Metodi Matematici - Ex. 2

Titolo nota

10 ottobre 2017 (8:30-9.15) - docente: Prof. **Emanuele Callegari** - Università di Roma ***Tor Vergata***

ES 9.B
LISTA 1

$f(x) = x^2$ $\quad V = C([0,1])$ con $\|\cdot\|_2$

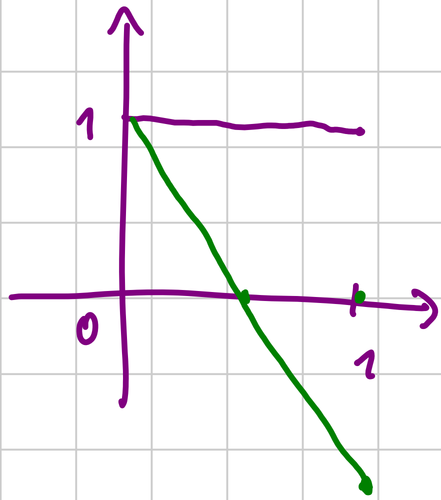
$H = \{ f \in V \mid f$ è pol. di grado $\leq 1 \}$

$\{v_1, v_2\}$

$v_1 = g(x) =$ costante $= \boxed{1}$ perché $\|\ \| = 1$

$v_2 = h(x) = (1-2x)C = \boxed{\sqrt{3}(1-2x)}$

scegliendo C in modo che $\|h\|_2 = 1$



$$\int_0^1 h^2(x)\,dx = \int_0^1 (1-2x)^2 \cdot C^2\,dx = C^2 \int_0^1 (2x-1)^2\,dx$$

$$= C^2 \left[\frac{(2x-1)^3}{6} \right]_0^1 = C^2 \left(\frac{1}{6} + \frac{1}{6} \right) = \boxed{\frac{C^2}{3}}$$

Perché $\|h\| = 1$ deve essere $C = \sqrt{3}$

$$\Pi_f(x) = \alpha_1 g(x) + \alpha_2 h(x)$$

$\alpha_1 = \langle f, g \rangle$

$\alpha_2 = \langle f, h \rangle$

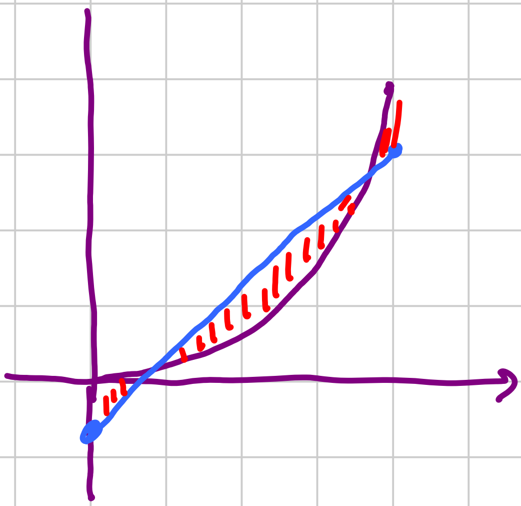
$$\alpha_1 = \int_0^1 x^2 \cdot 1\,dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3}$$

$$\alpha_2 = \int_0^1 x^2 \cdot \sqrt{3}(1-2x)\, dx = \int_0^1 \sqrt{3}\, x^2 - 2\sqrt{3}x^3\, dx =$$

$$= \sqrt{3}\left[\frac{x^3}{3} - \frac{x^4}{2}\right]_0^1 = \sqrt{3}\left(\frac{1}{3} - \frac{1}{2}\right) = -\frac{\sqrt{3}}{6}$$

$$\Pi_f(x) = \frac{1}{3}\cdot 1 - \frac{\cancel{\sqrt{3}}}{\cancel{6}_{2}} \cdot \cancel{\sqrt{3}}(1-2x) =$$

$$= \frac{1}{3} - \frac{1}{2} + x = \boxed{x - \frac{1}{6}}$$



$f(x) = x^2$

$\Pi_f(x) = x - \frac{1}{6}$

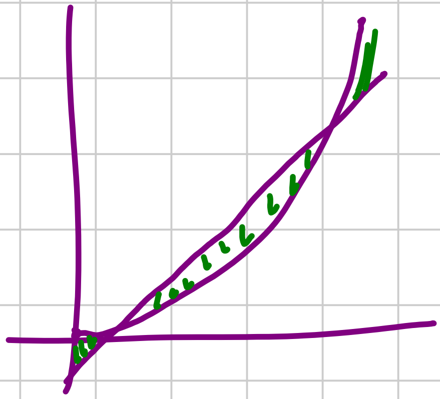
$$d(f, \Pi_f) = \| f(x) - \Pi_f(x) \|_2 = \sqrt{\int_0^1 \Big(\underbrace{x^2 - \left(x - \tfrac{1}{6}\right)}_{(f(x) - \Pi_f(x))^2}\Big)^2 dx} =$$

$= \ldots$ (finire semplice calcolo integrale)

ES. 8
LISTA 1

f, V e H come prima ma usando $\|\cdot\|_1$

$$d(f, H) = \inf_{u \in H} \underbrace{\|f - u\|_1}_{?}$$

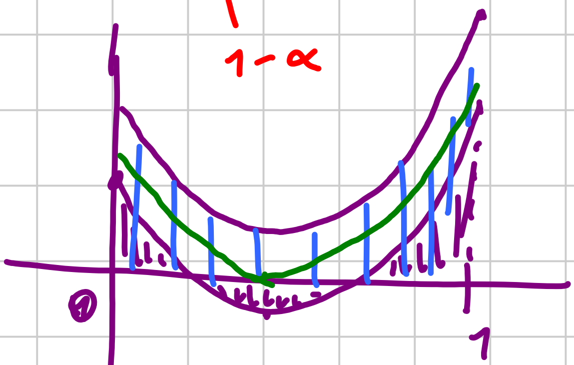


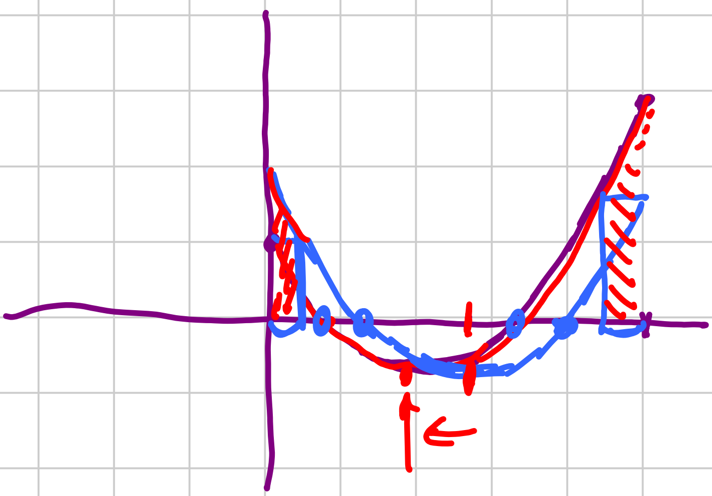
$u(x) = ax + b$

$f(x) = x^2$

$$\inf_{a,b \in \mathbb{R}} \int_0^1 \Big| \underbrace{x^2 - (ax+b)}_{(x-\alpha)(x-\beta)} \Big| \, dx$$

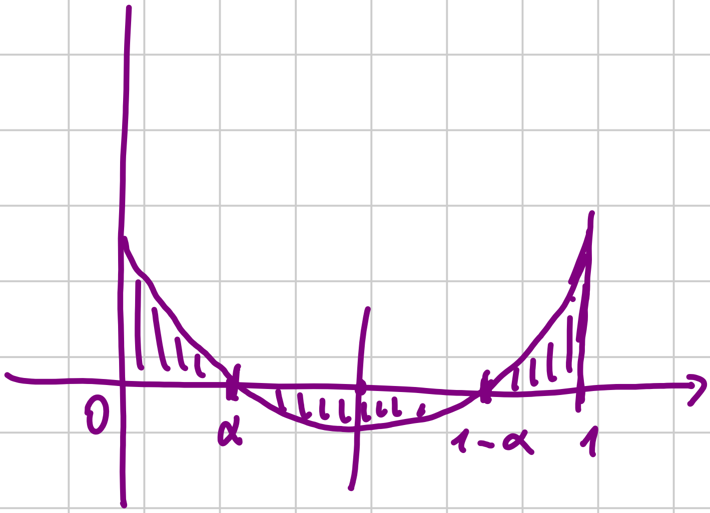
$\beta = 1-\alpha$





$$\inf_{\boxed{\alpha \in \mathbb{R}}} \int_0^1 \Big| \underbrace{(x-\alpha)(x-(1-\alpha))}_{} \Big| \, dx$$

$\alpha \in [0, \frac{1}{2}]$



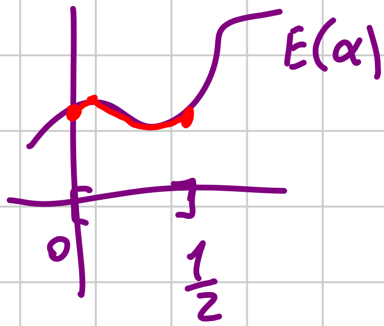
$$\int_0^1 \left|(x-\alpha)(x-(1-\alpha))\right|\,dx =$$

$$= 2\int_0^{\frac{1}{2}} \left| x^2 - x + \alpha(1-\alpha)\right|\,dx =$$

$$= 2\left(\int_0^{\alpha}\left(x^2 - x + \alpha(1-\alpha)\right)dx - \int_{\alpha}^{\frac{1}{2}}\left(x^2 - x + \alpha(1-\alpha)\right)dx\right) =$$

$$= 2 \cdot E(\alpha)$$

pol. di 3° grado in α



Trovando minimo (α_0) di $E(\alpha)$ al variare di α in $[0, \frac{1}{2}]$ si trova α_0 t.c.

$$\Big\| \underbrace{(x-\alpha_0)(x-(1-\alpha_0))}_{(x^2 + Ax + B)} \Big\|_1 \quad \text{è minima}$$

il polinomio di H che realizza minimo è $-Ax-B$