P.B. 41.6 Date (a_n) in $\mathbb{R}$ e (k_n) in $\mathbb{N}$ t.c. $k_n \to +\infty$

Supponiamo che $a_n \to l$, allora anche $a_{k_n} \to l$

(Usata alla fine della I^a parte della lezione

$$\frac{b^{\lfloor \log_a n \rfloor + 1}}{\lfloor \log_a n \rfloor + 1} \to +\infty$$

$$a_n = \frac{b^n}{n} \to +\infty \qquad k_n = \lfloor \log_a n \rfloor + 1$$

$$a_{k_n} = \frac{b^{\lfloor \log_a n \rfloor + 1}}{\lfloor \log_a n \rfloor + 1}$$

Dim $\left(\underset{1}{(a_n \to l)} \text{ e } \underset{2}{(k_n \to +\infty)} \right) \Rightarrow a_{k_n} \to l$

Facciamo caso $l \in \mathbb{R}$ (fare gli altri due casi $l = \pm\infty$)

1 $\forall \varepsilon > 0$ def. in n, $l - \varepsilon < a_n < l + \varepsilon$

$\exists n_0 \in \mathbb{N}$ t.c. se $n \geq n_0$ allora

Da 2 segue che def. in n $k_n \geq n_0$ quindi def. in n $\quad l-\varepsilon < a_{k_n} < l+\varepsilon$

$\boxed{\text{P.B. } 12.6}$ $\displaystyle\lim_{n\to+\infty} \frac{n^4 \log_5 n^2}{(n-1)^5} =$

$$= \lim_{n\to+\infty} \frac{n^5}{(n-1)^5} \cdot \frac{2\log_5 n}{n} =$$

$$= \lim_{n\to+\infty} \underbrace{\boxed{\frac{1}{\left(1-\frac{1}{n}\right)^5}}}_{\downarrow\, 1} \cdot \underbrace{\boxed{2}}_{\downarrow\, 2} \cdot \underbrace{\boxed{\frac{\log_5 n}{n}}}_{\downarrow\, 0} = 0$$

$\boxed{\text{P.B. } 19.6}$ $\displaystyle\lim_{n\to+\infty} \frac{\left(3+\frac{1}{n}\right)^n}{\left(4-\frac{1}{n}\right)^n} = 0$

def. in n

$$0 < \underbrace{\frac{\left(3+\frac{1}{n}\right)^n}{\left(4-\frac{1}{n}\right)^n}}_{\downarrow\, 0} < \frac{(3,1)^n}{(3,9)^n} = \left(\frac{31}{39}\right)^n \to 0$$

$\boxed{\text{P.B. } 21.6}$ $\displaystyle\lim_{n\to+\infty} \frac{(n-1)!}{2^{n+1}} = +\infty$

$$\underbrace{\frac{(n-1)!}{2^{n+1}}}_{\downarrow\, +\infty} = \underbrace{\frac{1}{2^2}}_{\downarrow\, \frac{1}{4}} \cdot \underbrace{\frac{(n-1)!}{2^{n-1}}}_{\downarrow\, +\infty}$$

$\boxed{\text{P.B. funzioni limite}}$

$$\lim_{n\to+\infty} \frac{n!}{2^{n^2}} = 0$$

def. in n

$$\underset{\downarrow\, 0}{0} < \boxed{\frac{n!}{2^{n^2}}}_{\downarrow\, 0} < \frac{n^n}{2^{n^2}} = \frac{n^n}{(2^n)^n} = \left(\frac{n}{2^n}\right)^n < \underbrace{\left(\frac{1}{2}\right)^n}_{\downarrow\, 0}$$

$\boxed{\text{P.B. 39.6}}$

$$\lim_{n\to+\infty} \frac{2^{n!}}{(2^n)!}$$

$$\boxed{n! - n\cdot 2^n} = \boxed{n}\underbrace{\left((n-1)! - 2^n\right)}_{1}$$

def. in n

def. in n

$$\underset{+\infty \swarrow}{\boxed{\frac{2^{n!}}{(2^n)!}}} > \frac{2^{n!}}{(2^n)^{2^n}} = \frac{2^{n!}}{2^{n\cdot 2^n}} = 2^{\boxed{n! - n\cdot 2^n}} > \underset{\downarrow\, +\infty}{2^n}$$

$$1 \quad \boxed{(n-1)! - 2^n} = \boxed{(n-1)!\left(1 - \frac{2^{n-1}}{(n-1)!}\cdot 2\right)} \to +\infty$$