Lezione 9: Il numero di Nepero

INDICE

... DALLA LEZIONE SCORSA

- CATENA DEGLI INFINITI VALE ANCHE CON $a_n \to +\infty$ AL POSTO DI n.

1) $\left(1+\frac{1}{n}\right)^n \to e$

2) PER QUALI f SI HA CHE $a_n \to \ell \Rightarrow f(a_n) \to f(\ell)$

3) $a_n^{b_n} \to \ell_1^{\ell_2}$

4) $\left(1+\frac{1}{a_n}\right)^{b_n}$ (VARI CASI)

5) ESERCIZI LISTA PRECEDENTE.

[...]

$$n^\alpha = o(A^n) \quad \boxed{\forall \alpha > 0 \quad \forall A > 1}$$

(?) $(a_n)^\alpha = o(A^{a_n})$ $\boxed{\forall \alpha > 0 \quad \forall A > 1}$ SE $a_n \to +\infty$

$$\boxed{\frac{A^{a_n}}{(a_n)^\alpha}} > \underbrace{\frac{A^{\lfloor a_n \rfloor}}{(\lfloor a_n \rfloor + 1)^\alpha} = \boxed{\frac{1}{A}} \cdot \underbrace{\boxed{\frac{A^{\lfloor a_n \rfloor + 1}}{(\lfloor a_n \rfloor + 1)^\alpha}}}_{\to +\infty}}_{\to +\infty}$$

$$\overset{\frown}{A^n} = o(n!) \qquad n! = o(\overset{\frown}{n^n})$$

$a_n \to +\infty$

$$A^{a_n} = o\left(a_n^{a_n}\right)$$

$$= \frac{a_n^{a_n}}{A^{a_n}} \to +\infty$$

$$= \left(\frac{a_n}{A}\right)^{a_n} > \frac{a_n}{A} \to +\infty$$

$$\boxed{\log_a a_n} \stackrel{?}{=} o\left(\boxed{(a_n)}^{\alpha}\right) \qquad a_n \to +\infty$$

$$a^{\log_a (a_n)^{\alpha}} = a^{\alpha \cdot \log_a a_n} =$$

$$= \left(a^{\alpha}\right)^{\log_a a_n}$$

A

$$= \boxed{A^{\log_a a_n}}$$

$$\boxed{n = o\left(A^n\right)} \qquad \boxed{\log_a(a_n) = o\left(A^{\log_a a_n}\right)}$$

$\forall M > 0$ DEF. $\log_a a_n > M$?

$$a_n > a^M \qquad \log_a a_n > \log_a\left(a^{M}\right) = M$$

$\boxed{T.1}$ DATE $a_n = \left(1+\frac{1}{n}\right)^n$ E $b_n = \left(1+\frac{1}{n}\right)^{n+1}$

ALLORA $\exists \ell \in \mathbb{R}$ CON $2<\ell<3$ t.c.

$a_n \to \ell$ CRESCENDO

$b_n \to \ell$ DECRESCENDO

DIM. b_n DEC.

$b_{n+1} < b_n$?

$$\left(1+\frac{1}{n+1}\right)\left(1+\frac{1}{n+1}\right)^{n+1} < \left(1+\frac{1}{n}\right)^{n+1} \quad ?$$

$$1+\frac{1}{n+1} < \left(\frac{1+\frac{1}{n}}{1+\frac{1}{n+1}}\right)^{n+1} \quad (?)$$

$$1+\frac{1}{n+1} < \left(\frac{(n+1)^2}{n(n+2)}\right)^{n+1} \quad (?)$$

$$1+\frac{1}{n+1} < \left(\frac{n^2+2n+1}{n^2+2n}\right)^{n+1} \quad (?)$$

$$\left(1+\frac{1}{n+1}\right)^{n+2} < \left(1+\frac{1}{n}\right)^{n+1} \quad ?$$

$$\left(\frac{n+2}{n+1}\right)^{n+2} < \left(\frac{n+1}{n}\right)^{n+1} \quad ?$$

$$\left(\frac{n+1}{n+2}\right)^{n+2} > \left(\frac{n}{n+1}\right)^{n+1} \quad ?$$

$$\left(\frac{n+1}{n+2}\right)^{\frac{n+2}{n+1}} > \frac{n}{n+1} \quad ?$$

$$\underbrace{\left(1-\frac{1}{n+2}\right)^{\frac{n+2}{n+1}}} > \underline{1-\frac{1}{n+1}}$$

$$> 1-\frac{1}{\cancel{n+2}}\cdot\frac{\cancel{n+2}}{n+1} = 1-\frac{1}{n+1}$$

$$(1+x)^n > 1+nx$$

$$(1+x)^\alpha > 1+\alpha x \qquad \alpha > 1$$

$$\underbrace{\left(1+\frac{1}{n^2+2n}\right)^{n+1}} > 1+\frac{1}{n+1} \quad ?$$

$$> 1+\boxed{\frac{(n+1)^2}{n^2+2n}}\,\frac{1}{n+1} > 1+\frac{1}{n+1}$$

a_n CRESCENTE

$$\boxed{a_{n+1} > a_n}$$

$$\left(1+\frac{1}{n+1}\right)^{n+1} > \frac{\left(1+\frac{1}{n}\right)^{n+1}}{1+\frac{1}{n}} \quad (?)$$