LEZ 16 - INTRODUZIONE A $\mathbb{C}$ (18/04/2023)

1) POTENZE DI NUMERI COMPLESSI
2) RADICI DI NUMERI COMPLESSI
3) ESEMPI + ESERCIZI

[...]

POTENZE IN $\mathbb{C}$

$$z = a + ib = \underline{\rho(\cos\theta + i\sin\theta)}$$

$\theta = \arg(z)$

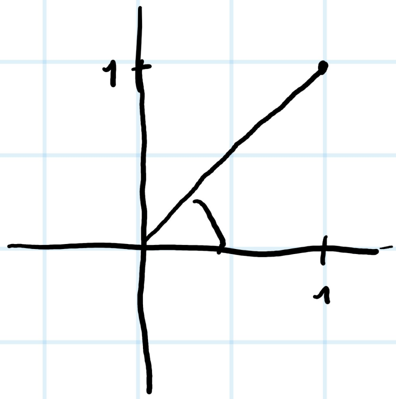
$\rho = |z|$

$$\arg(z^n) = n\theta \quad (\mathrm{MOD}\ 2\pi)$$

$$|z^n| = \rho^n$$

Es.

$$\underbrace{(1+i)}_{\substack{\parallel \\ z}}{}^{200}$$



$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\arg(z) = \frac{\pi}{4}$$

$$200 \cdot \frac{\pi}{4} = 50\pi \equiv 0$$

$$|z^{200}| = (\sqrt{2})^{200} = 2^{100}$$

$$\arg(z^{200}) = 0$$

$$(1+i)^{200} = 2^{100}$$

DEF $\omega \in \mathbb{C}$ È RADICE n-ESIMA DI $z \in \mathbb{C}$ se

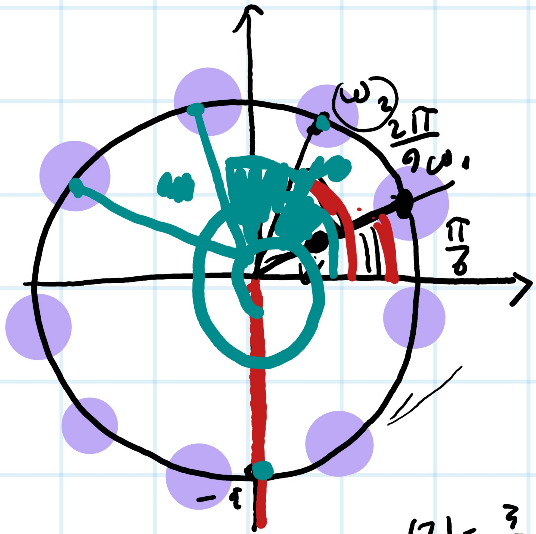
$$\omega^n = z$$

ES. $\sqrt[9]{z}$

$z = -i$

$\omega = ?$

$\omega^9 = z$



$\arg(z) = \frac{3}{2}\pi$

$1 = \rho = |\omega|$ $\quad\quad \rho^n = |z|$

$\frac{\pi}{6} = \theta = \arg(\omega)$ $\quad\quad n\theta \equiv \arg(z) \pmod{2\pi}$

$|\omega^n| = \rho^n$

$\arg(\omega^n) = n\theta \pmod{2\pi}$

ω_1 t.c. $\arg(\omega_1) = \frac{\pi}{6}$ $\quad |\omega| = 1$

ω_2 t.c. $\arg(\omega_2) = \left(\frac{\pi}{6} + \frac{2\pi}{9}\right)$

$$9\left(\frac{\pi}{6} + \frac{2\pi}{9}\right)$$

$$9\left(\frac{\pi}{6} + \frac{2\pi}{9}\right) = \boxed{\frac{3}{2}\pi + 2\pi}$$

$$\arg(\omega_k) = \frac{\pi}{6} + k \cdot \frac{2\pi}{9}$$

$$9 \cdot \bullet = \frac{3\pi}{2} + 2k\pi \equiv \frac{3}{2}\pi$$

$$\begin{cases} \omega_0 \\ \omega_1 \\ \vdots \\ \omega_8 \end{cases} \qquad \arg(\omega_k) = \frac{\pi}{6} + k \cdot \frac{2\pi}{9} \leftarrow$$

$$|\omega_k| = 1$$

PROP. 1 DATO $z \in \mathbb{C} - \{0\}$ E $n \in \mathbb{N} - \{0,1\}$ ALLORA z HA n RADICI n-ESIME DATE DA:

$\omega_0, \dots, \omega_k, \dots \omega_n$ TALI CHE

$$\arg(\omega_n) = \frac{1}{n}\arg(z) + k \cdot \frac{2\pi}{n}$$

$$|\omega_n| = \sqrt[n]{|z|}$$

DIM

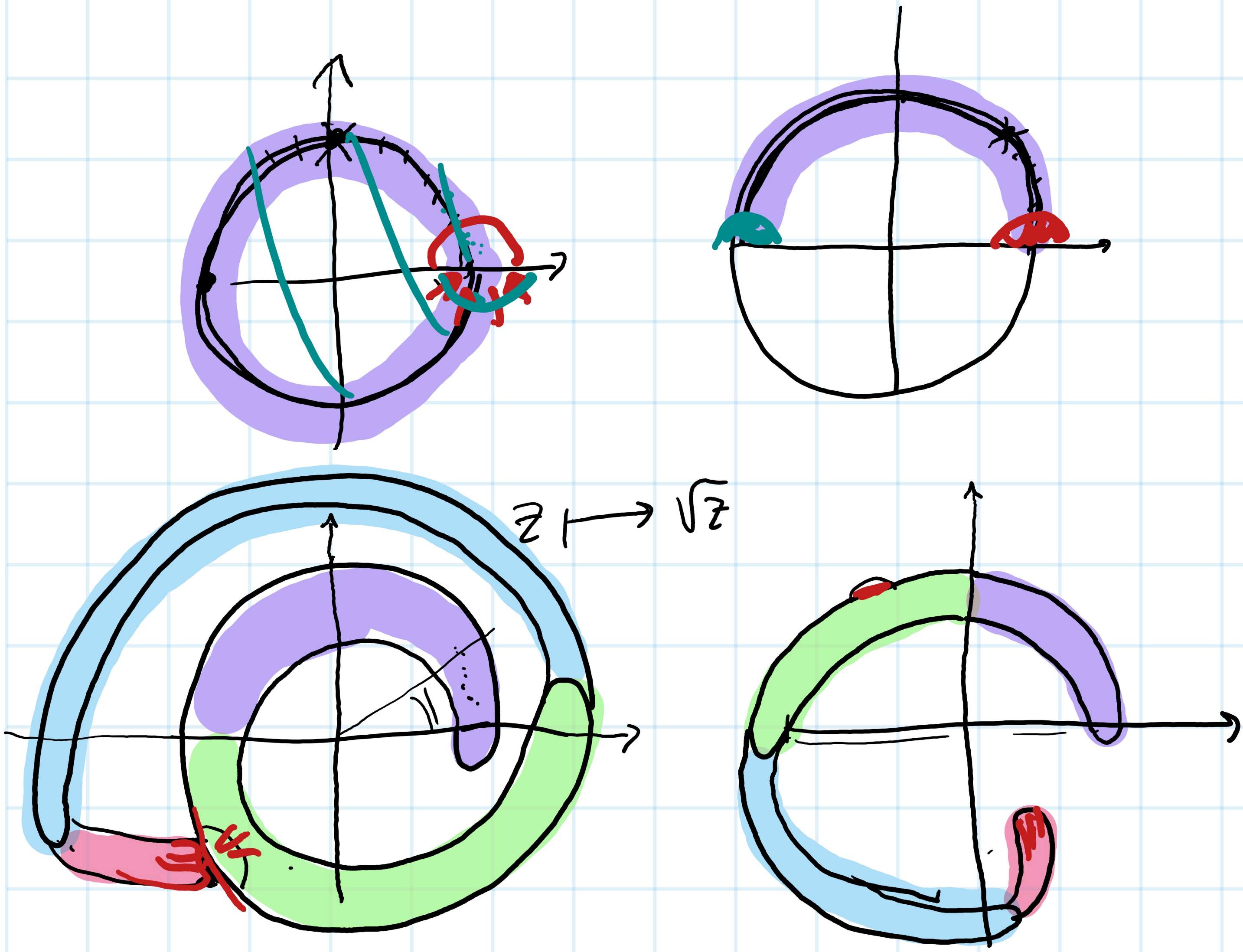
$$|\omega_k^n| \overset{\left(\sqrt[n]{|z|}\right)^n = |z|}{=} |z| \qquad \cancel{?}\ \text{SI}$$

$$\arg(\omega_k^n) \equiv \arg(z) \qquad ?$$

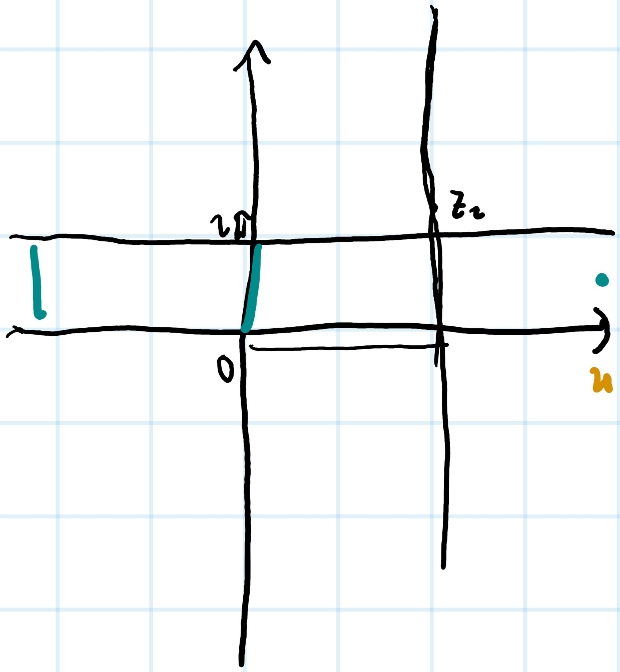
$$\|$$

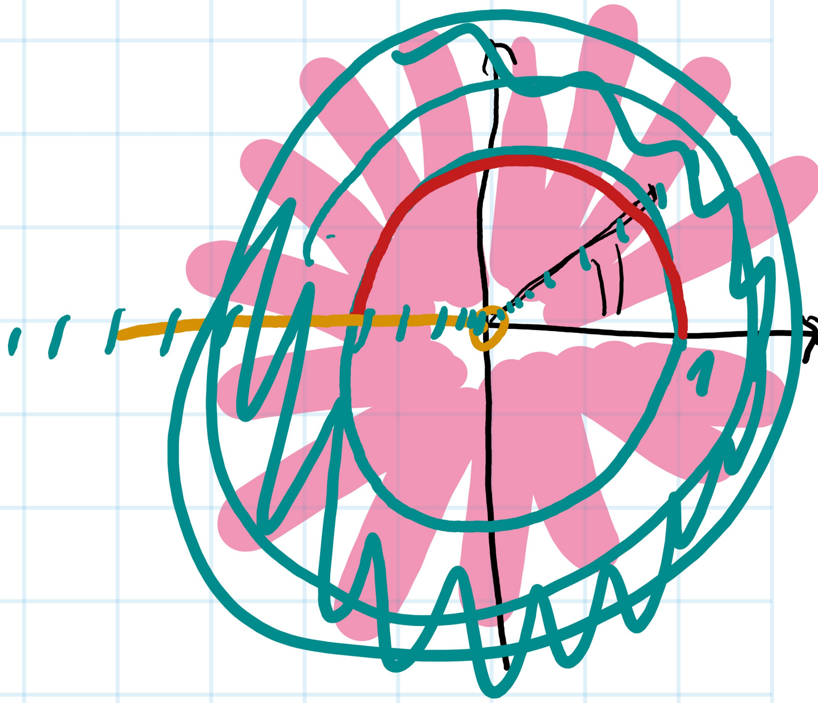
$$n \cdot \arg(w_n) \quad \text{MOD } 2\pi$$

$$\| \quad \cancel{n} \cdot \left(\frac{1}{\cancel{n}} \arg(z) + k \cdot \frac{2\pi}{\cancel{n}} \right) = \arg z + 2k\pi \equiv \arg(z) \quad (\text{MOD } 2\pi)$$



$$\overset{a+ib}{\overset{\parallel}{z}} \longmapsto e^{z} = e^{a}\cdot(\cos b + i \sin b)$$





$z_1 = a + ib$

$z_2 = a + i(b + 2\pi)$

$e^{z_1} = \boxed{e^{a}}(\underline{\cos b} + i \underline{\sin b})$

$e^{z_2} = e^{a}(\cos\underline{(b+2\pi)} + i \sin\underline{(b+2\pi)})$

3. Dato il numero complesso $\boxed{\left(\dfrac{500+250i}{11+2i}\right)^4}$ determinarne:

(**a**) tutte le radici quarte in forma cartesiana;

(**b**) tutte le radici ottave in forma cartesiana;

(**c**) la somma di tutte le radici sedicesime.

1 RISOLVERE IN $\mathbb{C}$ LE SEGUENTI EQUAZIONI: **a** $z^{10}\cdot\bar{z}^{8}=512i$

b $z^4=\dfrac{(i-\sqrt{3})^{16}}{(1+i)^{28}}$

$$z^4=\left(\frac{(i-\sqrt{3})^4}{(1+i)^7}\right)^4$$

$$\boxed{z=\left(\frac{500+250i}{11+2i}\right)^4}$$

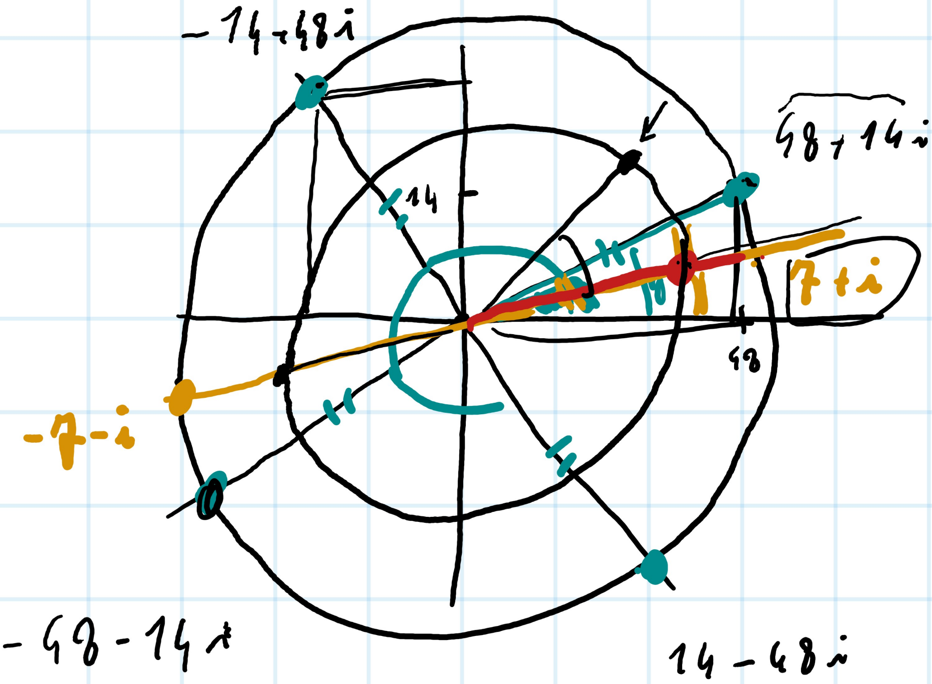
$$w=\boxed{\frac{500+250i}{11+2i}}=\frac{(500+250i)(11-2i)}{(11+2i)(11-2i)}=$$

$$=\frac{}{11^2-(2i)^2}=\frac{}{125}=$$

$$=\frac{(500+250i)(11-2i)}{125}=$$

$$=(4+2i)(11-2i)=44+4+(2\cdot 11-2\cdot 4)i$$

$$=\boxed{48+14i}$$



49

$(25-1)^2 = 625-49$

$\sqrt{48^2+14^2} = \sqrt{(24^2+7^2)2^2} =$

$= \sqrt{25^2 \cdot 2^2} = \sqrt{50^2}$

$$\cos\theta = \frac{48}{50} = \frac{24}{25}$$

$$\sin\theta = \frac{14}{50} = \frac{7}{25}$$

$$\theta = \arctan\left(\frac{7}{24}\right)$$

$$\frac{1}{2}\arctan\left(\frac{7}{24}\right)$$

$$(a+bi)^2 = 48+14i \qquad (7+i)^2 = \boxed{48+14i}$$

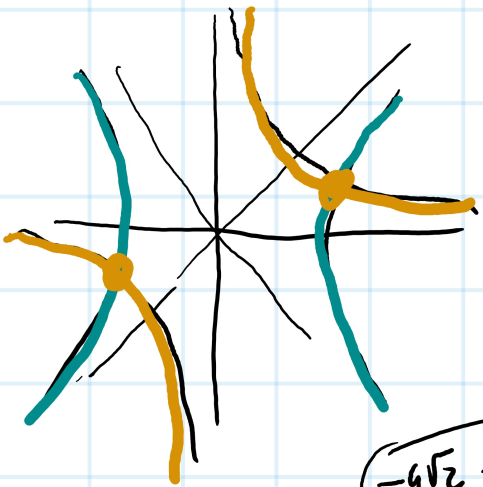
$$a^2-b^2+2abi = 48+14i \qquad (-7-i)^2 =$$

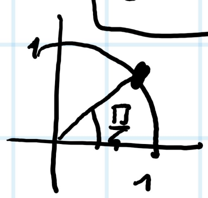
$$\begin{cases} a^2-b^2 = 48 \\ 2ab = 14 \end{cases}$$

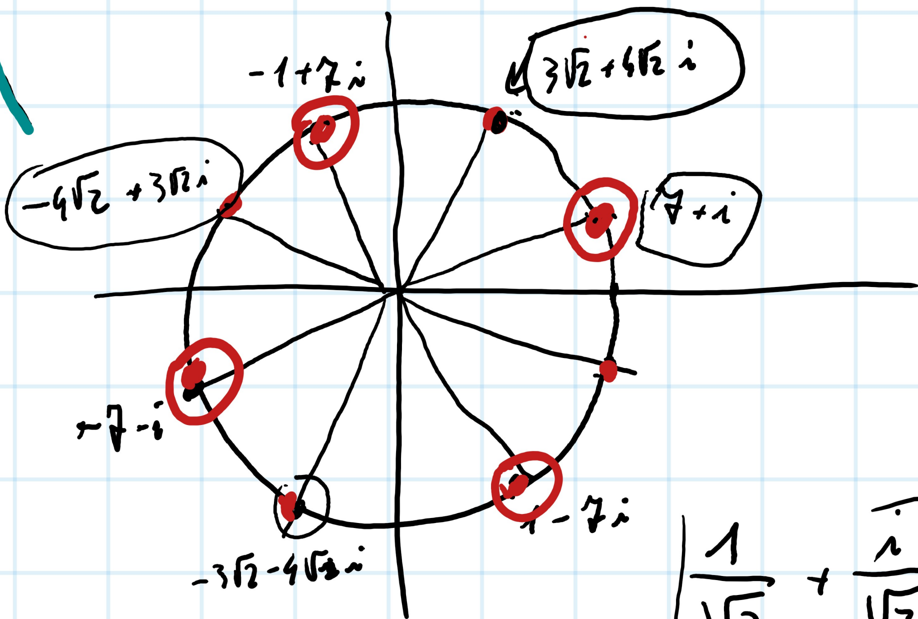
$$\boxed{\begin{aligned} a &= 7 \\ b &= 1 \end{aligned}}$$

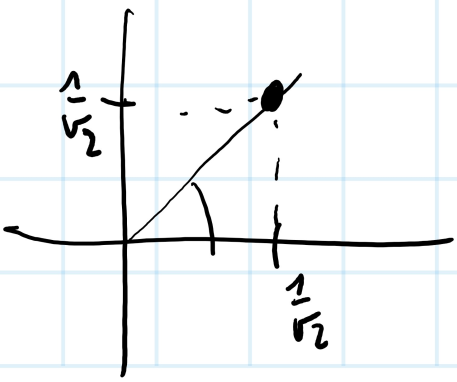
$$\boxed{\begin{aligned} a &= -7 \\ b &= -1 \end{aligned}}$$

$$\boxed{\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}}$$









$$\boxed{\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}}$$

$$(7+i)\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) =$$

$$\frac{1}{\sqrt{2}}\Big((7+i)(1+i)\Big) =$$

$$= \frac{1}{\sqrt{2}}(6+8i) = \boxed{3\sqrt{2} + 4\sqrt{2}\cdot i}$$

$$z_0 = \left(\quad\right)^4$$

$$(z-w_0)(z-w_1)\cdots(z-w_{15}) = 0$$

$$\boxed{z^{16} - z_0} = 0$$

$$\underbrace{(z-z_0)(z-z_1)(z-z_2)} = 0$$

$$z^3 - \underbrace{(z_0+z_1+z_2)}_{-\text{Summe}} z^2 + (z_0z_1+z_1z_2+z_0z_2)z - \overbrace{z_1 \cdot z_2 z_2}^{\text{Prod}} = 0$$

$$x^2 + \underset{-\text{Summe}}{a}\,x + \underset{\text{Produkt}}{b} = 0$$

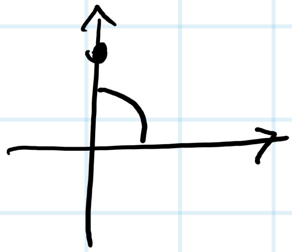
$$\boxed{z^{10} \cdot \bar{z}^{8}} = \boxed{512\, i}$$

ρ^{18} $\quad$ 512

512

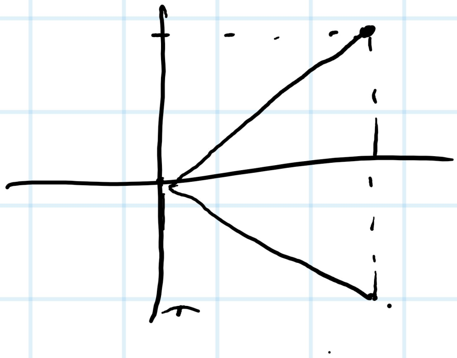
$|z| = \sqrt{2}$

$|z| = \rho = ?$



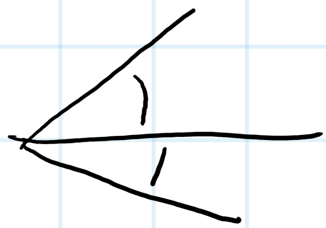
$|\bar{z}| = |z| = \rho$

$\rho^{18} = 512 = 2^{9}$



$$\boxed{\rho = \sqrt{2}}$$

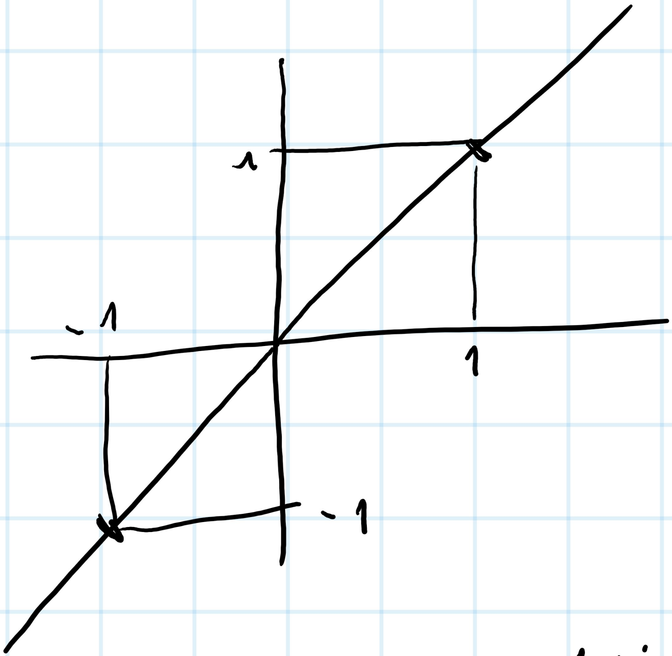
$$\arg(z \cdot \bar{z}) = 0$$



$$\arg(z^{10} \cdot \bar{z}^{8}) = \arg(z^{2} \cdot \underbrace{(z \cdot \bar{z})^{8}})$$

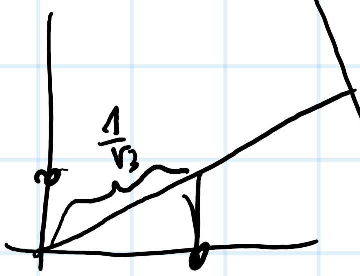
$$\arg(z^{2}) = \frac{\pi}{2}$$

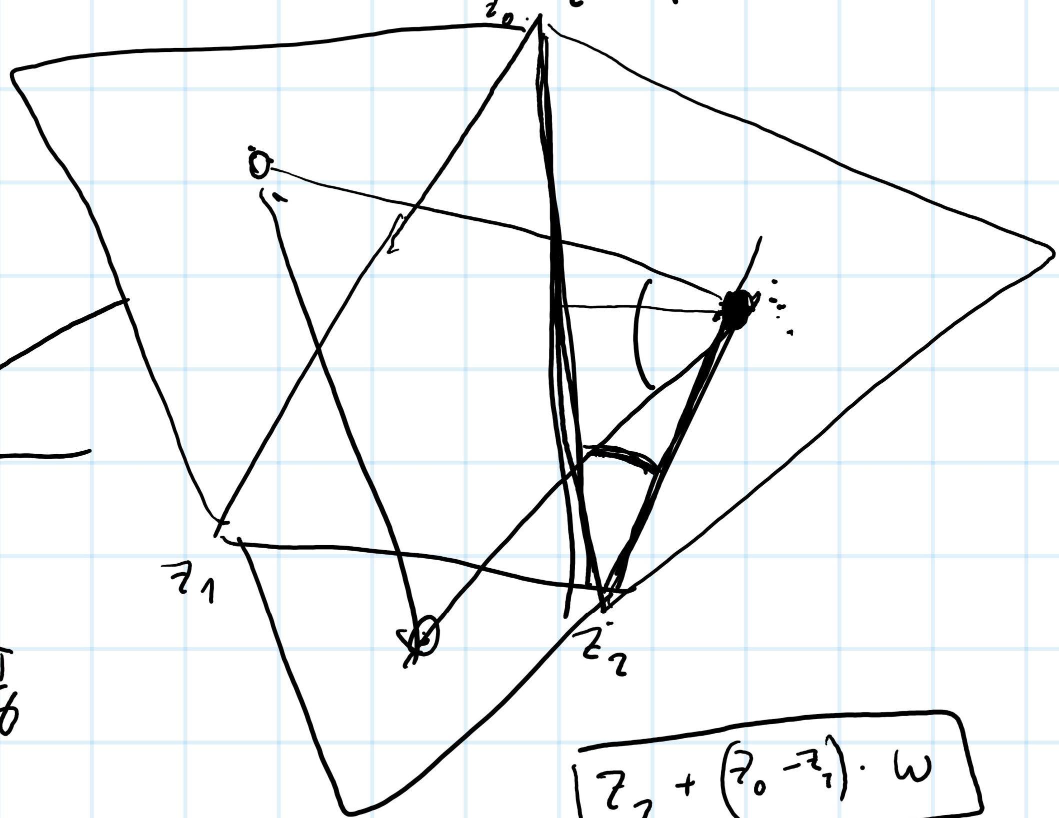
$\arg(z) = \frac{\pi}{4}$ $\qquad$ $\arg(z) = \frac{5}{4}\pi$



$z = 1 + i$

$z = -1 - i$





w

$-\frac{\pi}{6}$

$w =$

$\boxed{z_2 + (z_0 - z_1) \cdot w}$