

ESTREMI VINCOLATI PER FUNZIONI IN 2 VARIABILI

Trovare max. e min. relativi e/o assoluti di $f(x,y)$ ristretta al vincolo Γ a fianco indicato:

1) $f(x,y) = x^2 + 4y^2$

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid 4x^2 + y^2 = 4\}$$

2) $f(x,y) = xy$

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid 5x^2 + 5y^2 + 6xy = 4\}$$

3) $f(x,y) = y^3 e^x$

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid x^2 + 3y^2 = 4\}$$

4) $f(x,y) = y^4 e^x$

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid 4y^2 - x^2 = 3\}$$

5) $f(x,y) = x^2 y^2 + xy - 2$

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 - 2xy - x - y = 4\}$$

6) $f(x,y) = \sin^2 x + \sin^2 y$

$$\Gamma = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$$

Negli esercizi seguenti trovare sia i punti stazionari liberi di $f(x,y)$ interni a D sia i punti stazionari vincolati di $f(x,y)$ ristretta alla frontiera di D . Dire poi se i punti trovati sono di massimo o di minimo (relativo e/o assoluto). In particolare dire se ci sono punti che sono di estremo relativo per $f|_{\partial D}$ ma non per $f|_D$.

7) $f(x,y) = x^2 + 4y^2$

$$D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 25, x^2 - y^2 \leq 7\}$$

8) $f(x,y) = xy$

$$D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 + 12x + 11 \geq 0, x^2 + y^2 \leq 25\}$$

9) $f(x,y) = x(y+1)$

$$D = \{(x,y) \in \mathbb{R}^2 \mid x^2 + 3y^2 \leq 9, y \geq |x|\}$$

$$10) f(x, y) = x^2 + y^2$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid y^2 - x^2 \leq 9, 2x^2 - y^2 \leq 7\}$$

$$11) f(x, y) = x^4 + y^4 - 2xy - 2x^2 - 2y^2 + 2012$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$$

$$12) f(x, y) = x^4 + y^4 - x^3y - xy^3 + x^2y^2$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \geq 2\}$$

$$13) f(x, y) = x^3 + x^2y + 2xy - 11y$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 9\}$$

$$14) f(x, y) = (x^2 + y^2 - 4)y$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid 4 \leq x^2 + y^2 \leq 2x + 8\}$$

$$15) f(x, y) = 2x^3y + 2xy^3 + 5x^2y^2 - 6x^2y - 6xy^2 + 4xy$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid xy \geq \frac{1}{2}, x > 0\}$$

$$16) f(x, y) = 2x^3 + 2y^3 - 3x^2y - 3xy^2 - 2x^2 - 2y^2 - 5xy + 2012$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x + y + 1 \geq 0\}$$

$$17) f(x, y) = \frac{1}{1 + 9x^2 + 4y^2}$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 - 6x \geq 16, x^2 + y^2 + 6x \geq 16, 9x^2 + 4y^2 \leq 900\}$$

$$18) f(x, y) = \frac{x^2y^2}{x^2 + y^2 + 1}$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \geq 1, x \leq 2\}$$

$$19) f(x, y) = xy$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \geq 1, x^2 + y^2 \leq 5\}$$

$$20) f(x, y) = 6x^2 - 5xy + 6y^2$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^4 + y^4 \leq 1\}$$

$$21) f(x, y) = 2e^{xy-1} - x^2 - y^2$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$$

$$22) f(x, y) = xe^y$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid 4 \leq x^2 + y^2 \leq 16\}$$