A.M. 2 - LEZ 9 - ESERCIZI VARI SU INT. IMPROPRI (28 MARZO 2023)

Analisi Matematica 1 - Lista n. 26

Studio della Convergenza di Integrali Impropri

Titolo nota

www.problemisvolti.it

Studiare la convergenza dei seguenti integrali impropri:

1) $\displaystyle\int_1^{+\infty}\left(\frac{100}{1+x^2}\right)^x dx$ converge

2) $\displaystyle\int_1^{+\infty}\sin\sqrt{x}\, dx$ indeterminato

3) $\displaystyle\int_2^{+\infty}\left(1-\frac{1}{x}\right)^x dx$ diverge

4) $\displaystyle\int_2^{+\infty}\left(1-\frac{1}{x}\right)^{x^2} dx$ converge

5) $\displaystyle\int_0^{1}\frac{e^x-\cos x}{\tan x-\sin x}\, dx$ diverge

6) $\displaystyle\int_0^{+\infty}\frac{1}{\ln(1+x^x)}\, dx$ diverge

7) $\displaystyle\int_0^{+\infty}\ln\left(1+x^{-x}\right) dx$ converge

8) $\displaystyle\int_{-\infty}^{-2}\frac{x^2+5x+8}{x^6-1}\, dx$ converge

9) $\displaystyle\int_{-\infty}^{-1}\frac{x^2+5x+8}{x^6-1}\, dx$ diverge

10) $\displaystyle\int_3^{+\infty}\frac{\sin\left(\sin\frac{1}{x}\right)}{\ln(\ln x)}\, dx$ diverge

11) $\displaystyle\int_0^{1}\left(\ln\left(1+e^{-\frac{1}{x}}\right)+\ln^{-1}\left(1+e^{\frac{1}{x}}\right)\right)^{-\frac{1}{2}} dx$ converge

12) $\displaystyle\int_1^{+\infty}\left(\left(1+\frac{1}{x^2}\right)^x-e^{\frac{1}{x}}\right) dx$ converge

13) $\displaystyle\int_1^{+\infty}\frac{1}{x\ln(1+x)\ln^2(\ln(2+x))}\, dx$ converge

14) $\displaystyle\int_0^{+\infty}\frac{\sin x}{1+x\sqrt{x}}\, dx$ converge

15) $\displaystyle\int_3^{+\infty}\frac{100\sin x+\ln(\ln x)}{\sqrt{1+x^2}\cdot\left(\ln^2 x+\sin(x^2)\right)}\, dx$ converge

16) $\displaystyle\int_0^{+\infty}\frac{\sin x}{x}\, dx$ converge

17) $\displaystyle\int_0^{+\infty}\frac{\cos x}{\ln(3+x)}\, dx$ converge

18) $\displaystyle\int_1^{+\infty}\frac{\sin x+\sin\frac{1}{x}}{\ln(2+x)}\, dx$ diverge

19) $\displaystyle\int_0^{+\infty}\left|\sin(\sin x)\right|^x dx$ converge

20) $\displaystyle\int_0^{+\infty}\left(\frac{15+\sin x}{17+\cos x}\right)^x dx$ converge

21) $\displaystyle\int_0^{+\infty}\left(\frac{16+\sin x}{17+\cos x}\right)^x dx$ diverge

22) $\displaystyle\int_0^{+\infty}x\sin(x^4)\, dx$ converge

23) $\displaystyle\int_0^{+\infty}\left|\sin x\right|^{x^3} dx$ converge

24) $\displaystyle\int_0^{+\infty}\left|\sin x\right|^{x^2} dx$ diverge

Studiare la convergenza dei seguenti integrali impropri al variare dei valori dei parametri a fianco indicati e, se richiesto, calcolarli per particolari valori del parametro.

25) $\displaystyle\int_0^{+\infty} \frac{\arctan x}{1+x^\alpha}\,dx$ Studiare per $\alpha \in \mathbb{R}$, calcolare per $\alpha = 2$. [Converge per $\alpha > 1$; per $\alpha = 2$ vale $\frac{\pi^2}{8}$]

26) $\displaystyle\int_1^{+\infty} \frac{\arctan \sqrt{x}}{x^\alpha}\,dx$ Studiare per $\alpha \in \mathbb{R}$, calcolare per $\alpha = \frac{3}{2}$ [Converge per $\alpha > 1$; per $\alpha = \frac{3}{2}$ vale $\frac{\pi}{2} + \ln 2$]

27) $\displaystyle\int_0^{+\infty} \frac{\arctan \sqrt{x}}{x^\alpha}\,dx$ Studiare per $\alpha \in \mathbb{R}$ [Converge per $1 < \alpha < \frac{3}{2}$]

28) $\displaystyle\int_0^{1} \frac{\ln x}{|x-1|^\alpha}\,dx$ Studiare per $\alpha \in \mathbb{R}$, calcolare per $\alpha = \frac{3}{2}$ [Converge per $\alpha < 2$; per $\alpha = \frac{3}{2}$ vale $-4\ln 2$]

29) $\displaystyle\int_1^{2} \frac{\ln(x^\alpha)}{(x-1)^\alpha}\,dx$ Studiare per $\alpha \in \mathbb{R}$, calcolare per $\alpha = \frac{3}{2}$ [Converge per $\alpha < 2$; per $\alpha = \frac{3}{2}$ vale $-3\ln 2 + \frac{3}{2}\pi$]

30) $\displaystyle\int_0^{+\infty} \frac{1}{4x^{\frac{\alpha}{4}} + x^{\frac{5}{\alpha}}}\,dx$ Studiare per $\alpha > 0$, calcolare per $\alpha = 2$. [Converge per $0 < \alpha < 4$ e per $\alpha > 5$; per $\alpha = 2$ vale $\frac{\pi}{4}$]

31) $\displaystyle\int_{-\infty}^{+\infty} \frac{\sin x - \arctan x}{|x|^\alpha}\,dx$ Studiare per $\alpha > 0$, calcolare per $\alpha = \frac{7}{2}$. [Converge per $1 < \alpha < 4$; per $\alpha = \frac{7}{2}$ vale 0]

32) $\displaystyle\int_{e^e}^{+\infty} \frac{1}{x^\alpha(\ln x)^\beta(\ln(\ln x))^\gamma}\,dx$ Studiare per $\alpha, \beta, \gamma \in \mathbb{R}$, calcolare per $\boxed{\alpha = \beta = 1, \gamma = 2}$

[Converge nei seguenti 3 casi $\begin{cases} 1)\ \alpha > 1,\ \beta \text{ e } \gamma \text{ qualsiasi} \\ 2)\ \alpha = 1,\ \beta > 1,\ \gamma \text{ qualsiasi} \\ 3)\ \alpha = \beta = 1,\ \gamma > 1 \end{cases}$ NEL CASO RICHIESTO VALE 1

33) $\displaystyle\int_{2\alpha}^{+\infty} \left(1 - \frac{\alpha}{x}\right)^{x \ln x}\,dx$ Studiare per $\alpha > 0$. [Converge per $\alpha > 1$]

34) $\displaystyle\int_1^{+\infty} \frac{\sin x}{x^\alpha}\,dx$ Studiare per $\alpha > 0$. [Converge per $\alpha > 0$]

35) $\displaystyle\int_0^{+\infty} \sin(x^\alpha)\,dx$ Studiare per $\alpha \in \mathbb{R}$. [Converge per $|\alpha| > 1$]

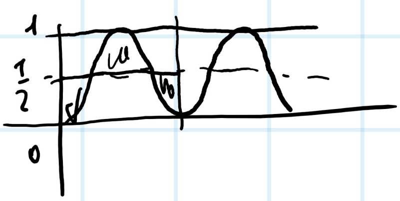
36) $\displaystyle\int_0^{+\infty} \left|\sin \sqrt{x}\right|^{x^\alpha}\,dx$ Studiare per $\alpha > 0$ [Converge per $\alpha > 2$]

ES. 1

$$\int_0^{+\infty} \frac{\arctan(\sin x)}{\ln(e + x + \sin x)}\, dx \quad \text{CONVERGE}$$

ES. 2

$$\int_\pi^{+\infty} \frac{\ln\left(1 + \frac{1}{2}\sin x\right)}{x}\, dx \quad \text{DIVERGE A } -\infty$$



1

$$\int_0^{+\infty} \frac{\arctan(\sin x)}{\ln(e+x+\sin x)}\,dx = \int_0^{+\infty} \arctan(\sin x)\cdot\boxed{\frac{1}{\ln(e+x+\sin x)}}\,dx$$

$\frac{11}{20}$

$$\int_{-\pi}^{\pi} \arctan(\sin x)\,dx = 0$$

$(x+\sin x)' =$

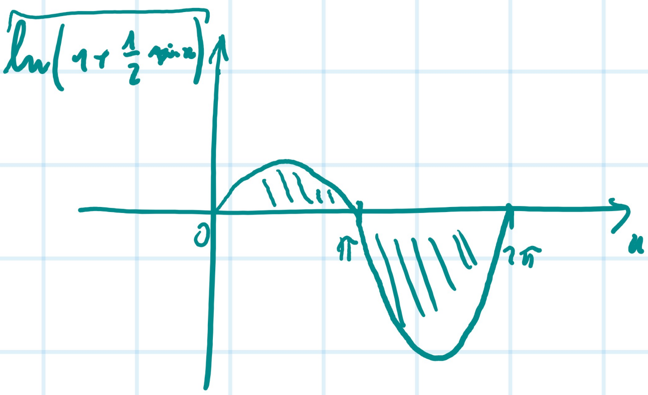
$= 1+\cos x \geq 0$

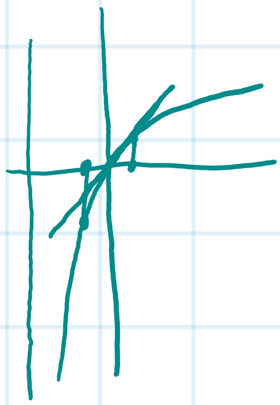
$$\rightarrow F(x) = \int_0^x \arctan(\sin t)\,dt$$

$$\int_{\pi}^{+\infty} \frac{\ln\left(1+\frac{1}{2}\sin x\right)}{x}\,dx$$

$\arctan \boxed{x}$ $\sin x$

$\ln(1+x)$





$x = \boxed{t+\pi}$

$$\boxed{\int_0^{2\pi} \ln\left(1+\frac{1}{2}\sin x\right)dx} = \int_0^{\pi} \boxed{} + \int_{\pi}^{2\pi} \ln\left(1+\frac{1}{2}\sin x\right)dx =$$

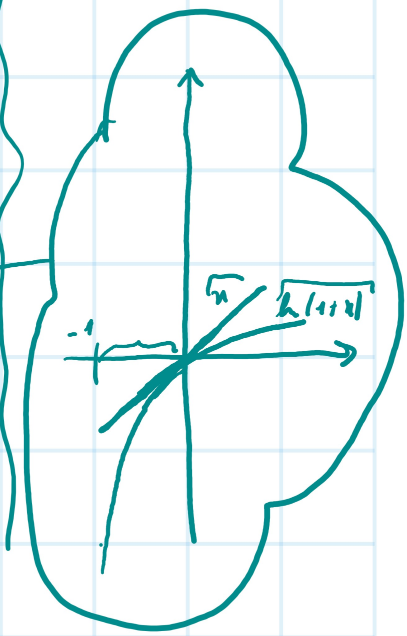
$$= \int_0^{\pi} \boxed{} + \int_0^{\pi} \ln\left(1+\frac{1}{2}\sin(t+\pi)\right)dt =$$

$$= \int_0^\pi \underbrace{\ln\left(1+\frac{1}{2}\sin x\right)}\,dx + \int_0^\pi \underbrace{\ln\left(1-\frac{1}{2}\sin x\right)}\,dx =$$

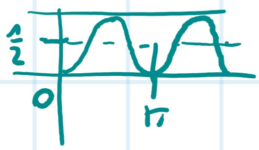
$$= \underbrace{\int_0^\pi \ln\left(1 - \underbrace{\frac{1}{4}\sin^2 x}\right) dx} \le \int_0^\pi - \underbrace{\frac{1}{4}\sin^2 x}\, dx =$$

$$= -\frac{1}{4} \cdot \frac{\pi}{2} = -\frac{\pi}{8}$$

$\ln(1+x) < x$
PER $-1<x<0$



$$\int_0^{2\pi} \ln\left(1+\frac{1}{2}\sin x\right) dx = C < 0$$



$$\int_\pi^{+\infty} \frac{\ln\left(1+\frac{1}{2}\sin x\right) - C + C}{x}\, dx =$$

Periodica a Media nulla

$$= \underbrace{\int_\pi^{+\infty} \frac{\ln\left(1+\frac{1}{2}\sin x\right) - C}{x} + \boxed{\frac{C}{x}}\, dx}$$

$$\int_\pi^{+\infty} \frac{C}{x}\, dx = -\infty$$

Periodica

$$\underbrace{\int \boxed{f(x)}\, \underbrace{g(x)}_{\searrow 0 \text{ Abel}}\, dx}$$

$$\int \boxed{C}\, g(x)\, dx$$

$$\int_0^{+\infty} \frac{\arctan(\sin x)}{\ln(e + x + \frac{11}{10}\sin x)} - \frac{\arctan(\sin x)}{\ln(e+x+\sin x)}\, dx$$

$$\int_2^{+\infty} \square \frac{-\ln\left(\frac{e + x + \sin x + \frac{1}{10}\sin x}{e+x+\sin x}\right)}{\ln(\square)\cdot\ln(\square)} =$$

$$-\ln\left(1 + \frac{\frac{1}{10}\sin x}{x}\right) \leq \frac{e}{x}$$

$$= \int_0^{+\infty} \arctan(\sin x)$$

$$|\,\circ\,| \leq \frac{\frac{e}{x}}{\ln^2 x} \leq \frac{e}{x\ln^2 x}$$

$\int_2^{+\infty} \square$ conv.

$$\int_0^{+\infty} \left(\frac{\alpha + \sin x}{17 + \cos x}\right)^x dx$$

$\alpha + \sin x \leq 17 + \cos x$

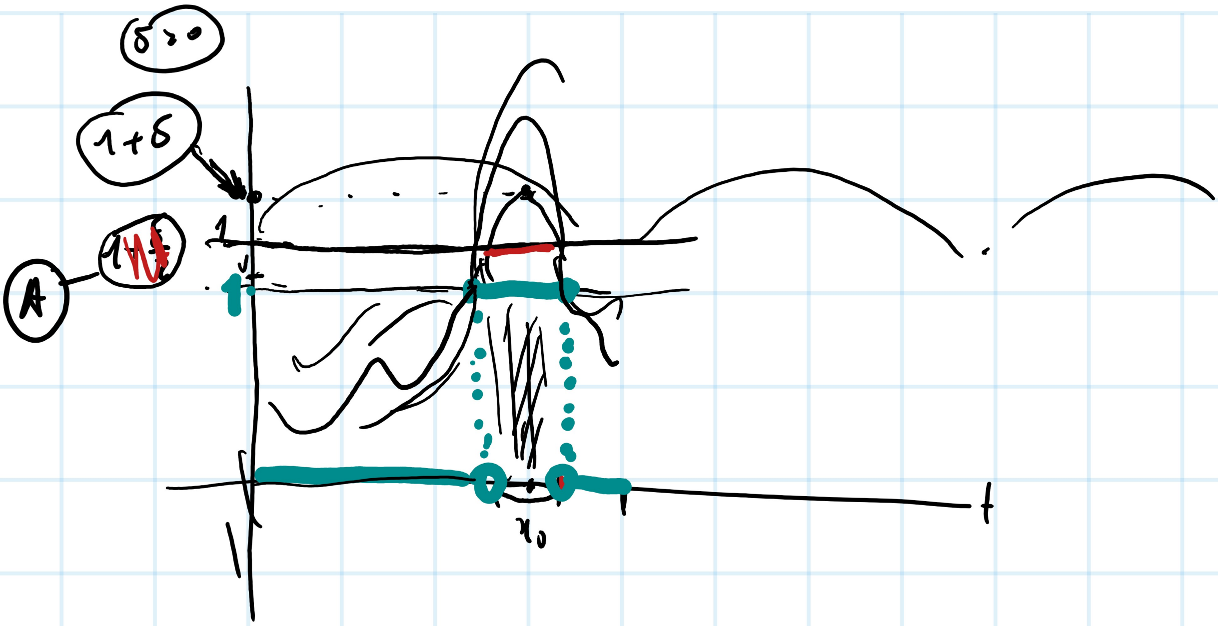
$\boxed{a < 1}$

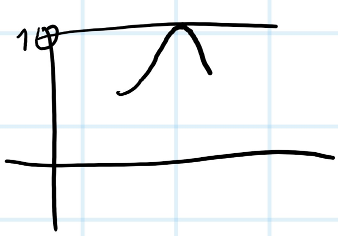
$$\left(\frac{15 + \sin x}{17 + \cos x}\right)^x \leq a^x \quad (< 1)$$

$\boxed{15 + \sin x < 17 + \cos x}$

$14 \leq 15 + \sin x \leq 16$

$16 \leq 17 + \cos x \leq 18$





$\left|\frac{3+\sin x}{4}\right|^{x^3}$

$a<1$

$\max\left(\frac{\sqrt{2}+\sin x}{17+\cos x}\right)=1$

$\alpha+\sin x \le 17+\cos x$

SE $\alpha = \boxed{17-\sqrt{2}}$

$\sin x - \cos x \le 17-\alpha$

$\sin\frac{\pi}{4}$

$\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x \le \frac{17-\alpha}{\sqrt{2}}$

$\cos\frac{\pi}{4}$

$-\cos\left(x+\frac{\pi}{4}\right) = \sin\frac{\pi}{4}\sin x - \cos\frac{\pi}{4}\cos x \le \frac{17-\alpha}{\sqrt{2}}$

$-\sqrt{2}$ $\quad \sqrt{2}$ $\qquad \sqrt{2}$

$$\underbrace{-\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)} \leq \boxed{17-\alpha}$$

$17-\alpha=\sqrt{2}$

$\alpha = 17-\sqrt{2}$

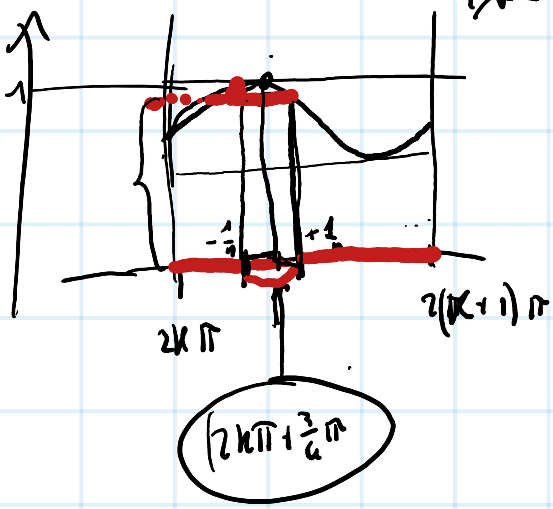
$$x+\frac{\pi}{4}=\pi+2k\pi$$

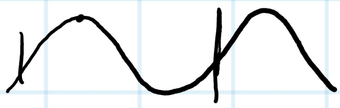
$$\int_0^{+\infty}\left(\frac{17-\sqrt{2}+\sin x}{17+\cos x}\right)^x dx \qquad \underline{\text{diverge}}\ (?)$$

$$17-\sqrt{2}+\sin x = 17+\cos x$$

$$\cos x-\sin x=-\sqrt{2}$$

$$\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x=-1$$





$$\cos\left(x+\frac{\pi}{4}\right)=-1$$

$$(b(k))^{k}\to 1\,??$$

$k\to+\infty$

$$x+\frac{\pi}{4}=\pi+2k\pi$$

$$\boxed{x=\frac{3}{4}\pi}$$

$$b(x)=\frac{17-\sqrt{2}+\sin x}{17+\cos x}$$

$$\boxed{\frac{1}{k}\cdot\left(b\left(2k\pi+\frac{3}{4}\pi-\frac{1}{k}\right)\right)^{2(k+1)\pi}} \approx$$

$$\frac{1}{k}\cdot\left(\left(\frac{17-\sqrt{2}+\sin\left(\cancel{2k\pi}+\frac{3}{4}\pi-\frac{1}{k}\right)}{17+\cos\left(\cancel{2k\pi}+\frac{3}{4}\pi-\frac{1}{k}\right)}\right)^{k+1}\right)^{2\pi} =$$

$$=\frac{1}{k}\left(\frac{17-\sqrt{2}+\sin\left(\frac{3}{4}\pi-\frac{1}{k}\right)-\cos\left(\frac{3}{4}\pi-\frac{1}{k}\right)+\cos\left(\frac{3}{4}\pi-\frac{1}{k}\right)}{\qquad}\right)^{x}$$

$$\frac{c}{k^2}$$

$$\boxed{\frac{1}{k}} \left(1 - \boxed{\frac{-\sin\left(\frac{3\pi}{4} - \frac{1}{n}\right) + \cos\left(\frac{3}{4}\pi - \frac{1}{k}\right) + \sqrt{2}}{\quad}} \right)^k$$

$$2$$

$$\frac{1}{n} \cdot \left(1 - \frac{c}{k^2} \right)^k$$