Lezione 29: Polinomio di Taylor (III)

66 $$\lim_{x\to 0} \frac{3\sin x - \sqrt{3}\sin\left(x\sqrt{3}\right)}{\arctan x - \arctan 2x + x}$$

62 $$\lim_{x\to 0} \frac{(x-\arctan x)\left(\ln\left(1+x^2\right) - x\arctan x\right) + \frac{x^7}{18}}{x^3 - \sin x^3}$$

72 $$\lim_{x\to 0} \frac{(x\cos x - \sin x)\left(x^2 - \sin x^2\right) + \frac{1}{18}x^9}{x^{11}}$$

73 $$\lim_{x\to 0} \frac{(\arctan x - x\cos x)\left(\sqrt{1+x^4} - 1\right) - \frac{1}{12}x^7}{x^9}$$

74 $$\lim_{x\to +\infty} \frac{x^4\sin\frac{1}{x^3} - 2x^3\left(1-\cos\frac{1}{x}\right)}{xe^{\frac{1}{x^2}} - x - \ln(1+x) + \ln x}$$

75 $$\lim_{x\to +\infty} \left(\frac{x^3}{x+2}e^{\frac{x}{x^2+2}} - x^2 + x\right)$$

79 $$\lim_{x\to 0^+} \frac{e^{-\frac{1}{x^2}} + 2 - \cos(x-\sin x) - \sqrt[72]{1+x^6}}{x^\alpha}$$

81 $$\lim_{x\to 0^+} \frac{x^x - \cos x - x\ln x}{x^\alpha \cdot |\ln x|^\beta}$$

84 $f(x) = \ln(\cos x)$, con $n=6$. ← CALCOLARE $f^{(n)}(0)$

93 $f(x) = \arctan(e^x - 1) - e^{\arctan x} + 1 + \frac{x^4}{6}$ ← (STABILIRE LA NATURA DI $x=0$)

SOLUZIONI

66 $$\lim_{x\to 0} \frac{3\sin x - \sqrt{3}\sin\left(x\sqrt{3}\right)}{\arctan x - \arctan 2x + x} =$$

$x + o(x) - [2x + o(x)] + x = o(x)$

$\arctan x = x - \frac{x^3}{3} + o(x^5)$

$\arctan(2x) = 2x - \frac{(2x)^3}{3} + o(x^5)$

$$= \lim_{x\to 0} \frac{3\left(x - \frac{x^3}{6} + o(x^5)\right) - \sqrt{3}\left(x\sqrt{3} - \frac{(x\sqrt{3})^3}{6} + o(x^5)\right)}{x - \frac{x^3}{3} + o(x^5) - \left(2x - \frac{(2x)^3}{3} + o(x^5)\right) + x} =$$

$$= \lim_{x\to 0} \frac{x^3 + o(x^5)}{\frac{7}{3}x^3 + o(x^5)} =$$

$$= \lim_{x\to 0} \frac{x^3}{\frac{7}{3}x^3} = \frac{3}{7}$$

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62 $$\lim_{x\to 0} \frac{(x - \arctan x)\cdot\left(\ln\left(1+x^2\right) - x\arctan x\right) + \frac{x^7}{18}}{x^3 - \sin x^3}$$

$x^3 - \sin x^3 \sim \frac{x^9}{6}$

$x^3 - \sin x^3 = x^3 - \left(x^3 - \frac{(x^3)^3}{6} + o(x^{15})\right) = \frac{x^9}{6} + o(x^{15})$

$$x - \operatorname{arctan} x = \cancel{x} - \left(\cancel{x} - \frac{x^3}{3} + \frac{x^5}{5} + o(x^7)\right) = \boxed{\frac{x^3}{3} - \frac{x^5}{5} + o(x^7)}$$

$$\to \quad \ln(1+x^2) - x \operatorname{arctan} x = \cancel{x^2} - \frac{(x^2)^2}{2} + o(x^6) - x\left(\cancel{x} - \frac{x^3}{3} + o(x^5)\right) =$$

$$= \boxed{-\frac{x^4}{6} + o(x^6)}$$

N0

$$\lim_{x \to 0} \frac{\left(\frac{x^3}{3} - \frac{x^5}{5}\right) \cdot \left(-\frac{x^4}{6}\right) + \frac{x^7}{18}}{\frac{x^9}{6}} =$$

$$= \lim_{x \to 0} \frac{\cancel{-\frac{x^7}{18}} + \frac{x^9}{30} + \cancel{\frac{x^7}{18}}}{\frac{x^9}{6}} = \frac{1}{5}$$

N1

$$\lim_{x \to 0} \frac{\left(\frac{x^3}{3} - \frac{x^5}{5} + o(x^7)\right)\left(-\frac{x^4}{6} + \boxed{o(x^6)}\right) + \frac{x^7}{18}}{\frac{x^9}{6}} =$$

$$= \lim_{x \to 0} \frac{-\frac{x^7}{18} + \boxed{\frac{x^9}{30}} + \overbrace{o(x^{13}) + o(x^9) + o(x^{11})}^{o(x^9)} + o(x^{13}) + \frac{x^7}{18}}{\frac{x^9}{6}} =$$

S1

$$\ln(1+x^2) - x \operatorname{arctan} x = \cancel{x^2} - \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} + o(x^8) - x\left(\cancel{x} - \frac{x^3}{3} + \frac{x^5}{5} + o(x^6)\right)$$

$$= -\frac{1}{6}x^4 + \frac{2}{15}x^6 + o(x^8)$$

$$\lim_{x\to 0} \frac{\left(\frac{x^3}{3} - \frac{x^5}{5} + O(x^7)\right)\left(-\frac{1}{6}x^4 + \frac{2}{15}x^6 + O(x^8)\right) + \frac{x^7}{18}}{\frac{x^9}{6}}$$

$$= \lim_{x\to 0} \frac{-\frac{x^7}{18} + \frac{x^9}{30} + \frac{2}{45}x^9 + O(x^{11}) + \frac{x^7}{18}}{\frac{x^9}{6}} =$$

$$= \lim_{x\to 0} \frac{\frac{7}{90}x^9}{\frac{1}{6}x^9} = \frac{7}{15}$$

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72 $\displaystyle \lim_{x\to 0} \frac{(x\cos x - \sin x)\left(x^2 - \sin x^2\right) + \frac{1}{18}x^9}{x^{11}}$

$$x\cos x - \sin x = x\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + O(x^6)\right) - \left(x - \frac{x^3}{6} + \frac{x^5}{120} + O(x^7)\right) =$$

$$= -\frac{x^3}{3} + \frac{x^5}{30} + O(x^7) \leftarrow$$

$$x^2 - \sin x^2 = x^2 - \left(x^2 - \frac{x^6}{6} + O(x^{10})\right) =$$

$$= + \frac{x^6}{6} + O(x^{10}) \leftarrow$$

$$\lim_{x\to 0}\frac{\left(-\frac{x^3}{3}+\frac{x^5}{30}+o(x^7)\right)\left(\frac{x^6}{6}+o(x^{10})\right)+\frac{1}{18}x^9}{x^{11}}=$$

$$=\lim_{x\to 0}\frac{-\frac{1}{18}x^9+\frac{1}{180}x^{11}+o(x^{13})+\frac{1}{18}x^9}{x^{11}}=$$

$$=\lim_{x\to 0}\frac{\frac{1}{180}x^{11}}{x^{11}}=\frac{1}{180}$$

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74 $$\lim_{x\to+\infty}\frac{x^4\sin\frac{1}{x^3}-2x^3\left(1-\cos\frac{1}{x}\right)}{xe^{\frac{1}{x^2}}-x-\ln(1+x)+\ln x}=$$

$$=\lim_{x\to+\infty}\frac{x^4\sin\frac{1}{x^3}-2x^3\left(1-\cos\frac{1}{x}\right)}{x\left(e^{\frac{1}{x^2}}-1\right)-\ln\left(1+\frac{1}{x}\right)}=$$

$$=\lim_{x\to+\infty}\frac{x^3\sin\frac{1}{x^3}-2x^2\left(1-\cos\frac{1}{x}\right)}{\left(e^{\frac{1}{x^2}}-1\right)-\frac{1}{x}\ln\left(1+\frac{1}{x}\right)}\begin{matrix}\leftarrow N(x)\\ \leftarrow D(x)\end{matrix}$$

$$D(x)=1+\frac{1}{x^2}+o\left(\frac{1}{x^4}\right)-1-\frac{1}{x}\left(\frac{1}{x}-\frac{1}{2x^2}+o\left(\frac{1}{x^3}\right)\right)=$$

$$=\boxed{\frac{1}{2x^3}+o\left(\frac{1}{x^4}\right)}$$

$$\lim_{x\to+\infty} \frac{\frac{1}{12}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^4}\right)}{\frac{1}{2x^3}+o\left(\frac{1}{x^4}\right)} = +\infty$$

$$x^3 \sin\frac{1}{x^3} - 2x^2\left(1-\cos\frac{1}{x}\right) =$$

$$= x^3\cdot\left(\frac{1}{x^3}-\frac{1}{6}\frac{1}{x^9}+o\left(\frac{1}{x^{12}}\right)\right) - 2x^2\cdot\left(1-\left(1-\frac{\left(\frac{1}{x}\right)^2}{2}+\frac{\left(\frac{1}{x}\right)^4}{24}+o\left(\frac{1}{x^4}\right)\right)\right)$$

$$= 1-\frac{1}{6}\frac{1}{x^6}+o\left(\frac{1}{x^{12}}\right)-1+\frac{1}{12\cdot x^2}+o\left(\frac{1}{x^4}\right)$$

$$= \frac{1}{12}\cdot\frac{1}{x^2}+o\left(\frac{1}{x^4}\right)$$

$$o\left(\frac{1}{x}\right)$$

75 $\displaystyle\lim_{x\to+\infty}\left(\frac{x^3}{x+2}e^{\frac{x}{x^2+2}}-x^2+x\right) = \frac{x^3}{x+2}\cdot o\left(\frac{1}{x^3}\right)$

$$= \lim_{x\to+\infty}\left(\frac{x^3}{x+2}\cdot\left(1+\frac{x}{x^2+2}+\frac{1}{2}\left(\frac{x}{x^2+2}\right)^2+o\left(\frac{1}{x^3}\right)\right)-x^2+x\right) =$$

$$= \lim_{x\to+\infty}\left(\frac{x^3}{x+2}+\frac{x^4}{(x+2)(x^2+2)}+\frac{1}{2}\frac{x^5}{(x+2)(x^2+2)^2}-x^2+x+o\left(\frac{1}{x}\right)\right)$$

$$= \lim_{x \to +\infty} \left(\frac{x^3(x^2+2)^2 + x^4(x^2+2) + \frac{1}{2}x^9 + (x-x^2)(x+2)(x^2+2)^2}{(x+2)(x^2+2)^2} + O\left(\frac{1}{x}\right) \right) =$$

$$= \lim_{x \to +\infty} \frac{x^7 + 4x^9 + \dots + x^6 + \dots + \frac{1}{2}x^9 - x^7 - x^6 - 2x^9 + \dots}{x^5 + (\dots)} =$$

$$= \lim_{x \to +\infty} \frac{\frac{5}{2}x^9 + \dots}{x^9 + \dots} = \frac{5}{2}$$

$$(x+2)(x^2+2)^2 =$$

$$= (x+2)(x^4+4x^2+4) =$$

$$= x^5 + 2x^4 + 4x^3 + \dots$$

$$(x-x^2)(x^5+2x^4+4x^3+\dots)$$

$$-x^7 + x^6 - 2x^6 + 2x^5 - 4x^5 \dots$$

$$-x^7 - x^6 - 2x^5 + \dots$$

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79 $\displaystyle \lim_{x\to 0^+} \frac{e^{-\frac{1}{x^2}} + 2 - \cos(x - \sin x) - \sqrt[72]{1+x^6}}{x^\alpha} =$

$$\sqrt[72]{1+x^6} = 1 + \frac{1}{72}x^6 + O(x^{12})$$

$$= \lim_{x \to 0^+} \frac{e^{-\frac{1}{x^2}} + 2 - }{}$$

$$\cos(\overbrace{x - \sin x}^{f(x)}) =$$

$$\boxed{x - \sin x} = x - \left(x - \frac{x^3}{6} + \dots\right) = \boxed{\frac{x^3}{6}} + \dots$$

$$= 1 - \frac{1}{2}(x - \sin x)^2 + o(x^{12})$$

$$= 1 - \frac{1}{2}\left(x - \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^7)\right)\right)^2 + o(x^{12}) =$$

$$= 1 - \frac{1}{2}\left(\frac{x^3}{6} - \frac{x^5}{120} + o(x^7)\right)^2 + o(x^{12}) =$$

$$= 1 - \frac{1}{2}\left(\frac{x^6}{36} - o(x^{10}) - \frac{x^8}{360}\right) + o(x^{12}) =$$

$$= \boxed{1 - \frac{x^6}{72} + \frac{x^8}{720} + o(x^{10})}$$

$$\lim_{x \to 0^+} \frac{e^{-\frac{1}{x^2}} + 1 - \left(1 - \frac{x^6}{72} + \frac{x^8}{720} + o(x^{10})\right) - \left(1 - \frac{1}{72}x^6 + o(x^{12})\right)}{x^\alpha} =$$

$$= \lim_{x \to 0^+} \frac{\overbrace{\boxed{e^{-\frac{1}{x^2}}}}^{o(x^8)} - \frac{1}{720}x^8 + o(x^{10})}{x^\alpha} =$$

$$= \lim_{x \to 0^+} \frac{-\frac{1}{720}x^8 + o(x^8)}{x^\alpha} = \begin{cases} 0 & \alpha < 8 \\ -\frac{1}{720} & \alpha = 8 \\ -\infty & \alpha > 8 \end{cases}$$