Università degli Studi di Roma Tor Vergata

Analisi Matematica 2

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Corso di Laurea in Matematica

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Integrali

1. Calcolare usando la definizione

   (a) $\int_0^1 x^3 dx$ (*Suggerimento:* $\sum_{j=0}^n j^3 = \frac{n^2(n+1)^2}{4}$)

   (b) $\int_0^1 e^x dx$ (*Suggerimento:* $\sum_{j=0}^n q^j = \frac{1-q^{n+1}}{1-q}$)

2. Calcolare i seguenti integrali per sostituzione

(1.a) $\displaystyle\int_0^{\sqrt{3}} \frac{\arctan x}{1+x^2} dx$

(1.b) $\displaystyle\int_{-2}^4 (x-1)e^{2x^2-4x+17} dx$

(1.c) $\displaystyle\int_0^1 5x\cos(x^2+6) dx$

(1.d) $\displaystyle\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

(1.e) $\displaystyle\int_{\pi/3}^{\pi} \sin^3 x \cos x dx$

(1.f) $\displaystyle\int_0^1 \frac{e^x}{e^{2x}+1}\, dx$

3. Calcolare i seguenti integrali per parti

(2.a) $\displaystyle\int_0^1 x\ln(x+1) dx$

(2.b) $\displaystyle\int_0^{\pi} x^2 \sin x\, dx$

(2.c) $\displaystyle\int_{-1}^1 xe^{2x} dx$

(2.d) $\displaystyle\int_2^3 \sqrt{x}\ln x\, dx$

4. Calcolare i seguenti integrali:

(3.a) $\displaystyle\int_{-1}^1 e^{2x}(1+e^x) dx$

(3.b) $\displaystyle\int_1^e \frac{2x-1}{x^2} dx$

(3.c) $\displaystyle\int_0^1 \frac{x^2}{1+x^6} dx$

(3.d) $\displaystyle\int_0^{e^2} \frac{\ln x}{x\sqrt{1+\ln^2 x}} dx$

(3.e) $\displaystyle\int_0^{15} \frac{36}{\sqrt{x+1}+\sqrt{x+49}} dx$

(3.f) $\displaystyle\int_0^1 \sqrt{1-x^2} dx$

(3.g) $\displaystyle\int_0^2 \sqrt{4x^2-1} dx$

(3.h) $\displaystyle\int_0^1 e^{\sqrt[3]{x}} dx$

(3.i) $\displaystyle\int_0^2 \cos\sqrt{2x+1} dx$

(3.j) $\displaystyle\int_0^1 (x^{24}+12x^{23})e^{2x-2} dx$

(3.k) $\displaystyle\int_4^{16} \frac{1}{\sqrt{x}e^{\sqrt{x}}} dx$

(3.l) $\displaystyle\int_{\pi/4}^{\pi/2} (4x+5)\cos x dx$

(3.m) $\displaystyle\int_0^1 e^{2x}\ln(1+e^x)dx$

(3.n) $\displaystyle\int_1^2 10x^2\ln x dx$

(3.o) $\displaystyle\int_1^3 9x^3(\ln x)^3 dx$

(3.p) $\displaystyle\int_0^{2\pi}(\cos\ x)^2 dx$

(3.q) $\displaystyle\int_0^{\pi} e^x\cos x dx$

(3.r) $\displaystyle\int_0^{\pi} xe^x\cos x dx$

(3.s) $\displaystyle\int_{-2}^{2}(5x^4+12x^2+5)\arctan(x^2+2)dx$

(3.t) $\displaystyle\int_{-\sqrt{3}}^{\frac{\sqrt{3}}{3}} \tan^3 x+\tan x dx$

(3.u) $\displaystyle\int_{-\frac{\sqrt{2}}{2}}^{1/2} \sin x e^{\cos x} dx$

(3.v) $\displaystyle\int_0^{\pi/4} \tan x\ln(\cos x) dx$

(3.w) $\displaystyle\int_0^1 \frac{xe^{\arctan x^2}}{1+x^4}dx$

(3.x) $\displaystyle\int_e^{e^{\pi/2}} \cos(\ln x)dx$

5. Calcolare il seguente intergrale indefinito senza utilizzare formule iterative

$$\int \cos^{1011} x dx.$$

Come si generalizza per ogni esponente dispari $\int \cos^{2n+1} dx$

Dimostrare per induzione che

$$\int \cos^{2n} x dx = \left(\prod_{k=1}^{n}\frac{2k-1}{2k}\right)x + \sum_{\ell=1}^{n}\frac{1}{2\ell-1}\prod_{k=\ell}^{n}\frac{2k-1}{2k}\cos^{2\ell-1} x\sin x + c$$

$$\int \sin^{2n} x dx = \left(\prod_{k=1}^{n}\frac{2k-1}{2k}\right)x - \sum_{\ell=1}^{n}\frac{1}{2\ell-1}\prod_{k=\ell}^{n}\frac{2k-1}{2k}\sin^{2\ell-1} x\cos x + c$$

6. Calcolare l'integrale

$$\int_{-\sqrt{13}}^{\sqrt{13}} \frac{x\sin(x)\ln(1+x^4)e^{-2x^4+3x^2+1}\arctan(x^3)}{x^2(\cos x+2)(\sin(x^4)+4)}dx$$