Metodi Matematici - Ex. 8

Titolo nota

21 novembre 2017 (8:30-9.15) - docente: Prof. **Emanuele Callegari** - Università di Roma ***Tor Vergata***

$\boxed{\text{ES. 1}}$ $f(x) = |x|$ $\quad f'(x) = \operatorname{sgn}(x) = g(x)$

$\forall \varphi \in C_0^\infty(\mathbb{R})$ $\quad \underbrace{\int_{-\infty}^{+\infty} f(x)\varphi'(x)\,dx}_{1} = -\underbrace{\int_{-\infty}^{+\infty} g(x)\varphi(x)\,dx}_{2}$

$1 = \int_{-\infty}^{+\infty} |x|\varphi'(x)\,dx =$

[Basta fare il caso in cui supp φ contiene sia punti positivi che negativi. (altri casi sono ovvi)]

Sia $[a,b]$ t.c. supp $\varphi \subset [\underset{<0}{a}, \underset{>0}{b}]$

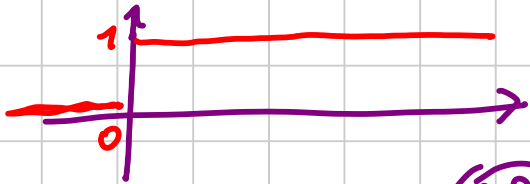
$= \int_a^b |x|\varphi'(x)\,dx = \int_a^0 -x\varphi'(x)\,dx + \int_0^b x\varphi'(x)\,dx =$

$= \cancel{\left[-x\varphi(x)\right]_a^0} + \int_a^0 \varphi(x)\,dx + \cancel{\left[x\varphi(x)\right]_0^b} - \int_0^b \varphi(x)\,dx =$

$= \int_a^b -\operatorname{sgn}(x)\varphi(x)\,dx = -\int_{-\infty}^{+\infty} \operatorname{sgn}(x)\varphi(x)\,dx$

ES 2

$$f(x) = \chi_{[0,+\infty)}(x)$$



$$\forall \varphi \in C_0^\infty(\mathbb{R}) \quad \underbrace{\int_{-\infty}^{+\infty} f(x)\,\varphi'(x)\,dx}_{} = -\int_{-\infty}^{+\infty} \overset{??}{g(x)}\,\varphi(x)\,dx$$

con b maggiorante per supp. φ

$$\int_{-\infty}^{+\infty} \chi_{[0,+\infty)}(x)\,\varphi'(x)\,dx = \int_0^{+\infty} \varphi'(x)\,dx = \int_0^b \varphi'(x)\,dx =$$

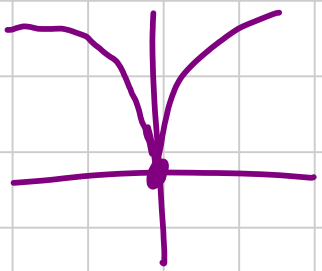
$$= \varphi(b) - \varphi(0) = -\varphi(0)$$

Non esiste $g \in L^1(\mathbb{R})$ che vada bene.

(Bisogna aspettare a definire distribuzioni)

ES.3

$$f(x) = \sqrt{|x|} \qquad \underbrace{g(x) = \operatorname{sgn}(x)\cdot\frac{1}{2\sqrt{|x|}}}_{}$$



$$\forall \varphi \in C_0^\infty(\mathbb{R}) \quad \underbrace{\int_{-\infty}^{+\infty} f(x)\,\varphi'(x)\,dx}_{(1)} = -\underbrace{\int_{-\infty}^{+\infty} g(x)\,\varphi(x)\,dx}_{(2)}$$

FACCIO SOLO PER φ con supporto "misto"

$$① = \int_{-\infty}^{+\infty} \sqrt{|x|}\, \varphi'(x)\, dx = \int_a^b \sqrt{|x|}\, \varphi'(x)\, dx =$$

$$[\underset{<0}{a}, \underset{>0}{b}] \supset \operatorname{supp} \varphi$$

$$= \int_a^0 \sqrt{-x}\, \varphi'(x)\, dx + \int_0^b \sqrt{x}\, \varphi'(x)\, dx =$$

$$= \lim_{\varepsilon \to 0^-} \int_a^{\varepsilon} \sqrt{-x}\, \varphi'(x)\, dx + \lim_{c \to 0^+} \int_c^b \sqrt{x}\, \varphi'(x)\, dx =$$

$$= \lim_{\varepsilon \to 0^-} \cancelto{0}{\left[\sqrt{-x}\, \varphi(x)\right]_a^{\varepsilon}} + \int_a^{\varepsilon} \frac{1}{2\sqrt{-x}}\, \varphi(x)\, dx +$$

$$+ \lim_{c \to 0^+} \cancelto{0}{\left[\sqrt{x}\, \varphi(x)\right]_c^{b}} - \int_c^b \frac{1}{2\sqrt{x}}\, \varphi(x)\, dx =$$

$$= -\int_a^0 -\frac{1}{2\sqrt{-x}}\, \varphi(x)\, dx - \int_0^b \frac{1}{2\sqrt{x}}\, \varphi(x)\, dx =$$

$$= -\int_a^b \frac{\operatorname{sgn}(x)}{2\sqrt{|x|}}\, \varphi(x)\, dx = ②$$

$\boxed{\text{ES. 2 ver.}}$

$$\underbrace{v(x,y) = u_x(x,y)}$$

$$\forall \varphi \in C_0^\infty(\mathbb{R}^2) \quad \int_{\mathbb{R}^2} u(x,y)\,\varphi_x(x,y)\,dx\,dy = -\int_{\mathbb{R}^2} v(x,y)\,\varphi(x,y)\,dx\,dy$$

$$u_{tt}(t,x) - u_{xx}(t,x) = 0$$

$$\forall \varphi \in C_0^\infty((0,+\infty)\times\mathbb{R})$$

$$\int_{\mathbb{R}^2} \big(u_{tt}(t,x) - u_{xx}(t,x)\big)\,\varphi(t,x)\,dt\,dx$$

$$= \int_{[a,b]\times[c,d]} u_{tt}(t,x)\,\varphi(t,x)\,dt\,dx - \int_{[a,b]\times[c,d]} u_{xx}(t,x)\,\varphi(t,x)\,dt\,dx =$$

contiene supp. φ

2 int. per parti

$$= \int_{\underbrace{[a,b]\times[c,d]}_{\mathbb{R}^2}} u(t,x)\,\varphi_{tt}(t,x)\,dt\,dx - \int_{\underbrace{[a,b]\times[c,d]}_{\mathbb{R}^2}} u(t,x)\,\varphi_{xx}(t,x)\,dt\,dx$$

$$= \int_{\mathbb{R}^2} u(t,x) \underbrace{\left(\varphi_{tt}(t,x) - \varphi_{xx}(t,x) \right)} \, dt \, dx$$