\frac{4k^2+4k+1}{4k^2+4k}\right) = \ln\left(1+\frac{1}{4k^2+4k}\right) \leq$$

$$\boxed{\lim_{b\to+\infty}\int_2^b \frac{(-1)^{[x]}}{x}\,dx} \qquad \leq \ln\left(1+\underbrace{\frac{1}{k(k+1)}}_{2}\right) \leq \underbrace{\frac{1}{k(k+1)}}_{2}$$

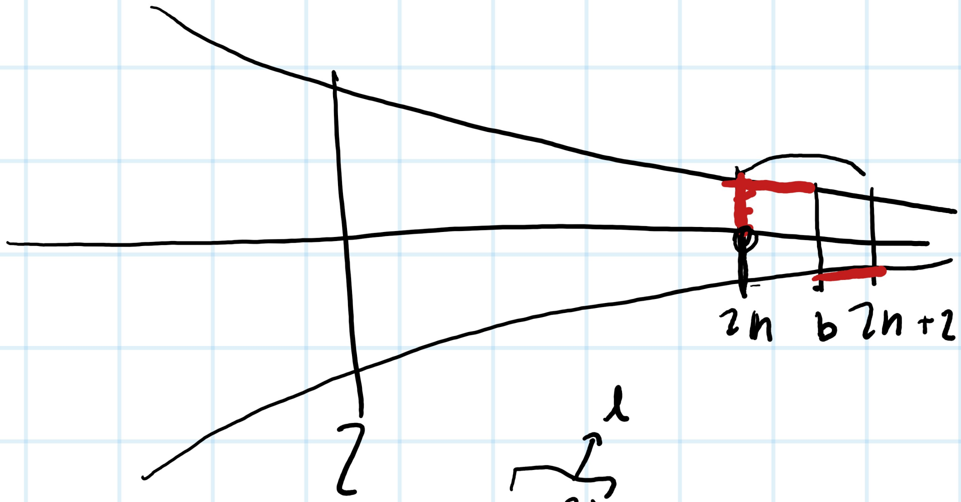
$$\boxed{\lim_{n\to+\infty}\int_2^{2n}\frac{(-1)^{[x]}}{x}\,dx} = \lim_{n\to+\infty}\left(\int_2^4+\int_4^6+\dots+\int_{2k}^{2k+2}+\dots+\int_{2n-2}^{2n}\right)$$

$$= \lim_{n\to+\infty}\left(\frac{\overset{2-1}{1}}{1\cdot 2}+\frac{\overset{3-2}{1}}{2\cdot 3}+\frac{\overset{4-3}{1}}{3\cdot 4}+\dots+\frac{\overset{n-(n-1)}{1}}{(n-1)\cdot n}\right) =$$

$$= \lim_{n\to+\infty}\left(\frac{\cancel{2}}{1\cdot\cancel{2}}-\frac{1}{1\cdot 2}+\frac{\cancel{3}}{2\cdot\cancel{3}}-\frac{\cancel{2}}{\cancel{2}\cdot 3}+\frac{4}{3\cdot 4}-\frac{\cancel{3}}{\cancel{3}\cdot 4}+\dots+\frac{n}{(n-1)n}-\frac{n-1}{(n-1)n}\right)$$

$$\lim_{n\to+\infty}\left(1-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}+\dots+\cancel{\frac{1}{n-1}}-\frac{1}{n}\right) =$$

$$= \lim_{n\to+\infty}\left(1-\frac{1}{n}\right) = 1$$



$$\underbrace{\int_0^{2n}}_{\ell}$$

$$\left|\int_0^b - \int_0^{2n}\right| = \left|\int_{2n}^b f(x)\right| \le \int_{2n}^b |f(x)|\,dx \le$$

$$\le \int_{2n}^b \frac{1}{2n}\,dx = \frac{1}{2n}\cdot(b-2n) \le \frac{1}{2n}\cdot 2 = \boxed{\frac{1}{n}}$$

$$\boxed{\int_a^{+\infty} f(x)\,dx}$$

$$\boxed{\int_a^{+\infty} g(x)\,dx \quad \text{conv.}}$$

$$\to \boxed{\int_0^{+\infty} f(x)+g(x)\,dx}$$

$$\Updownarrow$$

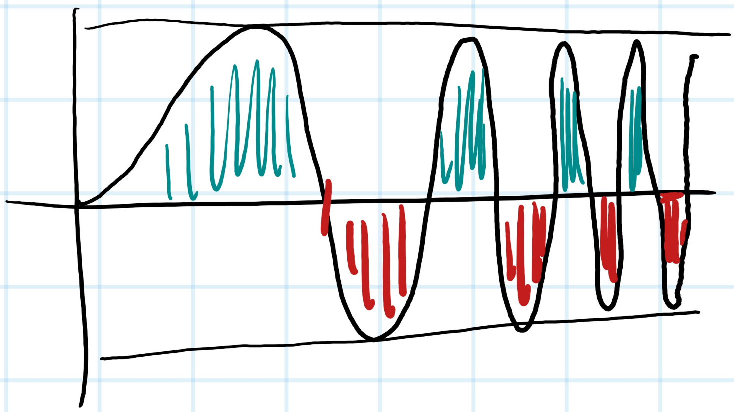
$$\lim_{b\to+\infty}\int_a^b f(x)+g(x)\,dx = \lim_{b\to+\infty}\left(\int_a^b f(x)\,dx + \underbrace{\int_a^b g(x)\,dx}_{\ell\in\mathbb{R}}\right)$$

$$\boxed{\int_2^{+\infty} \left(\boxed{\frac{(-1)^{\lfloor x \rfloor}}{x}} + \cancel{\frac{1}{x \ln x}} \right) dx}$$

DIVERGI

$$\boxed{\int_2^{+\infty} \frac{1}{x \ln x}\, dx}$$

$$\int_0^{+\infty} \sin(x^2)\, dx$$



$f(x) \not\to 0 \quad x \to \infty$

$$\lim_{b\to+\infty}\int_1^b \sin(\overset{x=\sqrt{t}}{x^2})\,dx = \lim_{b\to+\infty}\int_1^{\sqrt{b}}(\sin t)\cdot\frac{1}{2\sqrt{t}}\,dt =$$

$$= \boxed{\int_1^{+\infty}\frac{\sin t}{2\sqrt{t}}\,dt}$$ CONV.

$\frac{1}{2\sqrt{t}} \to 0$ decresc.

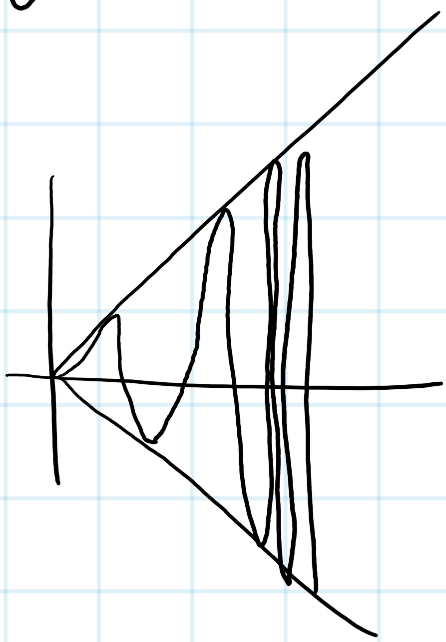
$$\int_0^{+\infty} x\sin(x^3)\,dx =$$

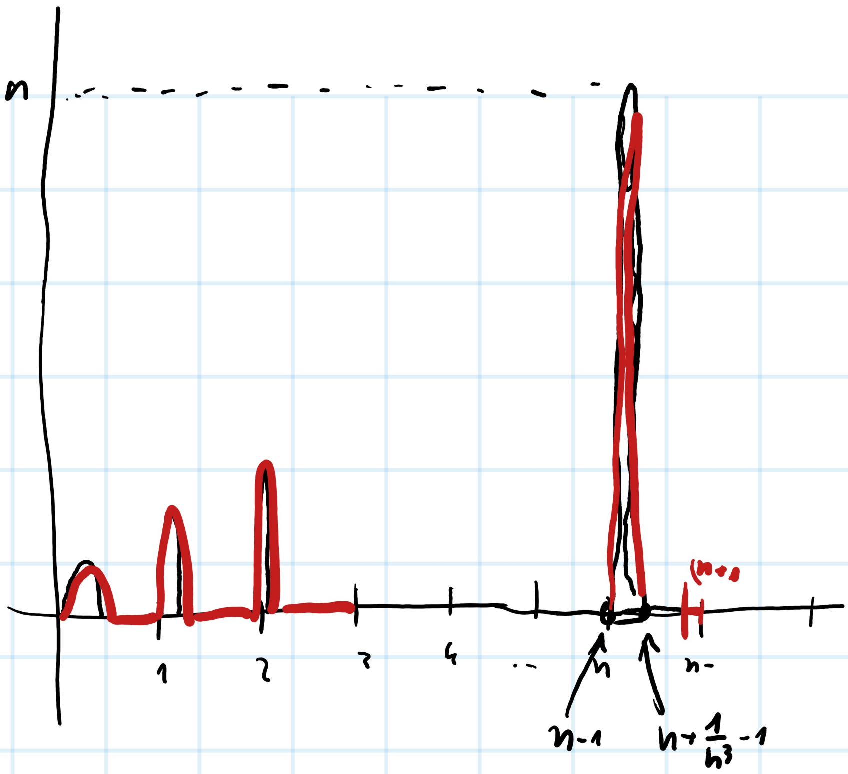
$x=\sqrt[3]{t}$

$$\lim_{b\to+\infty}\int_1^b x\sin(x^3)\,dx =$$

$$= \lim_{b\to+\infty}\int_1^{\sqrt[3]{b}} \cancel{\sqrt[3]{t}}\cdot\sin t\cdot\frac{1}{3\sqrt[3]{t^2}}\,dt =$$

$$= \int_1^{+\infty}\frac{\sin t}{3\sqrt[3]{t}}\,dt$$ CONV.





$$\overbrace{\lim_{n\to+\infty}\int_0^{n+1} f(x)\,dx}=\lim_{n\to+\infty}\left(\int_0^1+\int_1^2+\dots+\int_{n-1}^{n}\dots\right)\leq$$

$$\leq\lim_{n\to+\infty}\left(\boxed{1}+\overbrace{\frac{1}{2^2}+\dots+\frac{1}{n^2}}^{\leq 1}\right)\leq$$

$$\frac{1}{k^2}$$

$$\leq\left(\dots\frac{1}{k(k-1)}\right)\leq 2$$