$\boxed{2.3}$

$$\left(1+\frac{1}{a_n}\right)^{a_{n+1}} \longrightarrow e \ ?$$

con $a_n \to +\infty$

$\boxed{F}$

$$\left(1+\frac{1}{n!}\right)^{(n+1)!} = \left(\left(1+\frac{1}{n!}\right)^{n!}\right)^{(n+1)}$$

$\left(1+\frac{1}{n!}\right)^{n!} \to e$, $(n+1) \to +\infty$, $\to +\infty$

$$\left(1+\frac{1}{a_n}\right)^{a_n} \to e \quad \boxed{SI}$$

$\boxed{a_n \approx b_n}$ $\boxed{\frac{a_n}{b_n} \to 1}$

$$\left(1+\frac{1}{a_n}\right)^{b_n} = \left(\left(1+\frac{1}{a_n}\right)^{a_n}\right)^{\frac{b_n}{a_n}} \longrightarrow e^1$$

$\left(1+\frac{1}{a_n}\right)^{a_n} \to e$, $\frac{b_n}{a_n} \to 1$

$a_n = n!$

$a_{n+1} = (n+1)!$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{n!} = n+1 \longrightarrow +\infty$$

$\boxed{a_n \approx b_n}$ $\boxed{\begin{matrix} a_n \to +\infty \\ b_n \to +\infty \end{matrix}}$

$\boxed{\frac{a_n}{b_n} \to 1}$ $\overset{?}{\not\Rightarrow}$ $f(a_n) \approx f(b_n)$

$f = \log$ $\boxed{SI}$

$f = e$ sicuramente

$$\boxed{\frac{\ln(a_n)}{\ln(b_n)}} = \frac{\ln\left(\frac{a_n}{b_n}\cdot b_n\right)}{\ln(b_n)} = \frac{\ln\left(\frac{a_n}{b_n}\right) + \ln(b_n)}{\ln(b_n)} = \boxed{\frac{\ln\left(\frac{a_n}{b_n}\right)}{\ln(b_n)}} + 1 = \ldots$$

$\frac{\ln(a_n)}{\ln(b_n)} \to 1$; $\frac{a_n}{b_n} \to 1$, $\ln\left(\frac{a_n}{b_n}\right) \to 0$, $\ln(b_n) \to +\infty$, $\to 0$; $+1 \to 1$

$a_n = n^2 + n$

$b_n = n^2$

$$\frac{n^2+n}{n^2} = 1 + \frac{1}{n} \to 1$$

$$\frac{e^{a_n}}{e^{b_n}}$$

$$\frac{n^n + n! + \ln n}{n^n}$$