Analisi Matematica 1 - Lista n. 21
Calcolo della primitiva di funzioni razionali
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Titolo nota

Calcolare le seguenti primitive di Fratti Semplici:

1) $\displaystyle\int \frac{1}{x-3}\, dx = \ln|x-3|$

2) $\displaystyle\int \frac{7}{(4x+5)^9}\, dx = -\frac{7}{32\cdot(4x+5)^8}$

3) $\displaystyle\int \frac{x}{1+x^2}\, dx = \frac{1}{2}\ln\left(1+x^2\right)$

4) $\displaystyle\int \frac{8x}{(1+x^2)^5}\, dx = -\frac{1}{(1+x^2)^4}$

5) $\displaystyle\int \frac{1}{(1+x^2)^2}\, dx = \frac{1}{2}\left(\frac{x}{1+x^2} + \arctan x\right)$

6) $\displaystyle\int \frac{1}{(1+x^2)^3}\, dx = \frac{1}{8}\left(\frac{2x}{(1+x^2)^2} + \frac{3x}{1+x^2} + 3\arctan x\right)$

7) $\displaystyle\int \frac{2x+5}{9+4x^2}\, dx = \frac{1}{4}\ln\left(4x^2+9\right) + \frac{5}{6}\arctan\frac{2x}{3}$

8) $\displaystyle\int \frac{3x+5}{(1+x^2)^2}\, dx = \frac{5x-3}{2(1+x^2)} + \frac{5}{2}\arctan x$

9) $\displaystyle\int \frac{3-2x}{x^2+2x+2}\, dx = 5\arctan(x+1) - \ln\left(x^2+2x+2\right)$

10) $\displaystyle\int \frac{6x+1}{(x^2+2x+5)^2}\, dx = -\frac{5x+29}{8\cdot(x^2+2x+5)} - \frac{5}{16}\arctan\frac{x+1}{2}$

11) $\displaystyle\int \frac{10x-1}{(4x^2+12x+13)^2}\, dx = -\frac{8x+17}{4\cdot(4x^2+12x+13)} - \frac{1}{2}\arctan\left(x+\frac{3}{2}\right)$

12) $\displaystyle\int \frac{x+2}{(9x^2+30x+29)^2}\, dx = \frac{1}{72}\cdot\left(\frac{3x+1}{9x^2+30x+29} + \frac{1}{2}\arctan\left(\frac{3x+5}{2}\right)\right)$

Calcolare le seguenti primitive di funzioni razionali (spero è meglio usare dei trucchi)

13) $\int \dfrac{x^3 + x^2 + 5x + 9}{x^2 + 4}\, dx = \dfrac{x^2}{2} + x + \dfrac{1}{2} \ln(x^2 + 4) + \dfrac{1}{2} \arctan \dfrac{x}{2}$

14) $\int \dfrac{x^3 + x^2 + 11x + 12}{x^2 + 9}\, dx = \dfrac{x^2}{2} + x + \ln(x^2 + 9) + \arctan \dfrac{x}{3}$

15) $\int \dfrac{3x^3 + 7x^2 + 9x + 3}{x^4 + 2x^3 + 2x^2 + 2x + 1}\, dx = \ln|x+1| + \dfrac{1}{x+1} + \ln(x^2+1) + 3\arctan x$

16) $\int \dfrac{-x^3 + 6x^2 - 9x + 2}{x^4 - 2x^3 + 2x^2 - 2x + 1}\, dx = \ln|x-1| + \dfrac{1}{x-1} - \ln(x^2+1) + 4\arctan x$

17) $\int \dfrac{x^{11}}{x^{12} - 1}\, dx = \dfrac{1}{12} \ln|x^{12} - 1|$

18) $\int \dfrac{x^{11} + 2x^6 + 2x^5 + 2}{x^6 + 1}\, dx = 2x + \dfrac{x^6}{6} + \dfrac{1}{6} \ln(x^6 + 1)$

19) $\int \dfrac{x^3}{x^8 + 1}\, dx = \dfrac{1}{4} \arctan(x^4)$

20) $\int \dfrac{x}{x^6 + 3x^4 + 3x^2 + 2}\, dx = \dfrac{1}{6} \ln(x^6 + 3x^4 + 3x^2 + 2) - \dfrac{1}{6} \ln(x^4 + x^2 + 1) + \dfrac{1}{2\sqrt{3}} \arctan\left(\dfrac{2x^2 + 1}{\sqrt{3}}\right)$

21) $\int \dfrac{x^3}{x^8 + 4x^4 + 8}\, dx = \dfrac{1}{8} \arctan\left(\dfrac{x^4 + 2}{2}\right)$

22) $\int \dfrac{1}{x^7 - x}\, dx = \dfrac{1}{6} \ln|x^6 - 1| - \ln|x|$

23) $\int \dfrac{1}{x(x^3 - 1)(x^6 + 1)}\, dx = -\ln|x| + \dfrac{1}{6} \ln|x^3 - 1| + \dfrac{1}{12} \ln(x^6 + 1) - \dfrac{1}{6} \arctan(x^3)$

24) $\int \dfrac{1}{x(x^8 - 1)}\, dx = \dfrac{1}{8} \ln|x^8 - 1| - \ln|x|$

25) $\int \dfrac{x^3 - 2x^6}{x(x^2 + 1)(x^4 - x^2 + 1)}\, dx = \dfrac{1}{3} \arctan(x^3) - \dfrac{1}{3} \ln(x^6 + 1)$