Lez. 31, 32, 33: Esercizi su a(n+1)=f(a(n))

DESCRIVERE IL COMPORTAMENTO DELLE SUCCESSIONI DEFINITE DALLE SEGUENTI:

$\boxed{1}$
$$\begin{cases} a_{n+1} = \frac{2}{9}(a_n+1)^2 \\ a_0 = \alpha \in \mathbb{R} \end{cases}$$

$\boxed{2}$
$$\begin{cases} a_{n+1} = a_n^2 - a_n \\ a_0 = \alpha \in \mathbb{R} \end{cases}$$

$\boxed{3}$
$$\begin{cases} a_{n+1} = -\frac{4}{\pi}\arctan a_n \\ a_0 = \alpha \in \mathbb{R} \end{cases}$$

$\boxed{4}$
$$\begin{cases} a_{n+1} = \frac{2}{3}\left(a_n + \frac{1}{a_n}\right) + \frac{1}{3} \\ a_0 = \alpha \in \mathbb{R} - \{0\} \end{cases}$$

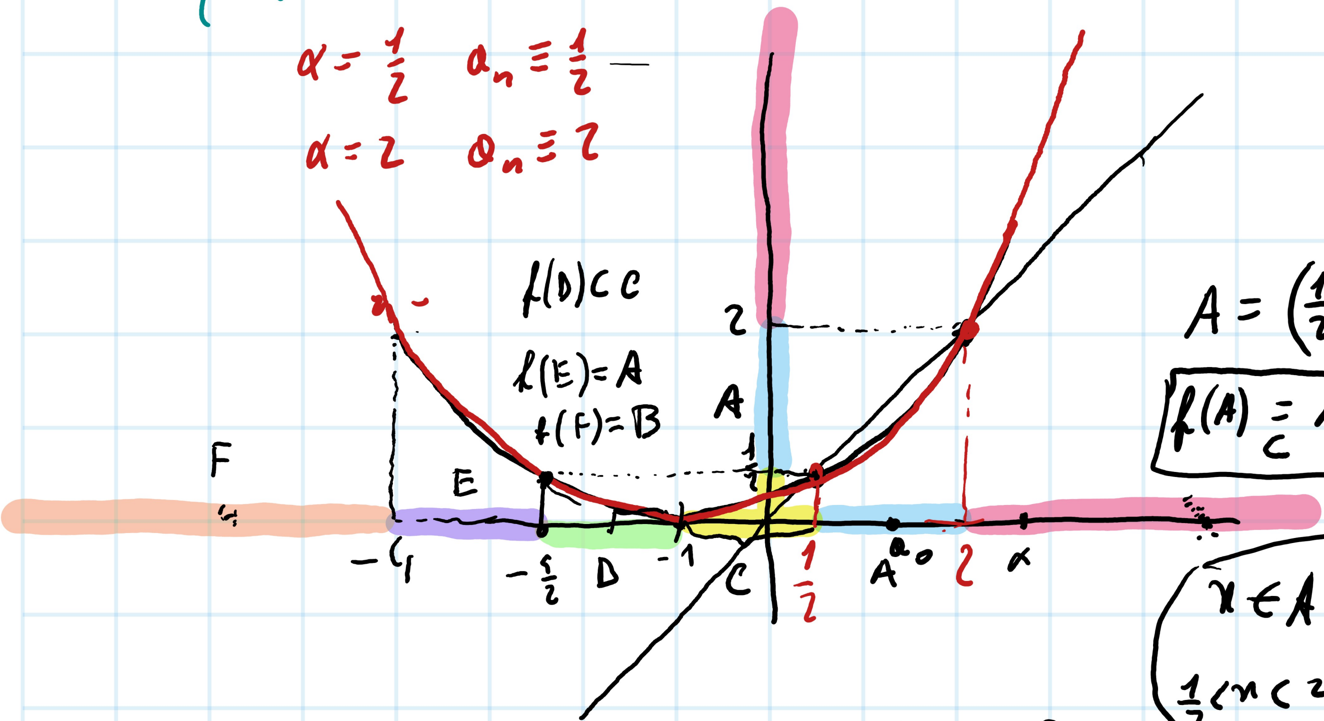
$\boxed{5}$
$$\begin{cases} a_{n+1} = \frac{2}{3}\left(a_n + \frac{1}{a_n}\right) - \frac{7}{6} \\ a_0 = \alpha \in \mathbb{R} - \{0\} \end{cases}$$

SOLUZIONI 18 DIC.

1 $\begin{cases} a_{n+1} = \frac{2}{9}(a_n+1)^2 \\ a_0 = \alpha \in \mathbb{R} \end{cases}$ $a_{n+1} = f(a_n)$ $\boxed{f(x) = \frac{2}{9}(x+1)^2}$

$\alpha = \frac{1}{2}$ $a_n \equiv \frac{1}{2}$

$\alpha = 2$ $a_n \equiv 2$



$A = \left(\frac{1}{2}, 2\right)$

$\boxed{f(A) \subseteq A}$

$\Uparrow$

$x \in A \Rightarrow f(x) \in A$

$\frac{1}{2} < x < 2$

$f\left(\frac{1}{2}\right) < f(x) < f(2)$

$\frac{1}{2} < f(x) < 2$

SU A 1) f CRESCENTE

2) $f(x) < x$

$\frac{2}{9}(x+1)^2 = x$

$2x^2 + 4x + 2 = 9x$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

$2x(x-2) - (x-2) = 0$

$(x-2)(2x-1) = 0$

$\boxed{\begin{array}{l} x = 2 \\ x = \frac{1}{2} \end{array}}$

$a_0 \in A$

$\boxed{\text{$a_n$ STA SEMPRE IN A E DECRESCE}}$

$a_1 = f(a_0) < a_0$

$a_1 < a_0 \Rightarrow f(a_1) < f(a_0)$

$a_2 < a_1$

$a_n < a_{n-1} \Rightarrow f(a_n) < f(a_{n-1}) \Rightarrow a_{n+1} < a_n$

$(a_n) \subset A \quad a_n$ DECRESCE

$\Downarrow$

(a_n) LIMITATA

$\Rightarrow a_n \to l$ finito

$$\left.\begin{array}{l} f(a_n) \to f(l) \\ \overset{\parallel}{a_{n+1}} \to l \end{array}\right\} \Rightarrow l = f(l)$$

$$\boxed{l = \frac{1}{2}}$$

$B = (2, +\infty) \qquad f(B) = B$

$$\left.\begin{array}{l} \to 1)\ f(x) \text{ CRESC.} \\ 2)\ \underline{f(x) > x} \end{array}\right\} \Rightarrow \underline{a_n \text{ CRESCENTE}}$$

$a_0 \in B$

$a_1 = f(a_0) > a_0$

$a_1 > a_0$

$f(a_1) > f(a_0)$

$a_2 > a_1$

$\vdots$

$a_n > a_{n-1} \Rightarrow f(a_n) > f(a_{n-1})$

$a_{n+1} > a_n$

$a_n \to l \in \mathbb{R} \cup \{+\infty\}$

P.A $l \in \mathbb{R} \qquad a_n \to l \Rightarrow$

$$\left.\begin{array}{l} a_{n+1} \to l \\ f(a_n) \to f(l) \end{array}\right\} \Rightarrow f(l) = l$$

$C = [-1, \frac{1}{2})$ $\qquad f(C) \subset C$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \| $

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad [0, \frac{1}{2})$

$$\left.\begin{array}{l} 1)\ f(x) \text{ CRESCENTE} \\ 2)\ f(x) > x \end{array}\right\} \Rightarrow a_n \text{ È CRESCENTE}$$

E LIMITATA PERCHÉ $a_n \in C$

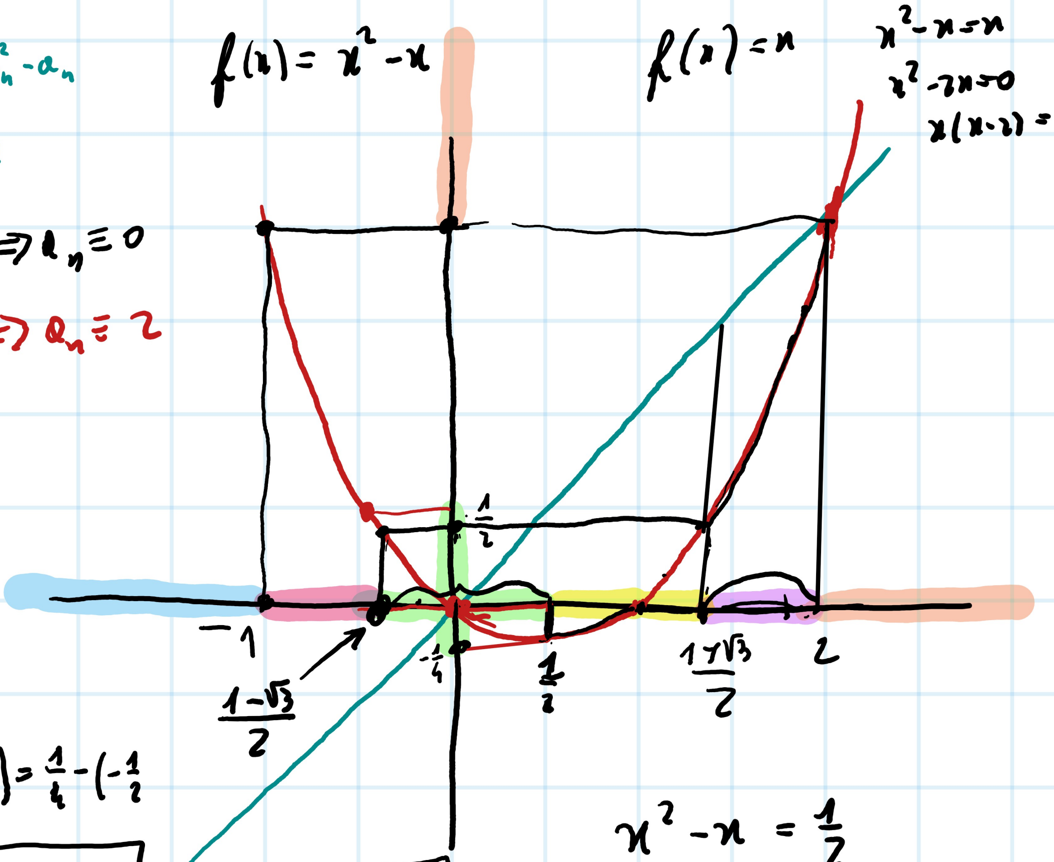
$a_n \to \ell$ finito

POICHÉ ℓ DEVE ESSERE P.T. FISSO DI $f(x)$ SI HA $\ell = \frac{1}{2}$

$\boxed{2}$ $\begin{cases} a_{n+1} = a_n^2 - a_n \\ a_0 = \alpha \in \mathbb{R} \end{cases}$ $\qquad f(x) = x^2 - x \qquad f(x) = x \qquad \begin{array}{l} x^2 - x = x \\ x^2 - 2x = 0 \\ x(x-2) = 0 \end{array}$

$\alpha = 0 \Rightarrow a_n \equiv 0$

$\alpha = 2 \Rightarrow a_n \equiv 2$



$f(x) = \frac{1}{2}$

$f(-\frac{1}{2}) = \frac{1}{4} - (-\frac{1}{2}$

$A = \overbrace{\left(\frac{1-\sqrt{3}}{2}, \frac{1}{2}\right)}$ $\qquad f(A) = \overbrace{\left[-\frac{1}{4}, \frac{1}{2}\right)}$

$x^2 - x = \frac{1}{2}$

$\overbrace{2x^2 - 2x - 1 = 0}$

$f(A)\subset A$

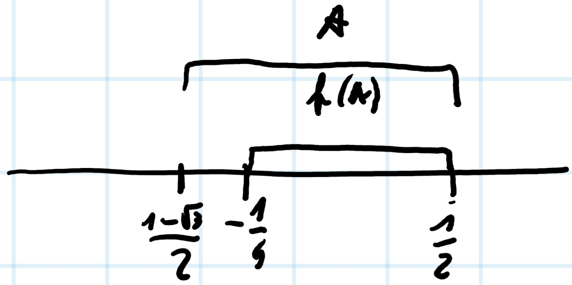
$$x = \frac{2\pm\sqrt{4+8}}{4} = \frac{2\pm 2\sqrt{3}}{4} = \boxed{\frac{1\pm\sqrt{3}}{2}}$$

$\frac{1-\sqrt{3}}{2} < -\frac{1}{4}$? SI

$2-2\sqrt{3} < -1$? SI

$3 < 2\sqrt{3}$ SI

$9 < 12$ SI



$A = \left(\frac{1-\sqrt{3}}{2}, \frac{1}{2}\right)$

$A = A_1 \cup A_2$ $\quad f(A_1)\subset A_2 \quad f(A_2)\subset A_1$

$A_1 = \left(\frac{1-\sqrt{3}}{2}, 0\right]$ $\quad A_2 = \left[0, \frac{1}{2}\right)$ ←

1) f DECR. SU A

$a_0 \in A_1$ $\quad a_{2n}\in A_1$, $\quad a_{2n+1}\in A_2$

$a_0\in A_2$ $\quad a_{2n}\in A_2$, $\quad a_{2n+1}\in A_1$

$a_0 \in A_1 \Rightarrow f(a_0) > f(0) = 0$

$\Downarrow$

$\boxed{a_0 < 0}$

$a_1 > 0$

$f(a_1) < f(0)$

$\boxed{a_2 < 0}$

$a_{2n} < 0$

$f(a_{2n}) > f(0) = 0$

$a_{2n+1} > 0$

$f(a_{2n+1}) < f(0) = 0$

$a_{2n+2} < 0$

$a_0 \in A_1 \Rightarrow a_{2k} \in A_1$

$a_{2k+1} \in A_2$

$$\begin{cases} a_{2k+2} = \overbrace{f(f}^{g}(a_{2k})) \\ a_0 = \alpha \end{cases}$$

A_1 INVARIANTE PER g

$$\left.\begin{matrix} g \text{ CRESCENTE} \\ g(x) > x \end{matrix}\right\} \Rightarrow a_{2k} \text{ CRESC.}$$

$$\begin{cases} a_{2k+3} = \overbrace{f(f}^{g}(a_{2k+1})) \\ a_1 \in \underline{A_2} \end{cases}$$

A_2

$$\left.\begin{matrix} g \text{ CRESC.} \\ \underline{g(x) < x} \end{matrix}\right\} \quad a_{2k+1} \text{ decrescente}$$

$g(x) = f(f(x)) =$

$= (x^2-x)^2 - (x^2-x) =$

$= x^4 - 2x^3 + \cancel{x^2} - \cancel{x^2} + x =$

$= \boxed{x^4 - 2x^3 + x}$

$\boxed{g(x) > x}$

$g(x) < x$

$x^4 - 2x^3 + x \overset{>}{?} x$

$x^4 - 2x^3 \; ? \; 0$

$x^3(x-2) > 0$

$\left(\frac{1-\sqrt{5}}{2}, 0\right)$

$\underline{\left(0, \frac{1}{2}\right)}$

$x^4 - 2x^3 + x \; ? \; x$

$x^4 - 2x^3 \; ? \; 0$

$x^3(x-2) < 0$

$\boxed{\text{LEZ. 37 — 20 DIC}}$

3

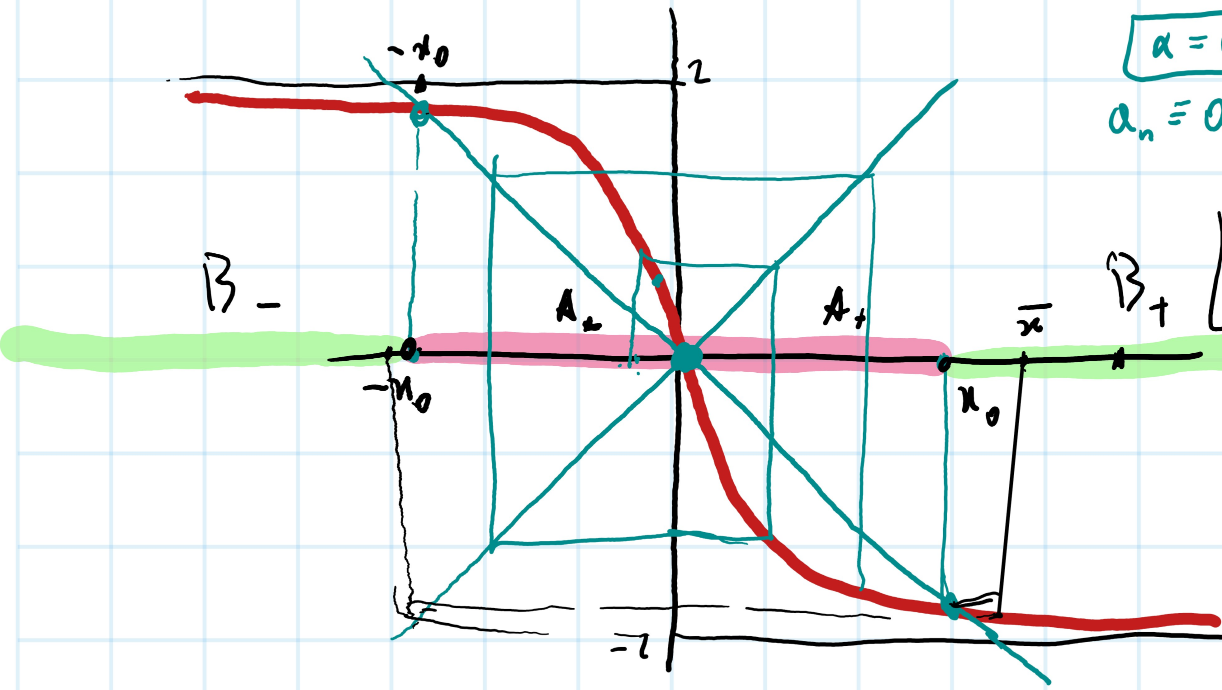
$$\begin{cases} a_{n+1} = -\frac{4}{\pi}\arctan a_n \\ a_0 = \alpha \in \mathbb{R} \end{cases}$$

$a_{n+1} = f(a_n)$

$f(x) = -\frac{4}{\pi}\arctan x$

$\boxed{\alpha = 0}$

$a_n \equiv 0 \quad \forall n$



$\boxed{-\frac{4}{\pi}\arctan x = -x}$

$\boxed{A = (-x_0, x_0)}$

$f(A_+) = A_- \qquad A_+ = (0, x_0)$

$f(A_-) = A_+ \qquad A_- = (-x_0, 0)$

$-\frac{4}{\pi}\arctan \bar{x} > -\bar{x}$

$f(\bar{x}) > -\bar{x}$

$f(B_+) = B_- \qquad B_+ = (x_0, +\infty)$

$f(B_-) = B_+ \qquad B_- = (-\infty, -x_0)$

$f(f(\bar{x})) < f(-\bar{x}) = -f(\bar{x}) < \bar{x}$

$\alpha \in A_+$

$$\begin{cases} b_{k+1} = \underbrace{f(f}_{g}(b_k)) \\ b_0 = \alpha \end{cases}$$

$$\begin{cases} a_{n+1} = f(a_n) \\ \end{cases}$$

$a_{n+2} = f(f(a_n))$

$a_{2k} = b_k$

$-f(x) > -(-x)$

$f(x) < -x$

$g(A_+) = A_+$

$\left[\begin{array}{l} g \text{ È CRESCENTE} \\ g(x) > x \end{array}\right]$ a_{2k} CRESCENTE, b_k CRESCENTE

$f(f(x)) > f(-x) = -f(x) > -(-x) = x$

$f(x) < -x$

$\underbrace{f(f(x))} > f(-x) = -f(x) > -(-x) = \underbrace{x}$

b_k CRESC. LIMIT. $\qquad a_{2k} \to x_0$

$b_k \to l$ finito

$g(b_k) \to g(l)$

$\|$

$b_{k+1} \longrightarrow l$

$\boxed{C_k = a_{2k+1}}$

$\alpha \in A_+ \quad a_1 = f(\alpha) \in A_-$

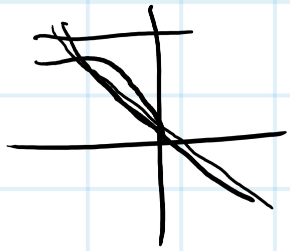
$C_{k+1} = a_{2(k+1)+1} = a_{2k+3} = f(a_{2k+2}) = f(f(a_{2k+1})) = g(a_{2k+1}) = g(C_k)$

$C_0 = f(\alpha) \overset{=\beta}{\in} A_-$

SU $\overline{A_-}$

$\begin{cases} C_{k+1} = g(C_k) \\ C_0 = \beta \in A_- \end{cases}$

$\left.\begin{array}{l} (1)\ g \text{ CRESC.} \\ (2)\ g(x) < x\ (?) \end{array}\right\}$ C_k DECR.



$x \in A_-$

$f(x) > -x \qquad -f(x) < -(-x)$

$\boxed{g(x)} = f(f(x)) < -f(x) < -(-x) = \boxed{x}$

c_n È DECR.

$c_n \to \ell$

$$\left.\begin{array}{l} g(c_n) \to g(\ell) \\ \quad \parallel \\ c_{n+1} \to \ell \end{array}\right\} \quad \ell = g(\ell) \qquad \ell = -x_0$$

$\alpha > x_0 \qquad \alpha \in B_+ \qquad g \qquad b_n = a_{2n}$

$$b_{n+1} = a_{2(n+1)} = a_{2n+2} = f(a_{2n+1}) = \overbrace{f(f(a_{2n}))}^{g} = f(f(b_n)) = g(b_n)$$

$$b_n \text{ DEC} \Longleftarrow \left(\begin{cases} b_{n+1} = g(b_n) \\ b_0 = \alpha \in B_+ \end{cases}\right)$$

SU B_+

1) g È CRESC.

2) $g(x) < x$ (?)

$\overline{f(x) > -x} \qquad \overline{-f(x) < x}$

$$g(x) = \underbrace{f(f(x))} < f(-x) = -f(x) < \underbrace{x}$$

$b_n \to \ell$ finito

$$\left.\begin{array}{l} g(b_n) \to g(\ell) \\ \quad \parallel \\ b_{n+1} \to \ell \end{array}\right\} \Rightarrow g(\ell) = \ell \Rightarrow \ell = x_0$$

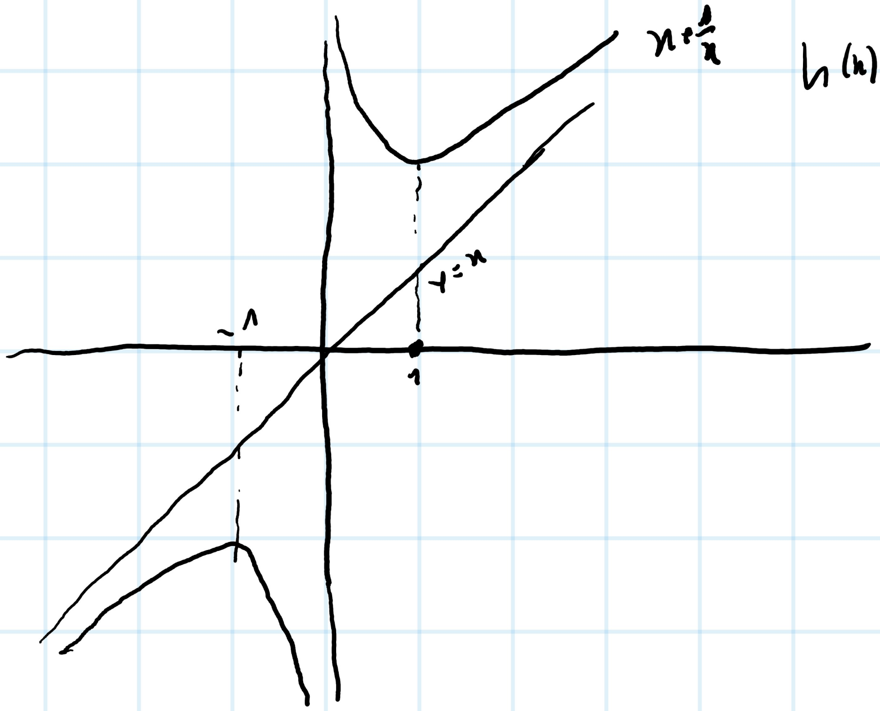
[. . .] $c_n = a_{2n+1} \to -x_0$ CRESC.

$\boxed{4}$

$$\begin{cases} a_{n+1} = \frac{2}{3}\left(a_n + \frac{1}{a_n}\right) + \frac{1}{3} \\ a_0 = \alpha \in \mathbb{R} - \{0\} \end{cases}$$

$a_{n+1} = f(a_n)$

$$\boxed{f(x) = \frac{2}{3}\left(x + \frac{1}{x}\right) + \frac{1}{3}}$$



$h(x) = x + \frac{1}{x}$

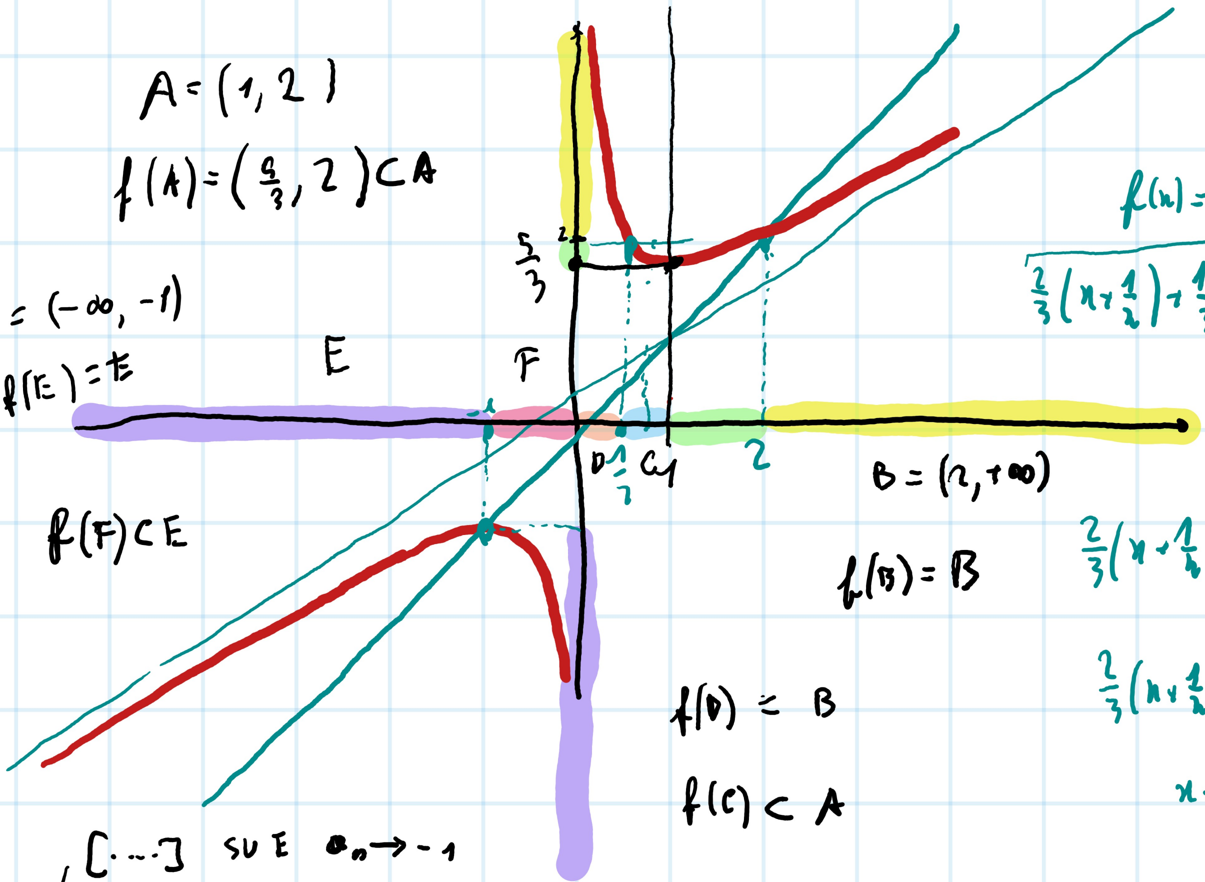
$\frac{2}{3}h(x) + \frac{1}{3}$

$A = (1, 2]$

$f(A) = \left(\frac{5}{3}, 2\right] \subset A$

$E = (-\infty, -1)$

$f(E) = E$



$f(F) \subset E$

$B = (2, +\infty)$

$f(B) = B$

$f(D) \subset B$

$f(C) \subset A$

$f(x) = x$

$\frac{2}{3}\left(x + \frac{1}{x}\right) + \frac{1}{3} = x$

$\frac{2}{3}\left(x + \frac{1}{x}\right) + \frac{1}{3} = 2$

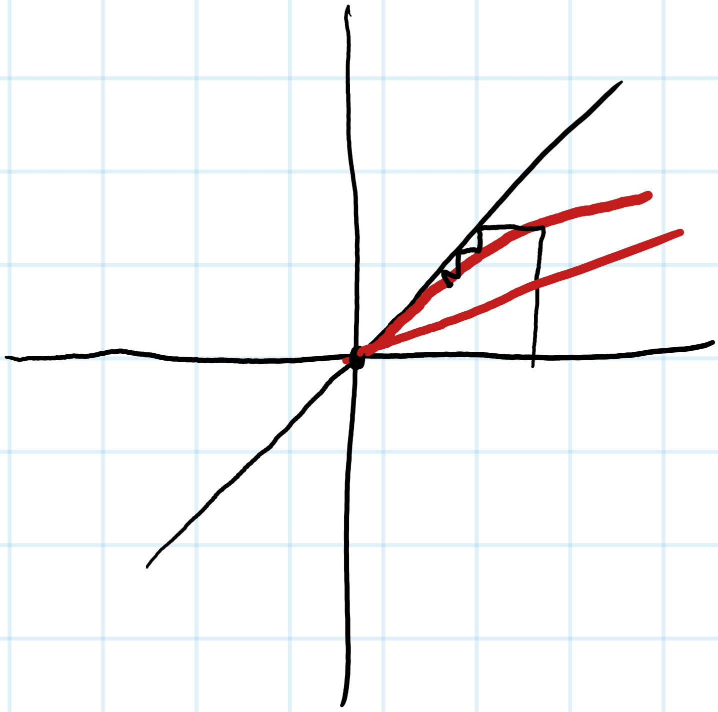
$\frac{2}{3}\left(x + \frac{1}{x}\right) = \frac{5}{3}$

$x + \frac{1}{x} = \frac{5}{2}$

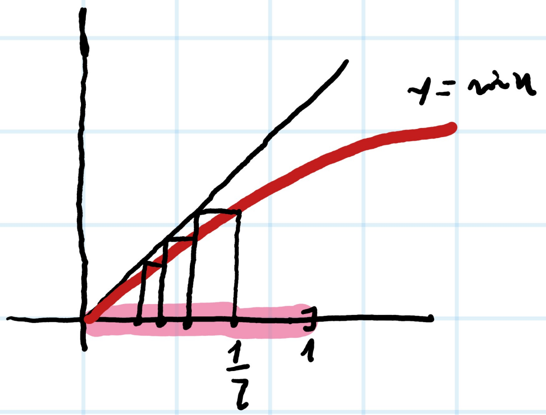
[...] su E $a_n \to -1$

A $\left.\begin{array}{l} f \text{ CRESC.} \\ f(x) > x \end{array}\right\} \Rightarrow a_n \text{ CRESC.} \to 2$

B $\left.\begin{array}{l} f \text{ CRESC.} \\ f(x) < x \end{array}\right\} \Rightarrow a_n \text{ DEC.} \to 2$



LEZ. 33 (21/12/2023)



$\boxed{f(x) = \sin x}$

$$\boxed{\begin{cases} a_{n+1} = \sin(a_n) \\ a_2 = \frac{1}{2} \end{cases}}$$

$A = (0,1)$

SU A 1) $f(x)$ CRESC. $\qquad f(A) \subset A$

2) $f(x) < x$

a_n DECRESC. LIMITATA $\qquad \sum a_n$

$a_n \to \ell \in \mathbb{R}$
$\quad\ \ \shortparallel$
$\quad\ \ 0$

$f(a_n) \to a_{n+1}$

$f\left(\frac{1}{n}\right) \qquad \frac{1}{n+1} \qquad \cos(\xi) \quad \boxed{0 < \xi < \frac{1}{n}}$

$$f\left(\frac{1}{n}\right) = \sin\frac{1}{n} = \frac{1}{n} - \frac{1}{6n^3} + \frac{\overline{f^{(5)}(\xi)}}{120} \cdot \left(\frac{1}{n}\right)^5$$

$$\longrightarrow \boxed{f\left(\frac{1}{n}\right) = \sin\frac{1}{n} > \frac{1}{n} - \frac{1}{6n^3} > \frac{1}{n+1}} \longleftarrow$$

$\frac{1}{n} - \frac{1}{6n^3} \ ? \ \frac{1}{n+1} \qquad \frac{1}{n} - \frac{1}{n+1} \ ? \ \frac{1}{6n^3} \qquad \frac{1}{n(n+1)} \ ? \ \frac{1}{6n^3}$

$\frac{1}{n+1} > \frac{1}{6n^2}$

$$\boxed{b_n = \frac{1}{n}}$$

$\forall n \in \mathbb{N} \setminus \{0\} \quad a_n \geq \frac{1}{n} \quad (?) \quad a_n =$

$a_2 = \frac{1}{2} \quad b_2 = \frac{1}{2}$

$a_3 = f\left(\frac{1}{2}\right) > \frac{1}{3} \quad b_3 = \frac{1}{3}$

$\vdots$

$\boxed{a_n > \frac{1}{n}} \overset{?}{\not\Rightarrow} a_{n+1} > \frac{1}{n+1}$

$$a_{n+1} = f(a_n) > f\left(\frac{1}{n}\right) > \frac{1}{n+1}$$

$$\begin{cases} a_{n+1} = \arctan a_n \\ a_2 = \frac{1}{2} \end{cases}$$

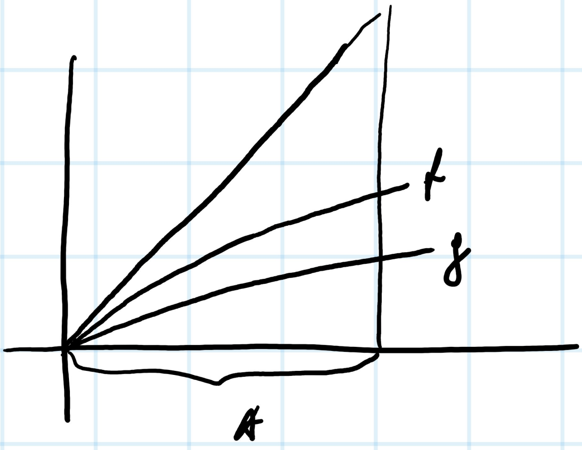
$a_n \ ? \ b_n = \frac{1}{n}$

$$\begin{cases} a_{n+1} = f(a_n) \\ a_0 = \alpha \end{cases} \qquad \begin{cases} b_{n+1} = g(b_n) \\ b_0 = \beta \end{cases}$$

$\beta \leq \alpha \quad \boxed{\alpha, \beta \in A}$

$g(x) \leq f(x) \leq x$

f, g CRESCENTI



TESI

$\forall n \in \mathbb{N} \ \boxed{b_n \leq a_n}$

DIMO

$n = 0$ OVVIO

$n = k \Rightarrow n = k+1$

$\boxed{a_{k+1}} = f(a_k) \geq g(a_k) \geq$

$\geq g(b_k) = \boxed{b_{k+1}}$

$\overline{f(x)} = \arctan x$

$$\to \begin{cases} a_{n+1} = \arctan a_n \\ a_2 = \frac{1}{2} \end{cases}$$

$$\boxed{b_n = \frac{1}{n}}$$

$$b_{n+1} = \frac{1}{n+1} = \frac{\frac{1}{n}}{1+\frac{1}{n}} = \frac{b_n}{1+b_n}$$

$a_n > b_n$

$$\boxed{a_n > \frac{1}{n}}$$

$$\begin{cases} b_{n+1} = \frac{n}{1+n} \\ b_2 = \frac{1}{2} \end{cases}$$

$$\begin{cases} b_{n+1} = \frac{b_n}{1+b_n} = g(b_n) \\ b_2 = \frac{1}{2} \end{cases}$$

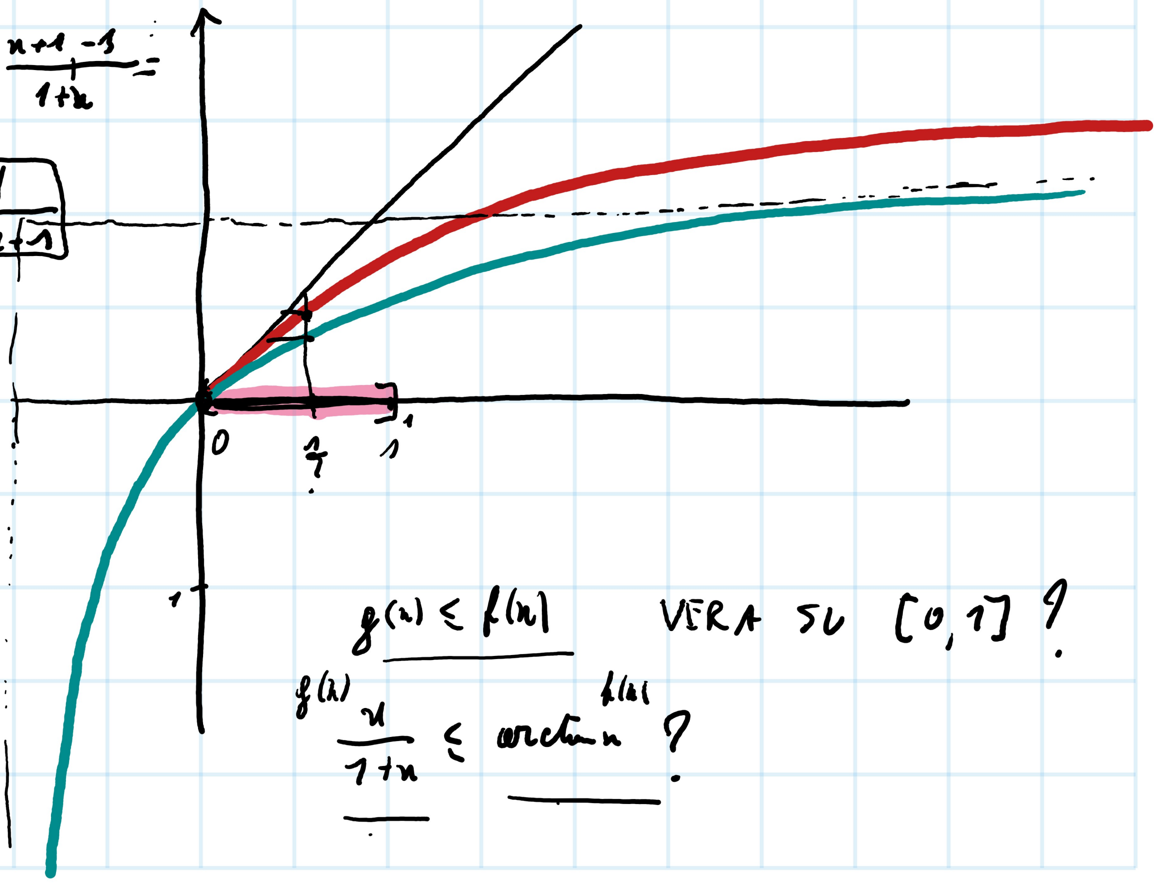
$$\boxed{g(x) = \frac{x}{1+x}}$$

$\frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

$$f\left(\frac{1}{3}\right) = \frac{\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{1}{\cancel{3}}}{\frac{4}{\cancel{3}}} = \frac{1}{4}$$

$$g(x) = \frac{x}{1+x} = \frac{x+1-1}{1+x} = \boxed{1 - \frac{1}{x+1}}$$



$\frac{1}{x}$

$-\frac{1}{x}$

$-\frac{1}{x+1}$ ←

$$\boxed{1 - \frac{1}{x+1}}$$ ↑

$g(x) \leq f(x)$ VERA SU $[0,1]$?

$$\underbrace{\frac{x}{1+x}}_{g(x)} \leq \underbrace{\arctan x}_{f(x)} \; ?$$

$$g'(x) = \left(\frac{x}{1+x}\right)' = \left(1 - \frac{1}{1+x}\right)' = + \frac{1}{(1+x)^2}$$

$$f'(x) = (\arctan x)' = \frac{1}{1+x^2}$$

$$x > 0 \qquad 1+x^2 < (1+x)^2$$

$$\frac{1}{1+x^2} > \frac{1}{(1+x)^2}$$

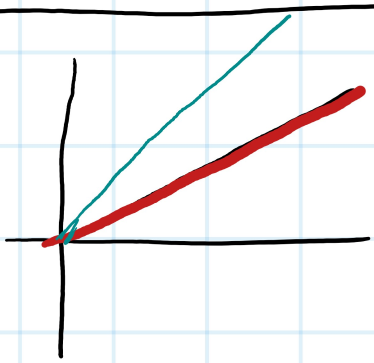
$$f'(x) > g'(x)$$

$$(f(x)-g(x))' > 0$$

$$f(x)-g(x) > 0 \Leftarrow \begin{cases} f(x)-g(x) \text{ CRESCENTI STRETT.} \\ f(0)-g(0)=0 \end{cases}$$

$$\Downarrow$$

$$f(x) > g(x)$$



$$\boxed{a_n = \cancel{\left(\frac{1}{2}\right)^n}}$$

$$a_{n+1} = \left(\frac{1}{2}\right)^{n+1} = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^n = \frac{1}{2} a_n$$

$$f(x) = \frac{x}{2} \qquad \begin{cases} a_{n+1} = \frac{1}{2} a_n \\ a_0 = 1 \end{cases}$$

$$0 < A < 1$$

$$a_n = A^n \qquad \begin{cases} a_{n+1} = A \cdot a_n \\ a_0 = 1 \end{cases}$$

$$\begin{cases} a_{n+1} = f(a_n) \\ a_0 = \alpha \end{cases}$$

SE $f'(0) = \lambda < 1$ ALLORA $\frac{f(x)}{x} \to \lambda$ QUINDI

PRESO $\lambda < \mu < 1$ $\exists\ \delta > 0$ T.C. $\forall x \in [0,\delta]$

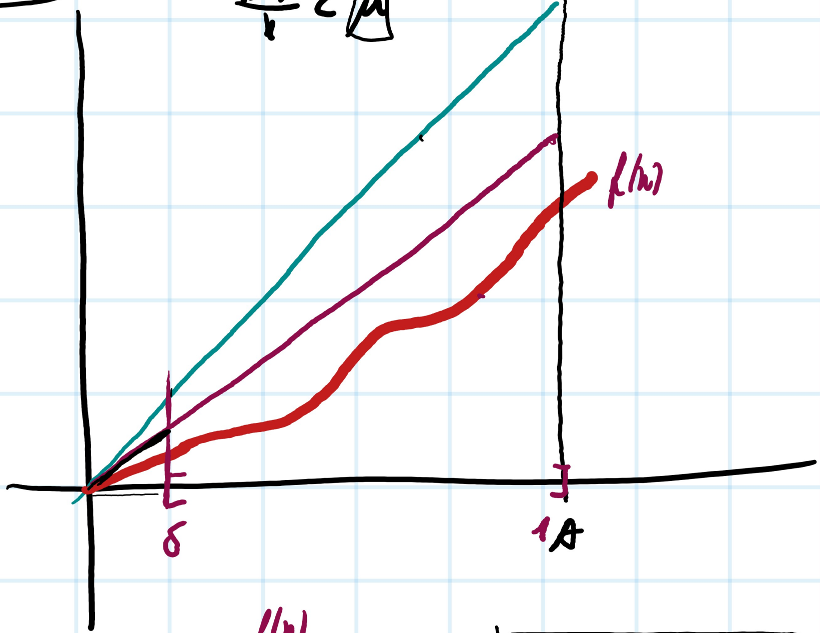
$\frac{f(x)}{x} < \boxed{\mu}$

$\frac{f(x)}{x}$ CONTINUA SU $[\delta, 1]$

$\exists\ x_0 \in [\delta, 1]$ t.c.

$\frac{f(x_0)}{x_0} = \max_{x \in [\delta,1]} \frac{f(x)}{x}$

< 1



$\frac{f(x)}{x}$

$\nu = \max\{\mu, \frac{f(x_0)}{x_0}\} < 1$

$\boxed{\frac{f(x)}{x}} < \nu \quad \forall x \in [0,1]$

quindi $y = \nu \cdot x$ SODDISFA

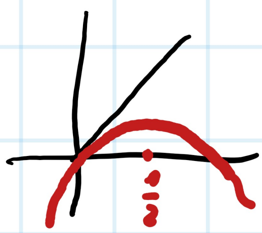
$f(x) \leq \nu x < x$

$$\begin{cases} b_{n+1} = \nu \cdot b_n \\ b_0 = 1 \end{cases}$$

$a_n \leq b_n = \nu^n$

$$\begin{cases} a_{n+1} = a_n^2 - a_n \\ a_0 = \frac{1}{2} \end{cases}$$

$f(x) = x^2 - x$



$$x^2 - 5x + 6$$

$$\left\{\begin{array}{l}\boxed{a_{n+2} = 5a_{n+1} - 6a_n} = F(a_{n+1}, a_n) \\ a_0 = \alpha \\ a_1 = \beta\end{array}\right.$$

$$\boxed{F(x,y) = 5x - 6y}$$

n	a_n
0	0
1	1
2	5
3	19
4	$\vdots$
5	
6	

$$a_n = f(n)$$

$\rightarrow a_0, a_1, \dots, a_n, \dots$

$\rightarrow b_0, b_1, \dots, b_n, \dots$

$\rightarrow \alpha a_0 + \beta b_0, \alpha a_1 + \beta b_1, \dots, \alpha a_n + \beta b_n, \dots$

$$\boxed{\alpha a_{n+2} + \beta a_{n+2}} \overset{?}{=} F(\alpha a_{n+1} + \beta b_{n+1}, \alpha a_n + \beta b_n) =$$

$$= 5(\alpha a_{n+1} + \beta b_{n+1}) - 6(\alpha a_n + \beta b_n) =$$

$$= \alpha(\underbrace{5a_{n+1} - 6a_n}_{a_{n+2}}) + \beta(\underbrace{5b_{n+1} - 6b_n}_{b_{n+2}}) =$$

$$= \alpha a_{n+2} + \beta b_{n+2}$$

$E_0 = (1, 0, \ldots\ldots\ldots)$

$E_1 = (0, 1, \ldots\ldots\ldots)$

$\underline{\alpha E_0 + \beta E_1} = (\alpha, \beta, \underline{\ldots\ldots\ldots})$

$a_{n+2} = 5a_{n+1} - 6a_n$ $\qquad a_{n+2} - 5a_{n+1} + 6a_n$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda = 2$
$\lambda = 3$

$(1, 2, 4, \ldots, 2^n, \ldots)$ $\qquad p(\lambda) \qquad \lambda$ i.e $p(\lambda) =$

$(1, 3, 9, \ldots, 3^n, \ldots)$

$a_n = 3^n$ $\qquad a_{n+2} \overset{?}{=} 5a_{n+1} - 6a_n$

$3^{n+2} \overset{?}{=} 5 \cdot 3^{n+1} - 6 \cdot 3^n$

$A a_{n+2} = -B a_{n+1} + C a_n$

$3^{n+2} - 5 \cdot 3^{n+1} + 6 \cdot 3^n \overset{?}{=} 0$

$\boxed{p(\lambda) = A\lambda^2 + B\lambda + C}$

$3^2 - 5 \cdot 3 + 6 = 0$

$a_n = \lambda^n$ $\qquad A \cdot \lambda^{n+2} \overset{?}{=} -B\lambda^{n+1} - C\lambda^n \qquad ?$

$$\overline{A\lambda^2 + B\lambda + C = 0} \qquad ?$$

$1 \quad 2 \quad 4 \ \dots \ 2^n \dots$

$1 \quad 3 \quad 9 \ \dots \ 3^n \dots$

$$a_n = \boxed{\alpha\, 2^n + \beta \cdot 3^n}$$

$(1, 1, -1, \dots$

$(1, 2, \dots$

$(1, 3, \dots$

$a_0 = 1$

$a_1 = 1$

$$\begin{cases} \alpha + \beta = 1 \\ \alpha \cdot 2 + \beta \cdot 3 = 1 \end{cases}$$

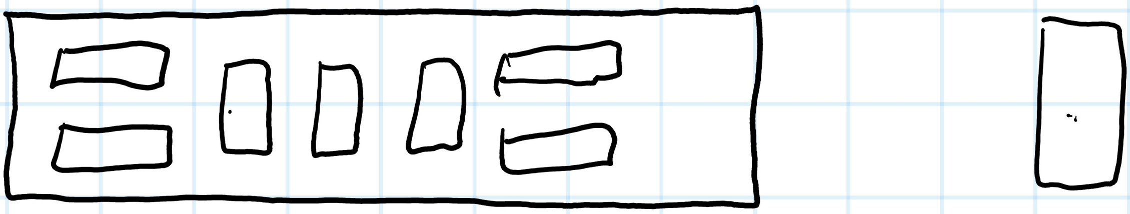
$\beta = -1$

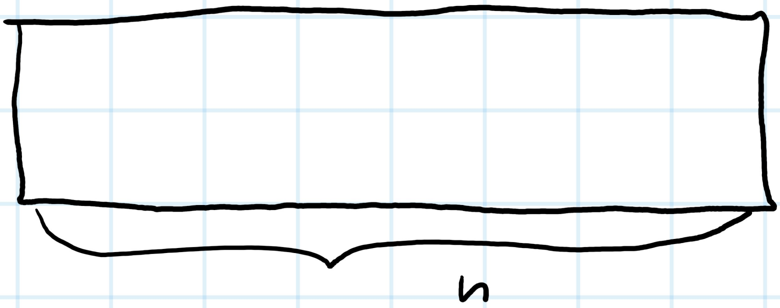
$\alpha = 2$

$$\boxed{a_n = 2 \cdot 2^n - 3^n}$$

$\underline{a_2 = 2 \cdot 4 - 9 = -1}$

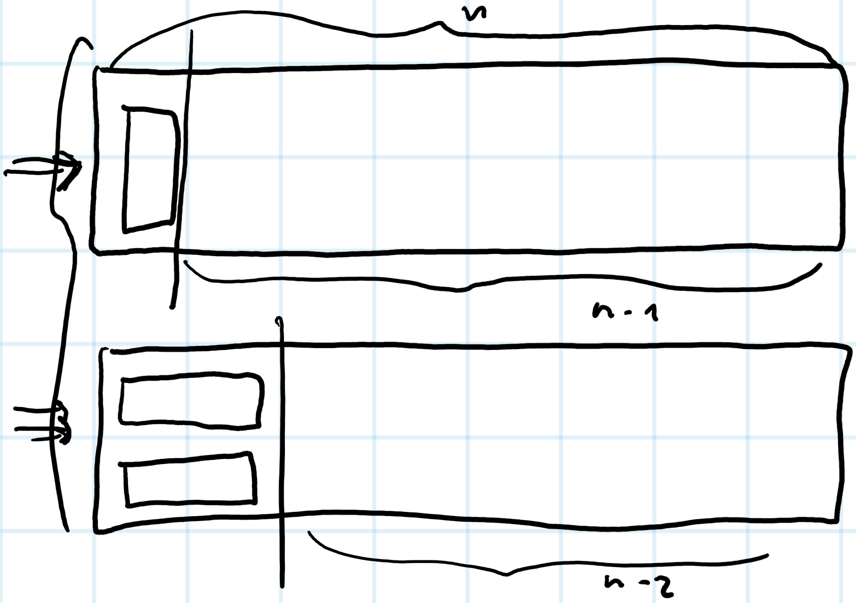
a_{100}





$$X_n = X_{n-1} + X_{n-2}$$

$$\begin{cases} X_{n+2} = X_{n+1} + X_n \\ X_1 = 1 \\ X_2 = 2 \end{cases}$$



X_{n-1}

X_{n-2}

$$\lambda^2 - \lambda - 1 = 0$$

$$\lambda = \frac{1 \pm \sqrt{5}}{2}$$

$$\begin{cases} a_0 = 1 \\ a_1 = 1 \end{cases}$$

$a_2 = 2$

$$a_n = \alpha\left(\frac{1+\sqrt{5}}{2}\right)^n + \beta\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\begin{cases} \alpha + \beta = 1 \\ \alpha \cdot \frac{1+\sqrt{5}}{2} + \beta \cdot \frac{1-\sqrt{5}}{2} = 1 \end{cases}$$

$$(1-\beta)\frac{\cancel{1}+\sqrt{5}}{2} + \beta\frac{\cancel{1}-\sqrt{5}}{2} = 1$$

$$\cancel{1}+\sqrt{5} - \beta(\cancel{1}+\sqrt{5}) + \beta(\cancel{1}-\sqrt{5}) = \cancel{2}^{1}$$

$$-2\sqrt{5}\beta = 1-\sqrt{5}$$

$$\beta = \boxed{\frac{\sqrt{5}-1}{2\sqrt{5}}} =$$

$$\alpha = 1-\beta = \frac{2\sqrt{5}-\sqrt{5}+1}{2\sqrt{5}} = \frac{\sqrt{5}+1}{2\sqrt{5}}$$

$$\boxed{a_n = \frac{\sqrt{5}+1}{2\sqrt{5}} \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{\sqrt{5}-1}{2\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$