A.M.2 - LEZ. 8 - STUDIO DELLA FUNZIONE INTEGRALE (27/03/2023)

STUDIARE LE SEGUENTI FUNZIONI INTEGRALI

1) $$F(x) = \int_2^x \frac{t^2+100}{(t+1)\cdot\sqrt{|t^2-1|}}\,dt$$

2) $$F(x) = \int_3^x \frac{t}{t+1}\sqrt{\left|\frac{t^2+2t}{t^2-1}\right|}\,dt$$

3) $$F(x) = \int_2^x \arctan\frac{1}{\sqrt{t^4-t^2}}\,dt$$

4) $$F(x) = \int_5^x \sqrt[n]{1-\frac{2}{t^5}+\frac{2}{t^5+1}}\,dt$$

PER $n = 3, \boxed{4, 11}, 12$

5) SIANO $f, g \in C(\mathbb{R})$ TALI CHE $f(0)=g(0)=0$ E, PER OGNI $x \in \mathbb{R}$, SIANO $F(x) = \int_0^x f(t)\,dt$ E $G(x) = \int_0^x g(t)\,dt$. MOSTRARE CHE:

a) $f(t) \approx g(t)$ PER $t \to 0 \Rightarrow F(x) \approx G(x)$ PER $x \to 0$

b) $f(t) = o(g(t))$ PER $t \to 0 \Rightarrow F(x) = o(G(x))$ PER $x \to 0$

6) SIANO $h, f, g \in C(\mathbb{R})$ TALI CHE $f(0)=g(0)=h(0)=0$ E, PER OGNI $x \in \mathbb{R}$, SIANO $F(x) = \int_0^{f(x)} h(t)\,dt$ E $G(x) = \int_0^{g(x)} h(t)\,dt$.

È VERO CHE $\left(f(x) \approx g(x) \text{ PER } x \to 0\right) \Rightarrow \left(F(x) \approx G(x) \text{ PER } x \to 0\right)$?

MOTIVARE LA RISPOSTA CON DIMOSTRAZIONE O CONTROESEMPIO.

$$F(x) = \int_5^x \underbrace{\sqrt[4]{1 - \frac{2}{t^5} + \frac{2}{t^5+1}}}_{f(t)}\, dt$$

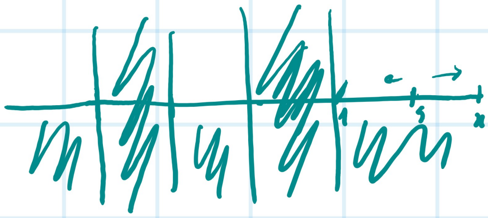
PER $n = 3, 4, 11, 12$

$$\boxed{D = [1, +\infty)}$$

$$\frac{1}{t^5} - \frac{1}{t^5+1} = \frac{1}{t^5(t^5+1)}$$

$$1 - \frac{2}{t^5} + \frac{2}{t^5+1} \geq 0$$

$$1 - \frac{2}{t^5(t^5+1)} \geq 0$$

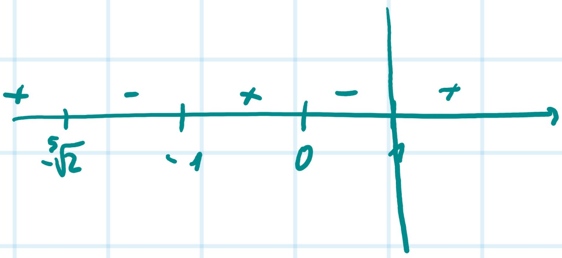


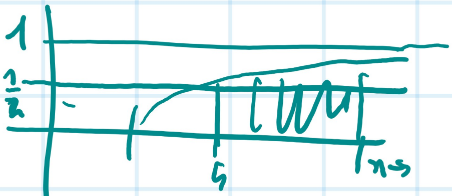
$m = t^5$

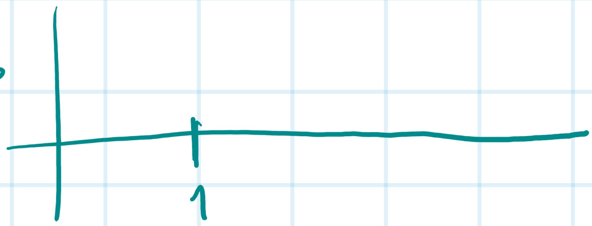
$$\frac{m^2 + m - 2}{m(m+1)} \geq 0$$

$$f(t) = \sqrt[4]{1 - \underbrace{\frac{1}{t^5} + \frac{1}{t^5+1}}}$$

$$\frac{(m+2)(m-1)}{m(m+1)}$$







$$\lim_{x \to +\infty} F(x) = \underbrace{\lim_{x \to +\infty} \int_5^x f(t)\,dt}_{} = \underbrace{\int_5^{+\infty} f(t)\,dt}_{} = +\infty$$

$$f(t) \geq \frac{1}{2}$$

$$\int_5^{+\infty} \frac{1}{2}\, dt = +\infty$$

$y = mx + q$

$F(x)$

$$\boxed{\lim_{x\to+\infty} \frac{F(x)}{x}} = m \quad (?)$$

$$\lim_{x\to+\infty} \left(F(x) - mx\right) = q \quad (?)$$

$$\lim_{x\to+\infty} \frac{F(x)}{x} = \lim_{x\to+\infty} \frac{\int_5^x \sqrt[4]{1-\frac{2}{t^5}+\frac{2}{t^5+1}}\,dt}{x} =$$

$$= \lim_{x\to+\infty} \frac{\sqrt[4]{1-\frac{2}{x^5}+\frac{2}{x^5+1}}}{1} = 1$$

$\int_0^5 + \int_5^x$

$$\lim_{x\to+\infty}\left(F(x) - x\right) = \lim_{x\to+\infty}\left(\int_5^x \sqrt[4]{1-\frac{2}{t^5}+\frac{2}{t^5+1}}\,dt - \int_0^x 1\,dt\right) =$$

$$= \lim_{x\to+\infty}\left(\ \ldots\ - \int_5^x 1\,dt - 5\right)$$

$$= -5 + \lim_{x\to+\infty} \int_5^x \left(\sqrt[4]{1-\frac{2}{t^5}+\frac{2}{t^5+1}} - 1\right) dt = \underline{\text{E. FINITO.}}$$

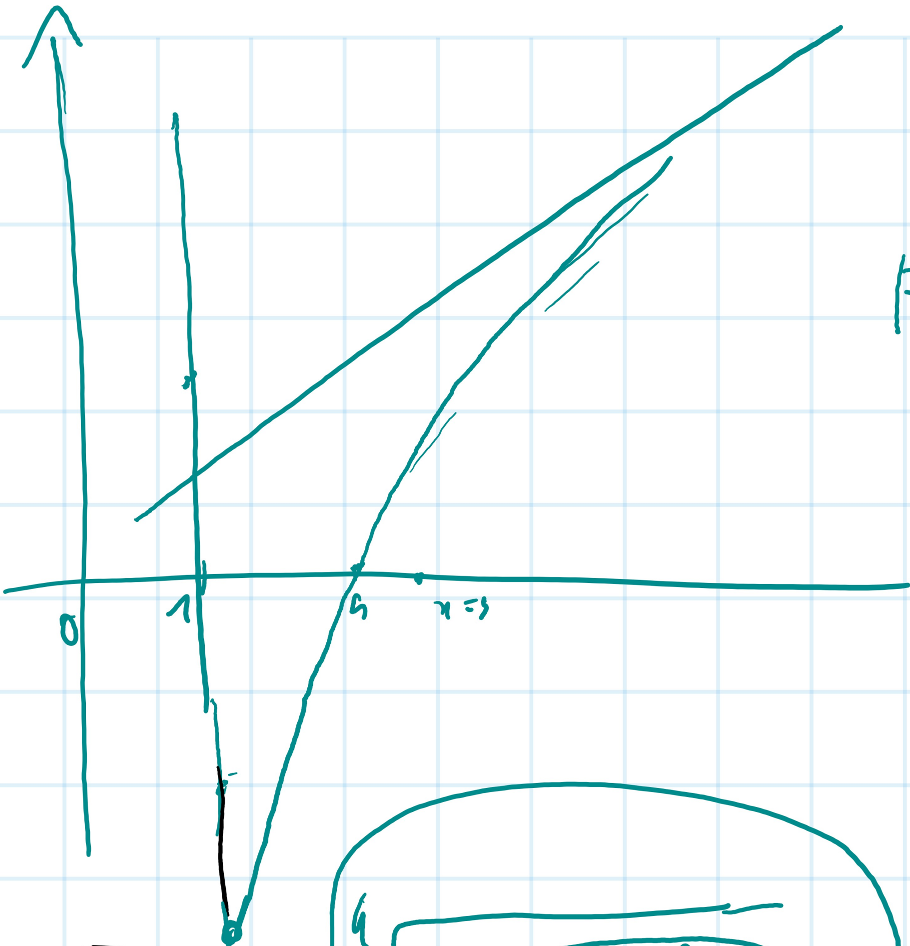
$\boxed{g(t)\to 0}$

$$\sqrt[4]{1+g(t)} - 1 \approx \frac{1}{4} g(t)$$

$$(1+x)^\alpha - 1 \approx \alpha x \quad \boxed{x\to 0}$$

$$\sqrt[4]{1 - \frac{2}{t^5\cdot(t^5+1)}} - 1 \approx \boxed{-\frac{1}{2t^5\cdot(t^5+1)}}$$

$$\int_5^{+\infty} \ldots\, dt \ \boxed{\text{CONV.}}$$



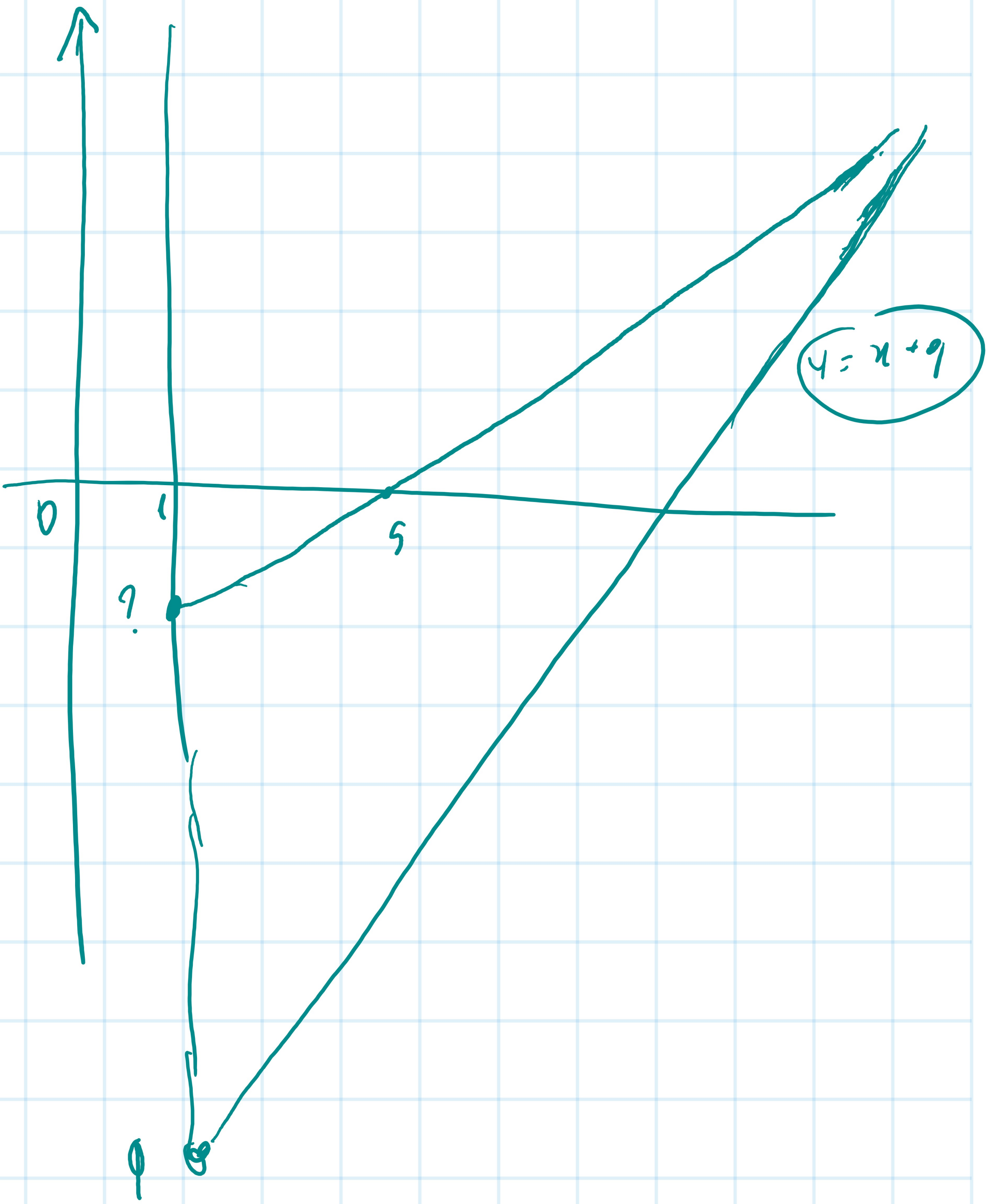
$F'(x) = f(x)$

$\sqrt[4]{1 - \dots}$

$$\boxed{F'(x)} = \sqrt[4]{1 - \frac{2}{x^4(x^4+1)}}$$

$x^4(x^4+1)$ CRESC.

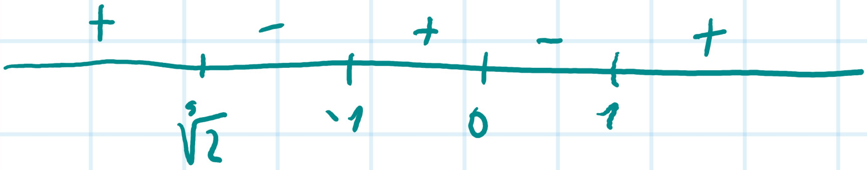
SU $[1, +\infty)$ (S1)



y = x + 9
0
1
5
?
9

$$\frac{(x+2)(x-1)}{x(x+1)}$$

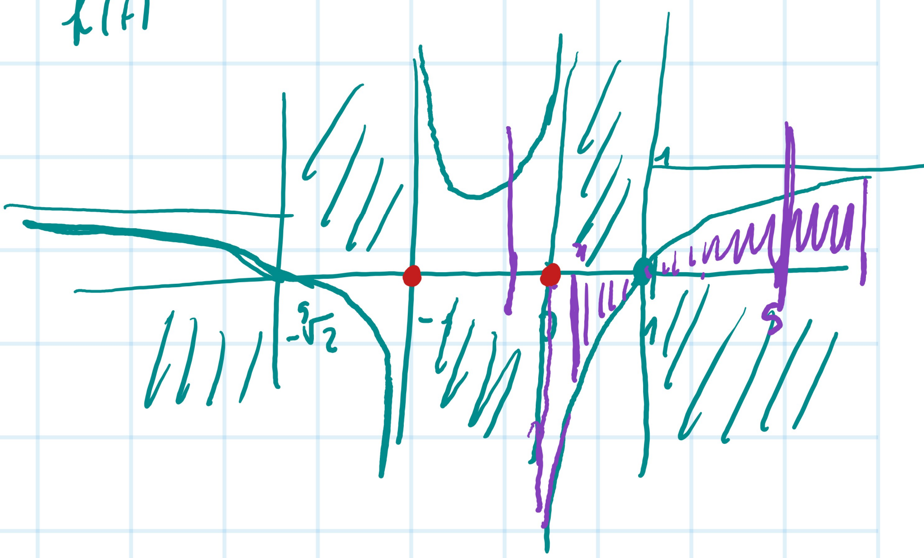
$$F(x)=\int_5^x \sqrt[11]{1-\frac{2}{t^5(t^5+1)}}\,dt$$



$$\sqrt[11]{\frac{(t^5+2)(t^5-1)}{t^5(t^5+1)}} = \sqrt[11]{\;} \quad \frac{1}{t^5}$$

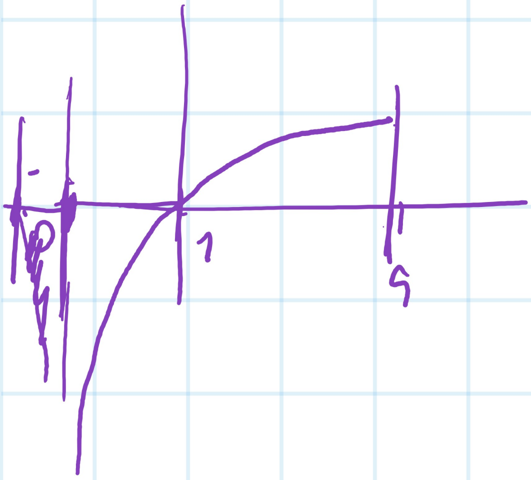
$f(t)$

$\int_5^0 f(t)\,dt$ CONV. DIV.



$x \to 0^+$

$$f(t) = \sqrt[11]{\frac{(t^5+2)(t^5-1)}{(t^5+1)}} \cdot \frac{1}{\sqrt[11]{t^5}} \approx \boxed{-\sqrt[11]{2} \cdot \frac{1}{t^{\frac{5}{11}}}}$$

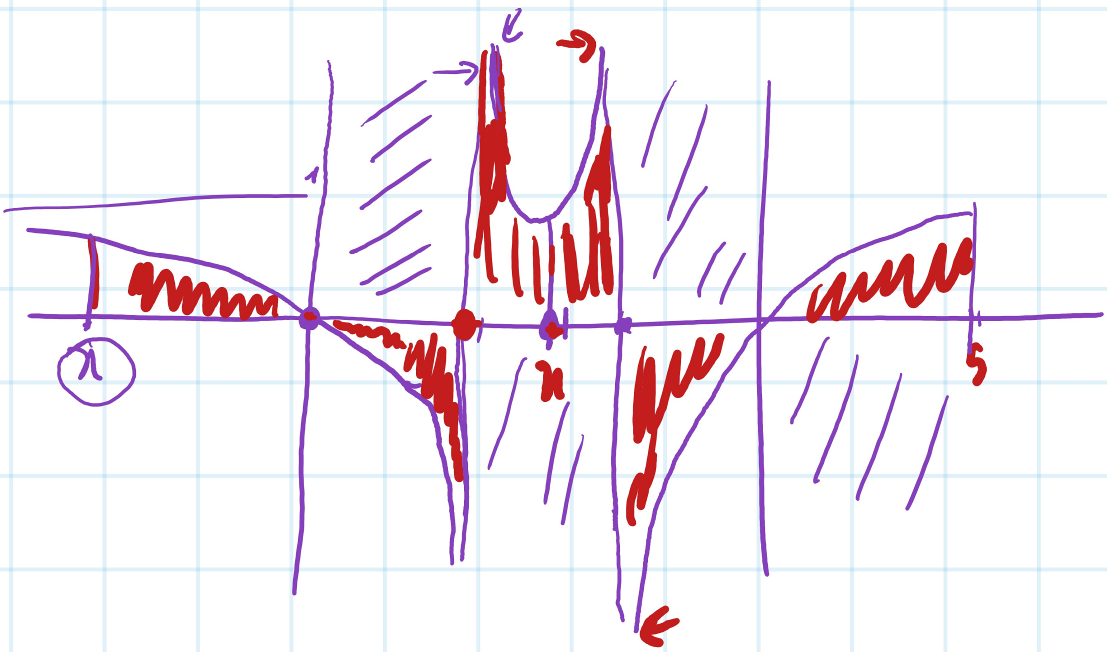


$$\int_0^5 \circ \, dt \quad \text{CONV.}$$

$$\Downarrow$$

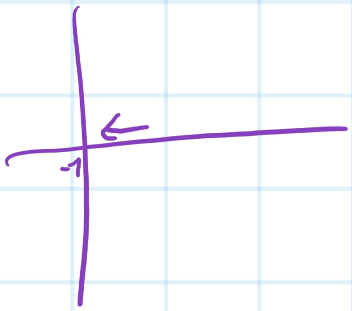
$$\int_0^5 f(t)\, dt. \quad \text{CONV.}$$

$x \to -1^+$



$x \to -1^+$

$$f(t) = \sqrt[11]{\frac{(t^5+2)(t^5-1)}{t^5 \cdot \underbrace{(t+1)(t^4-t^3+t^2-t+1)}_{t^5+1}}} =$$



$\int_{-1}^{-\frac{1}{2}}$ conv.

$$= \boxed{k} \cdot \frac{1}{\sqrt[11]{t+1}} \approx \boxed{\sqrt[11]{\frac{2}{5}} \cdot \frac{1}{(t+1)^{\frac{1}{11}}} \quad t \to -1}$$

$$F(x) = \int_5^x \sqrt[11]{1 - \frac{1}{t^5} + \frac{1}{t^5+1}}\, dt$$

$\boxed{D = \mathbb{R}}$

$$\lim_{x \to +\infty} F(x) = \lim_{x \to +\infty} \int_5^x f(t)\,dt = +\infty$$

$\int_0^x 1\,dt$

$$m = \lim_{x \to +\infty} \frac{F(x)}{x} \overset{H}{=} \lim_{x \to +\infty} \frac{f(x)}{1} = 1$$

$\|$

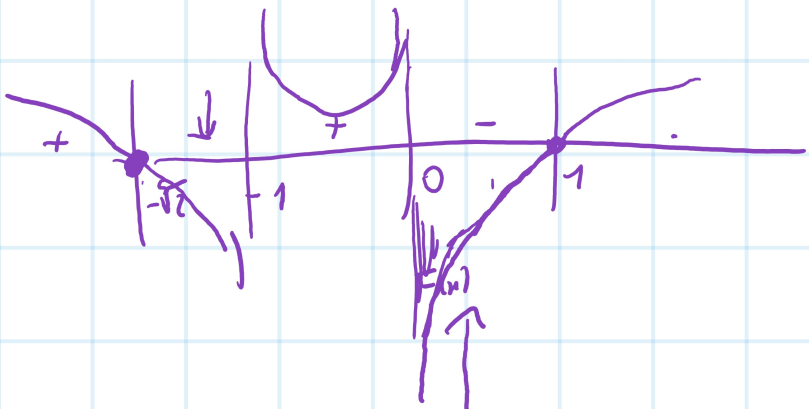
$\boxed{\int_0^5} + \int_5^x 1\,dt$

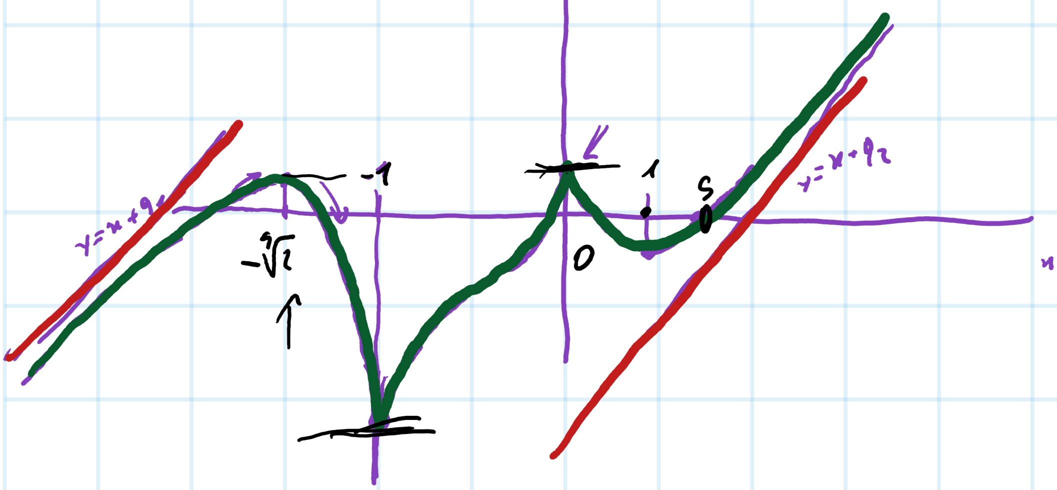
$+5$

$$q = \lim_{x \to +\infty} \left(F(x) - x\right) = \lim_{x \to +\infty} \left(\int_5^x f(t)\,dt - 5 - \int_5^x 1\,dt \right) =$$

$$= -5 + \lim_{x \to +\infty} \int_5^{x/11} \left(\sqrt{1 - \frac{2}{t^9(t^9+1)}} - 1 \right) dt$$

$$\underset{t \to +\infty}{\sim} -\frac{2}{11} \cdot \frac{1}{t^9(t^9+1)} \sim -\frac{2}{11} \cdot \frac{1}{t^{10}}$$





SIANO $f, g \in C(\mathbb{R})$ TALI CHE $f(0)=g(0)=0$ E, PER OGNI $x \in \mathbb{R}$, SIANO

$F(x)=\int_0^x f(t)\,dt$ E $G(x)=\int_0^x g(t)\,dt$. MOSTRARE CHE:

$\to$ a) $f(t) \approx g(t)$ PER $t \to 0 \Rightarrow F(x) \approx G(x)$ PER $x \to 0$

b) $f(t) = o(g(t))$ PER $t \to 0 \Rightarrow F(x) = o(G(x))$ PER $x \to 0$

$$\to F(x) = \int_0^x f(t)\,dt$$

$$\to G(x) = \int_0^x g(t)\,dt$$

$$F(x) = \int_0^x \sin(\sin t)\,dt \approx \int_0^x t = \frac{x^2}{2}$$

$f(t) \approx g(t) \quad t \to 0$

$$\lim_{x\to 0} \frac{F(x)}{G(x)} \overset{H}{=} \lim_{x \to 0} \frac{f(x)}{g(x)} = 1$$

$\frac{0}{0}$

$$F(x)=\int_0^{f(x)} \underbrace{h(t)}\,dt$$

$$G(x)=\int_0^{g(x)} \underbrace{h(t)}\,dt$$

$$f(x)\simeq g(x) \overset{?}{\not\Rightarrow} F(x)\simeq G(x)$$

$(\ )^2$

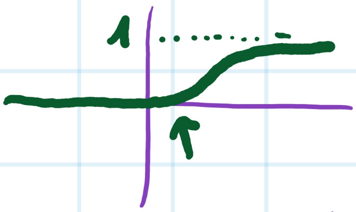
$$\boxed{H(g(x))}$$

$$\boxed{H(f(x))}$$

$$h(t)=t \qquad ($$

$$(f(x))^{\alpha}\simeq(g(x))^{\alpha}$$

$$f(x)\simeq g(x)$$



$$h(t)=\begin{cases} e^{-\frac{1}{t}} & t>0 \\ 0 & t\leq 0\end{cases}$$

$$\boxed{F(x)=\int_0^{x^2+x^3} \underbrace{h(t)}\,dt}$$

$$\boxed{G(x)=\int_0^{x^2} \underbrace{h(t)}\,dt}$$

$$\lim_{x\to 0^+}\frac{F(x)}{G(x)} \overset{H}{=} \lim_{x\to 0^+}\frac{e^{-\frac{1}{x^2+x^3}}}{e^{-\frac{1}{x^2}}} =$$

$$=\lim_{x\to 0^+} e^{\frac{1}{x^2}-\frac{1}{x^2+x^3}}$$

$$= \lim_{x \to 0^+} e^{\frac{x^3}{\underline{x^2(x^2+x^3)}}} = +\infty$$