Metodi Matematici - Ex. 7

Titolo nota

14 novembre 2017 (8:30-9.15) - docente: Prof. **Emanuele Callegari** - Università di Roma ***Tor Vergata***

CALCOLO TRASF. DI FOURIER

0 $f(x) = \boxed{\dfrac{1}{1+x^2}}$ $\qquad \hat{f}(\lambda) = \pi e^{-|\lambda|}$

1 $g(x) = \dfrac{1}{x^2+9} = \dfrac{1}{9} \cdot \boxed{\dfrac{1}{\left(\frac{x}{3}\right)^2+1}}$

$$\mathcal{F}\left(\frac{1}{9} \cdot \frac{1}{\left(\frac{x}{3}\right)^2+1}\right) = \frac{1}{9} \cdot \mathcal{F}\left(\frac{1}{\left(\frac{x}{3}\right)^2+1}\right) = \frac{1}{9} \cdot \frac{1}{\frac{1}{3}} \cdot \hat{f}\left(\frac{\lambda}{\frac{1}{3}}\right) =$$

$$= \frac{1}{3} \hat{f}(3\lambda) = \boxed{\frac{1}{3}\pi \cdot e^{-|3\lambda|}} \leftarrow$$

2 $h(x) = \dfrac{1}{x^2-2x+10} = \dfrac{1}{(x-1)^2+9} = g(x-1)$

$$\mathcal{F}\big(g(x-1)\big) = e^{-i\lambda} \hat{g}(\lambda) = \frac{\pi}{3} e^{-|3\lambda| - i\lambda}$$

3 $\varphi(x) = \dfrac{1}{(x^2+1)^2} = \dfrac{x^2+1-x^2}{(x^2+1)^2} = \dfrac{1}{x^2+1} - \dfrac{x^2}{(x^2+1)^2} =$

$$= \frac{1}{x^2+1} + \frac{x}{2} \cdot \boxed{\frac{-2x}{(x^2+1)^2}} =$$

$$= \frac{1}{x^2+1} - \frac{x}{2} \cdot \left(\frac{1}{x^2+1}\right)' =$$

$\boxed{3a}$ calcolo di: $\mathcal{F}\left(\frac{-2x}{(x^2+1)^2}\right) =$

$$= \mathcal{F}\left(\left(\frac{1}{x^2+1}\right)'\right) = \underline{i\lambda\pi e^{-|\lambda|}}$$

$\boxed{3b}$ $\mathcal{F}\left(-\frac{x^2}{(1+x^2)^2}\right) = \mathcal{F}\left(\frac{x}{2}\cdot\frac{-2x}{(1+x^2)^2}\right) =$

$$= \mathcal{F}\left(\frac{i}{2}(-ix)\cdot\frac{-2x}{(1+x^2)^2}\right) =$$

$$= \frac{i}{2}\,\mathcal{F}\left((-ix)\,\frac{-2x}{(1+x^2)^2}\right) =$$

$$= \frac{i}{2}\cdot\left(i\lambda\pi e^{-|\lambda|}\right)' =$$

$$= -\frac{\pi}{2}\left(\lambda e^{-|\lambda|}\right)' =$$

$$= \left\{\begin{array}{ll} -\frac{\pi}{2}\left(e^{-\lambda}-\lambda e^{-\lambda}\right) & \boxed{\lambda>0} \\ -\frac{\pi}{2} & \lambda=0 \\ -\frac{\pi}{2}\left(e^{\lambda}+\lambda e^{\lambda}\right) & \boxed{\lambda<0} \end{array}\right\} =$$

$$= -\frac{\pi}{2}\left(e^{-|\lambda|}-|\lambda|e^{-|\lambda|}\right)$$

$$\mathcal{F}\left(\frac{1}{(1+x^2)^2}\right)=\mathcal{F}\left(\frac{1}{x^2+1}-\frac{x}{2}\cdot\left(\frac{1}{x^2+1}\right)'\right)=$$

$$=\mathcal{F}\left(\frac{1}{1+x^2}\right)+\mathcal{F}\left(-\frac{x}{2}\cdot\left(\frac{1}{x^2+1}\right)'\right)=$$

$$=\pi e^{-|\lambda|}-\frac{\pi}{2}\left(e^{-|\lambda|}-|\lambda|e^{-|\lambda|}\right)=$$

$$=\frac{\pi}{2}e^{-|\lambda|}+\frac{\pi}{2}|\lambda|e^{-|\lambda|}$$

$\boxed{4}$ $$\psi(x)=\frac{1}{(1+x^2)^3}=\frac{1+x^2-x^2}{(1+x^2)^3}=$$

$$=\frac{1}{(1+x^2)^2}+\frac{x}{4}\cdot\frac{-4x}{(1+x^2)^3}=$$

$$=\frac{1}{(1+x^2)^2}+\frac{x}{4}\cdot\left(\frac{1}{(1+x^2)^2}\right)'=\ldots\ldots$$

$\boxed{5}$ $$g(x)=\frac{5x^3+3x^2+2}{(1+x^2)^3}=\frac{5x^3+5x-5x+3x^2+3-1}{(1+x^2)^3}=$$

$$=5\cdot\frac{x}{(1+x^2)^2}+\frac{3}{(1+x^2)^2}-5\cdot\frac{x}{(1+x^2)^3}-\frac{1}{(1+x^2)^3}=$$

$$=-\frac{5}{2}\cdot\frac{-2x}{(1+x^2)^2}+3\cdot\frac{1}{(1+x^2)^2}+\frac{5}{4}\cdot\frac{-4x}{(1+x^2)^3}-\frac{1}{(1+x^2)^3}=$$

$$= -\frac{5}{2}\cdot\left(\frac{1}{1+x^2}\right)' + 3\cdot\frac{1}{(1+x^2)^2} + \frac{5}{4}\left(\frac{1}{(1+x^2)^2}\right)' - \frac{1}{(1+x^2)^3} =$$

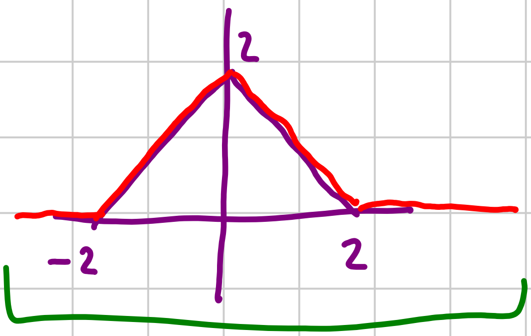
$$= \cdots$$

Stesso metodo per $h(x) = \dfrac{P(x)}{(1+x^2)^k}$ se $gr(P(x)) \leq 2(k-1)$

$$\Updownarrow$$

$$\varphi(x) = \frac{P(x)}{(ax^2+bx+c)^k} \quad \text{se } gr(P(x)) \leq 2(k-1)$$

$\boxed{8}$ $f(x) = (2-|x|)^+$

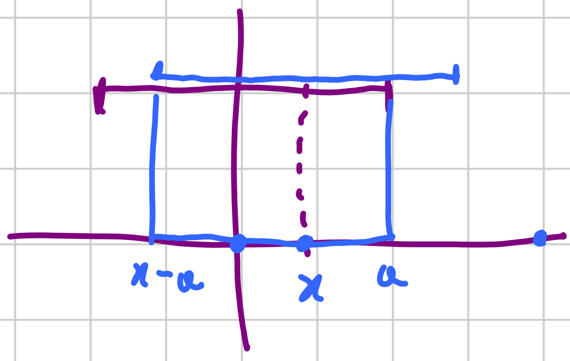


Ricordare che

$$\mathcal{F}\left(\chi_{[-a,a]}(x)\right) = \begin{cases} 2a & \text{se } \lambda = 0 \\ 2\dfrac{\sin(a\lambda)}{\lambda} & \lambda \neq 0 \end{cases}$$

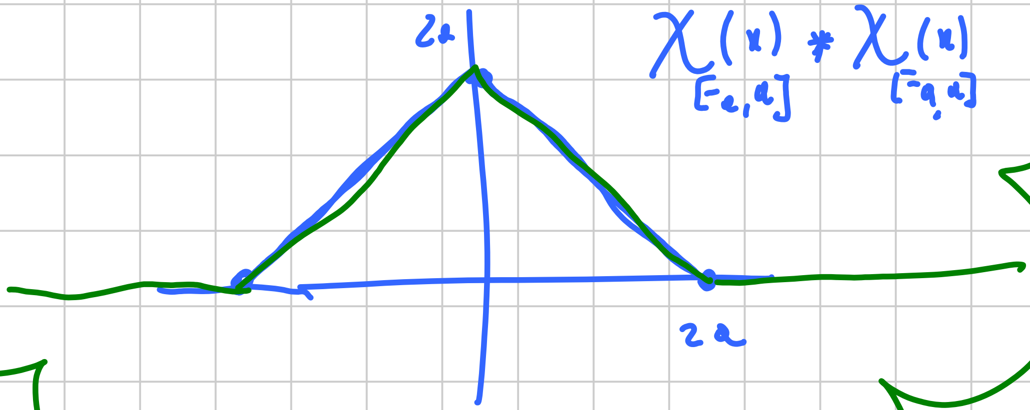
$$f(x) = \left(\chi_{[-1,1]} * \chi_{[-1,1]}\right)(x)$$

$$(g_1 * g_2)(x) = \int_{-\infty}^{+\infty} g_1(y)\cdot g_2(x-y)\,dy$$



$$a-(x-a)=\underline{2a-x}$$

$$\chi_{[-a,a]}(x) * \chi_{[-a,a]}(x)$$

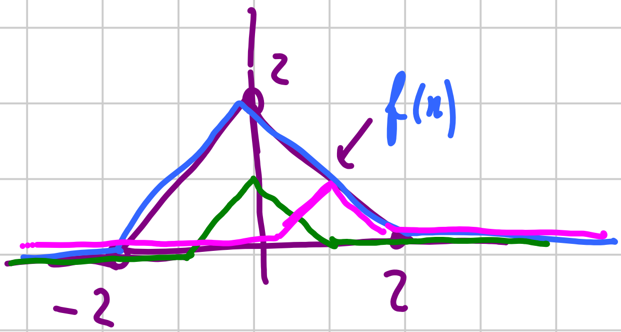


$$\mathcal{F}(f(x)) = \Big(\underbrace{\mathcal{F}(\chi_{[-1,1]}(x))}\Big)^2 = \begin{cases} 4 & \lambda = 0 \\ 4\dfrac{\sin^2\lambda}{\lambda^2} & \lambda \neq 0 \end{cases}$$

$$\begin{cases} 2 & \text{se } \lambda = 0 \\ \dfrac{2\sin\lambda}{\lambda} & \text{se } \lambda \neq 0 \end{cases}$$

$\boxed{9}$

$g(x) =$



$$h(x) = \frac{1}{2} f(2x)$$

$$\varphi(x) = h(x-1) = \frac{1}{2} f(2(x-1))$$

$$\boxed{g(x)} = \varphi(x) - \varphi(-x) =$$

$$= \boxed{\frac{1}{2} f(2(x-1)) - \frac{1}{2} f(2(-x-1))} =$$

$$=$$

$$f(x) = (2-|x|)^+$$